A

AQA

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I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Statistics

7367/3S

Time allowed: 2 hours

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

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- **You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.**
- **You should have a scientific calculator that meets the requirements of the specification.**
- **You must ensure you have the other optional Question Paper/Answer Book for which you are entered (EITHER Discrete OR Mechanics). You will have 2 hours to complete BOTH papers.**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).**
- **Do NOT write on blank pages.**
- **Show all necessary working; otherwise marks for method may be lost.**
- **Do all rough work in this book. Cross through any work that you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 50.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO

SECTION A

Answer ALL questions in the spaces provided.

1 The random variable T follows a discrete uniform distribution and can take values

1, 2, 3, …, 16

Find the variance of T

Circle your answer. [1 mark]

1.25

18.75

21.25

21.33

2 The random variable X has probability density function

$$f(x) = \begin{cases} 1 & 0 < x \leq \frac{1}{2} \\ \frac{3}{8}x^{-2} & \frac{1}{2} < x \leq \frac{3}{2} \\ 0 & \text{otherwise} \end{cases}$$

Find P($X < 1$)

Circle your answer. [1 mark]

$\frac{1}{8}$

$\frac{3}{8}$

$\frac{5}{8}$

$\frac{7}{8}$

[Turn over]

3 The random variable X has an exponential distribution with probability density function $\mathrm{f}(x) = \lambda \mathrm{e}^{-\lambda x}$ where $x \geq 0$

3 (a) Show that the cumulative distribution function, for $x \geq 0$, is given by $\mathrm{F}(x) = 1 - \mathrm{e}^{-\lambda x}$

[3 marks]

3 (b) Given that $\lambda = 2$, find $P(X > 1)$, giving your answer to three decimal places. [2 marks]

4 Daisies and dandelions are the only flowers growing in a field.

The number of daisies per square metre in the field has a mean of 16

The number of dandelions per square metre in the field has a mean of 10

The number of daisies per square metre and the number of dandelions per square metre are independent.

4 (a) Using a Poisson model, find the probability that a randomly selected square metre from the field has a total of at least 30 flowers, giving your answer to three decimal places. [3 marks]

[Turn over]

4 (b) **A survey of the entire field is taken.**

The standard deviation of the total number of flowers per square metre is 10

State, with a reason, whether the model used in part (a) is valid. [2 marks]

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5 The mass, X, in grams of a particular type of apple is modelled using a normal distribution.

A random sample of 12 apples is collected and the summarised results are

$$\sum x = 1038 \quad \text{and} \quad \sum x^2 = 90\,100$$

5 (a) A 99% confidence interval for the population mean of the masses of the apples is constructed using the random sample.

Show that the confidence interval is (81.7, 91.3) with values correct to three significant figures. **[4 marks]**

[Turn over]

5 (b) Padraig claims that the population mean mass of the apples is 85 grams.

He carries out a hypothesis test at the 1% level of significance using the random sample of 12 apples.

The hypotheses are

$H_0: \mu = 85$

$H_1: \mu \neq 85$

State, with a reason, whether the null hypothesis is accepted or rejected. [1 mark]

5 (c) **Interpret, in context, the conclusion to the hypothesis test in part (b). [1 mark]**

6 The discrete random variable X has probability distribution function

$$\mathrm{P}(X = x) = \begin{cases} a & x = 0 \\ b & x = 1 \\ c & x = 2 \\ 0 & \text{otherwise} \end{cases}$$

where a, b and c are constants.

The mean of X is 1.2 and the variance of X is 0.56

6 (a) Deduce the values of a, b and c **[6 marks]**

6 (b) **The continuous random variable Y is independent of X and has variance 15**

Find Var($X - 2Y - 11$) [2 marks]

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[Turn over]

7 A scientist is investigating the air quality in two countries, *A* and *B*

Measurements of the air quality are taken at sites throughout both countries and assigned a rank.

The air quality is given Rank 1 if the level of air pollution is lower than a specified threshold.

The air quality is given Rank 2 if the level of air pollution is higher than the specified threshold.

A random sample of 500 measurements is collected.

The results are summarised in the following table.

		Air Quality		
		Rank 1	Rank 2	Total
Country	*A*	87	101	188
	B	167	145	312
	Total	254	246	500

The scientist claims that there is an association between country and air quality.

7 (a) **Test the scientist's claim, using the 10% level of significance. [8 marks]**

7 (b) **For the context of the test carried out in part (a), state the meaning of a Type I error. [1 mark]**

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[Turn over]

8 The continuous random variable X has cumulative distribution function $\mathrm{F}(x)$ where

$$\mathrm{F}(x) = \begin{cases} 0 & x = 0 \\ \mathrm{e}^{kx} - 1 & 0 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

8 (a) Show that $k = \frac{1}{5}\ln 2$ [2 marks]

8 (b) Show that the median of X is $a\dfrac{\ln b}{\ln 2} - c$, where a, b and c are integers to be found. [3 marks]

8 (c) Show that the mean of X is $p - \frac{q}{\ln 2}$, where p and q are integers to be found. **[6 marks]**

9 Lianne models the maximum time in hours that a rechargeable battery can be used, before needing to be recharged, with a rectangular distribution with values between 8 and 12

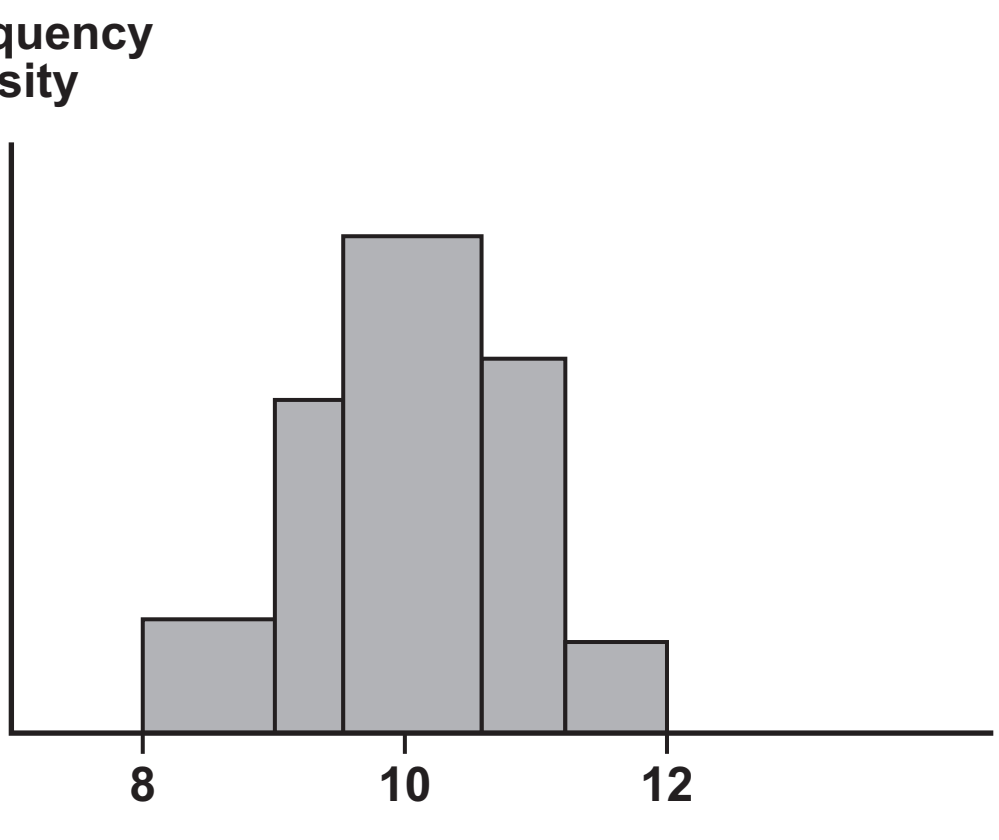
9 (a) The probability that the maximum time the battery can be used before needing to be recharged is more than 10.5 hours is equal to p

Lianne will only buy the battery if p is more than 0.4

Determine whether Lianne will buy the battery. **[2 marks]**

[Turn over]

9 (b) **A histogram is plotted for 100 recharges showing the maximum time the battery can be used before needing to be recharged.**



Explain why the model used in part (a) may not be valid and suggest the name of a different distribution that could be used to model the maximum time between recharges. [2 marks]

END OF QUESTIONS

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For Examiner's Use	
Examiner's Initials	
Question	Mark
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