AQA

# A-LEVEL
# FURTHER MATHEMATICS

7367/2
Report on the Examination

7367
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## General

This was the first summer examination series since 2019, when this specification was examined for the first time. We expected students' performance to be adversely affected by the likely interruption to their education caused by the pandemic and lockdown. However, achievement was broadly in line with summer 2019. It seems probable that this was partly because students and teachers made good use of the advance information released by AQA when planning revision.

Students performed well on the straightforward parts of the paper, scoring highly on most of the AO1 marks. This indicated that they generally had a good grounding in the topics examined.

The question parts that caused the most problems to students were those of a non-standard nature, or those requiring detailed explanation. On questions 6, 9(c), 11(b)(ii), 12(a),13(c)(i), 13(c)(ii) and 14(c), the mean mark was below 30% of the total available.

Some parts of the paper were very well-answered. Around 70% of students scored full marks on 11(a) (finding eigenvalues and eigenvectors). There was a marked improvement, compared with 2019, in solving the question on coupled differential equations (14(b)), with over half gaining 6 or more marks out of 9.

## Question 1

Over 90% answered this correctly.

## Question 2

Over 90% answered this correctly.

## Question 3

Nearly 80% answered this correctly, with the third option the most popular among the remaining students.

## Question 4

Around 70% answered this correctly, with the second option the most popular among the remaining students.

## Question 5

This was answered well, with about 60% of students gaining 3 or 4 marks. Most took out a factor of $(k + 1)^2$, which gave a relatively easy route to at least 3 marks. Those who multiplied out the brackets and then factorised their result risked making an error, but some managed to use this method successfully.

Quite a few students dropped the final mark because they omitted either "by induction" or "for all integers $n \geq 1$."

**Question 6**

Very few students gained more than 1 mark here. This was because they did not state that the area found using the mid-ordinate rule is equal to the area of the trapezium.

Around half the students gained the first mark by stating that the area of the trapezium was greater than the area under the curve (or equivalent). However, those who did not mention the trapezium at all were unable to gain any marks.

**Question 7**

Over 90% answered part (a) correctly.

Most then went on to use a correct approach to solving part (b). Some sketched and referred to a graph, while others multiplied the inequality by the square of the denominator. Others found the set of critical values and examined the sign of the function between these values. All these approaches led to a correct solution in many cases.

Some students considered the cases $x < \frac{9}{2}$ and $x > \frac{9}{2}$ separately. While this could lead to a correct solution, it was not the most efficient approach.

Where marks were lost, it was usually because students did not spot that a strict inequality was required for $x < \frac{9}{2}$, because $x = \frac{9}{2}$ is an asymptote.

**Question 8**

Around three-quarters of students gained the first two marks in part (a)(i), but it was difficult to get the next mark as this required correct differentiation four times. Even so, around 35% scored full marks.

In part (a)(ii) the number gaining full marks was about 65%, as they were able to use the result given in part (a)(i).

Part (b) was challenging, with only just over half the students gaining 1 mark or more. Many were unable to obtain a correct Maclaurin expansion for cosh $x$. Most of those who showed a correct sequence of calculations leading to the answer were unable to gain the final mark because they did not show evidence of higher powers until the limit was taken.

**Question 9**

Use of the midpoint method in part (a) was generally very accurate, especially considering that this has not been examined previously. Around 75% gained 2 or more marks; those who dropped marks generally substituted the wrong value into one of the formulae.

In part (b)(i) over 70% were able to use the integrating factor method. Those whose integrating factor was $(x^2 - 9)$ were generally unable to score more than 2 marks. A further group of students found the correct integrating factor, and correctly multiplied the differential equation by it, but did not use inverse cosh to integrate the right-hand side.

Some obtained a correct solution, but then went further and obtained a solution with the constant of integration in the wrong place. They were able to gain full marks for this part, but in many cases it caused them to lose marks in the later parts.

Many students were able to gain method marks in part (b)(ii) despite having an incorrect answer to part (b)(i), as long as they used the new information to evaluate their constant of integration and substituted into their expression for $y$.

Most students who answered parts (a) and (b) correctly also got the mark for part (c).

**Question 10**

In part (a), nearly all students were able to manipulate the equation of $C_2$ to get it into a form where it was apparent which transformations should be used. After that, around a quarter arrived at a completely correct solution. Over 30% scored 3 marks out of 4, usually because they had made a sign error in their translation vector, or had the wrong scale factor (eg 2 or $\frac{1}{4}$), or had the order of transformations wrong.

In part (b), around two-thirds scored one or more mark, and nearly 30% reached a correct solution. A common error was to have equations of the form $25y + x + k = 0$.

**Question 11**

Over 70% answered part (a) correctly, and 80% gained 4 or more marks. This is clearly a topic which is very well-understood by students. A few lost marks by not indicating which eigenvalue corresponded to which eigenvector.

About half of the students gained 1 or 2 marks in part (b)(i). In order to gain both marks, it was not sufficient to state that the lines were perpendicular: a reason had to be given (normally that the scalar product of the vectors was zero, or that the gradients of the lines were negative reciprocals.)

Part (b)(ii) was not answered well, with only around 6% gaining any marks. Clearly this is an area of the course which is not as familiar to students.

**Question 12**

Students found part (a) very challenging. Under 10% gained 3 or 4 marks. This is a proof that students are likely to have encountered in their learning, but they were evidently not expecting to have to reproduce it. Some students summed the areas of circles, rather than the volumes of discs, but this was not accepted as a valid method.

Over half the students made some progress in finding partial fractions in part (b). In general, students were very good at finding the logarithmic integrals, then substituting and manipulating logs. Where they often struggled was in integrating $\frac{1}{(1+x)^2}$, with some finding a logarithmic expression for this.

**Question 13**

Many different approaches were used, often successfully, in part (a). Around two-thirds gained 1 mark or more, but only a fifth of students gained full marks. Some left out key details, such as stating the trigonometric identities they were using.

Around a quarter of students answered part (b) correctly. Some used, without justification, statements such as $(\mathbf{BA})^2 = \mathbf{A}^2\,\mathbf{B}^2$ which are not generally true for matrix multiplication. In order for this to be acceptable, they would have to explain why the statement holds for these particular matrices.

Part (c) was an unusual type of question, and about half the students attempted both parts. In order to gain marks in part (ii) they had to refer to lines they had drawn in part (i).

Part (d) was done well, with half the students gaining 3 or 4 marks. Most students set up and solved two equations, then identified the common solution.

**Question 14**

Over 80% gained at least one mark for part (a).

Part (b) was done very well, with a mean mark of nearly 60%. Most students were able to obtain a correct general solution for either $x$ or $y$. This shows an impressive level of improvement since the topic was last examined in 2019.

In part (c), very few students appreciated that, as $x$ and $y$ are both positive for $0 \le t \le 5$, they should be considering values of $t$ which are greater than 5. This made it very difficult for them to pick up any marks, with only around 7% scoring 1 mark or more.

## Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.