



Surname _____

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I declare this is my own work.

A-level

MATHEMATICS

Paper 1

7357/1

Tuesday 6 June 2023 Afternoon

Time allowed: 2 hours

At the top of the page, write your surname and forename(s), your centre number, your candidate number and add your signature.

[Turn over]



MATERIALS

For this paper you must have:

- the AQA Formulae for A-level Mathematics booklet
- a graphical or scientific calculator that meets the requirements of the specification.

INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do NOT write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.



INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO



Answer ALL questions in the spaces provided.

- 1 Find the coefficient of x^7 in the expansion of $(2x - 3)^7$**

Circle your answer. [1 mark]

–2187

128

2

128

- 2 Given that $y = 2x^3$ find $\frac{dy}{dx}$**

Circle your answer. [1 mark]

$$\frac{dy}{dx} = 5x^2$$

$$\frac{dy}{dx} = 6x^2$$

$$\frac{dy}{dx} = \frac{x^4}{2}$$

$$\frac{dy}{dx} = 6x^3$$



- 3 The curve with equation $y = \ln x$ is transformed by a stretch parallel to the x -axis with scale factor 2

Find the equation of the transformed curve.

Circle your answer. [1 mark]

$$y = \frac{1}{2} \ln x$$

$$y = 2 \ln x$$

$$y = \ln \frac{x}{2}$$

$$y = \ln 2x$$

- 4 Given that y is a small angle, find an approximation for $\cos 2\theta$

Circle your answer. [1 mark]

$$1 - \frac{\theta^2}{2}$$

$$2 - 2\theta^2$$

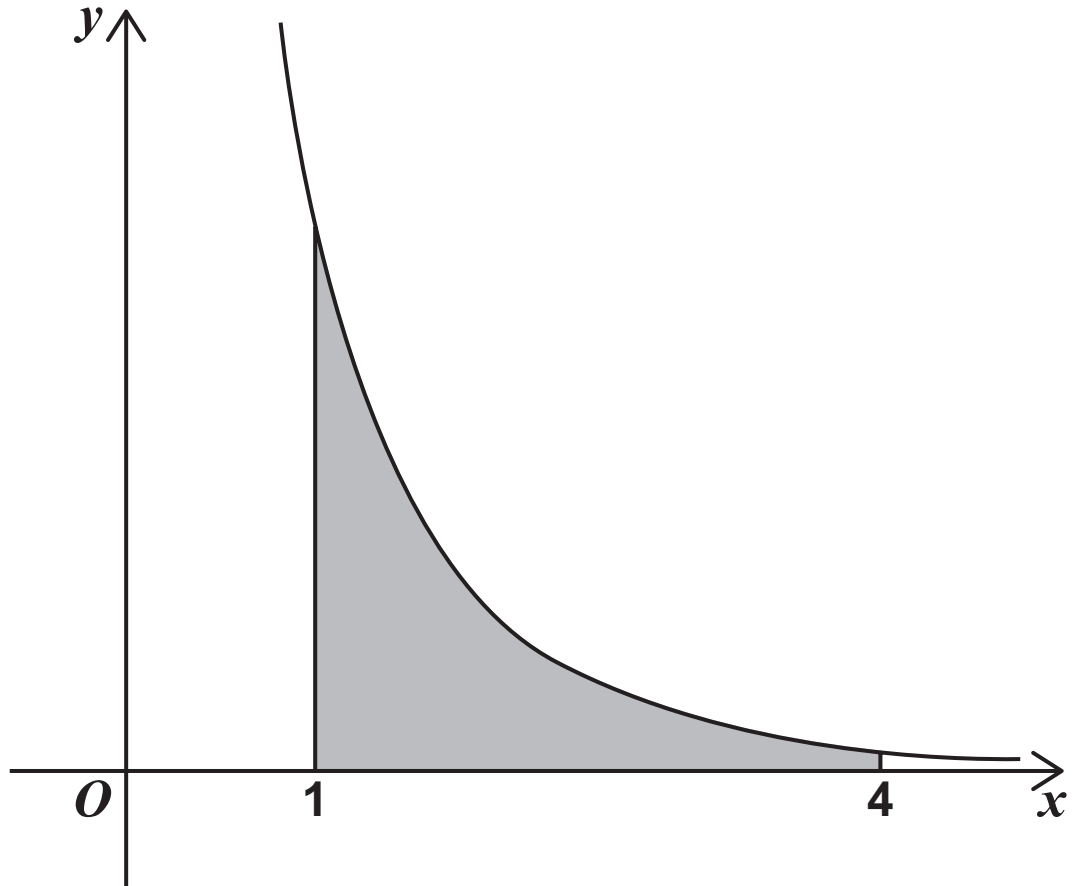
$$1 - 2\theta^2$$

$$1 - \theta^2$$

[Turn over]



- 5 The graph of $y = \frac{5}{e^x - 1}$ is shown in the diagram below.



The trapezium rule with 6 ordinates (5 strips) is to be used to find an approximation for the shaded area.

The values required to obtain this approximation are shown in the table below.

| | | | |
|-----|---------|---------|---------|
| x | 1 | 1.6 | 2.2 |
| y | 2.90988 | 1.26485 | 0.62305 |

| | | | |
|-----|---------|---------|---------|
| x | 2.8 | 3.4 | 4 |
| y | 0.32374 | 0.17263 | 0.09329 |



5(a) Use the trapezium rule with 6 ordinates (5 strips) to find an approximate value for the shaded area.

**Give your answer to four decimal places.
[3 marks]**

[illegible]

[Turn over]



5 (b) Using your answer to part (a) deduce an

estimate for $\int_1^4 \frac{20}{e^x - 1} dx$ [1 mark]



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[Turn over]



6

$$2\log_{10} x = \log_{10} 4 + \log_{10} (x + 8)$$

has exactly one solution.

Fully justify your answer. [5 marks]

[illegible]

[Turn over]



7 (a) Given that n is a positive integer, express

$$\frac{7}{3 + 5\sqrt{n}} - \frac{7}{5\sqrt{n} - 3}$$

**as a single fraction not involving surds.
[3 marks]**

[illegible]

7 (b) Hence, deduce that

$$\frac{7}{3 + 5\sqrt{n}} - \frac{7}{5\sqrt{n} - 3}$$

is a rational number for all positive integer values of n [1 mark]

[Turn over]



8

Show that

$$\int_0^{\frac{\pi}{2}} (x \sin 4x) dx = -\frac{\pi}{8} \quad [6 \text{ marks}]$$

[illegible]

- 9 The points P and Q have coordinates $(-6, 15)$ and $(12, 19)$ respectively.

- 9 (a) (i) Find the coordinates of the midpoint of PQ
[1 mark]

- 9 (a) (ii) Find the equation of the perpendicular bisector of PQ

Give your answer in the form $ax + by = c$ where a , b and c are integers. [4 marks]



[Turn over]



9 (b) (i) A circle passes through the points P and Q

The centre of the circle lies on the line with equation $2x - 5y = -30$

Find the equation of the circle. [3 marks]

[illegible]

[illegible]

[Turn over]



9 (b) (ii) The circle intersects the coordinate axes at n points.

State the value of n [1 mark]



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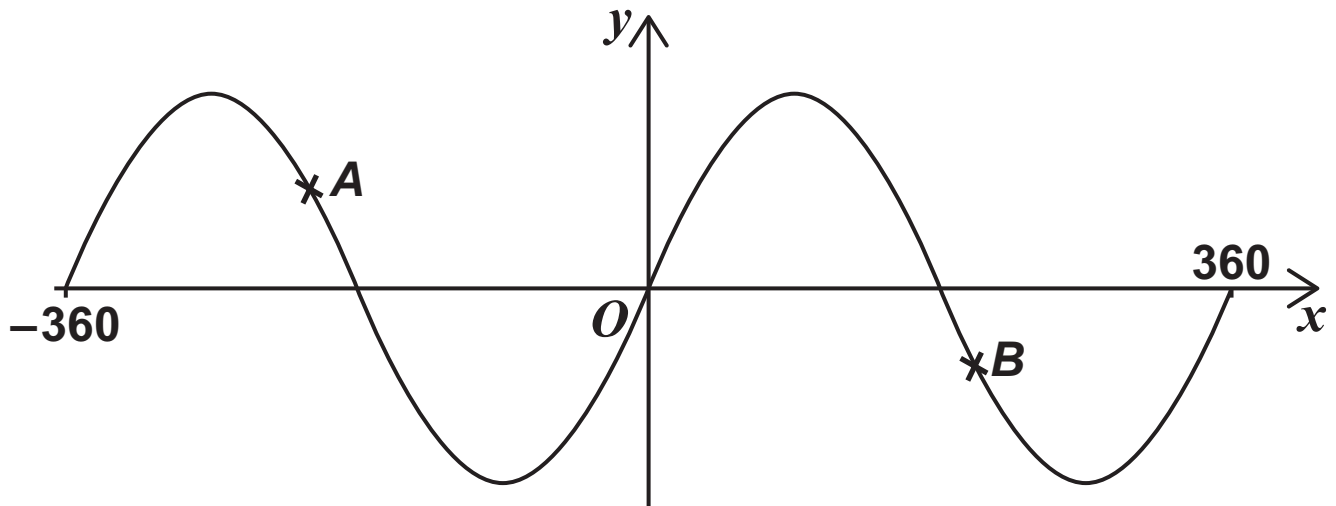
[Turn over]



10 The curve with equation

$$y = \sin x^\circ$$

for $-360 \leq x \leq 360$ is shown below.



10 (a) Point A on the curve has coordinates $(a, 0.5)$

10 (a) (i) Find the value of a [2 marks]



10 (a) (ii) State the value of $\sin (180^\circ - a^\circ)$ [1 mark]

[Turn over]



10 (b) Point B on the curve has coordinates $\left(b, -\frac{3}{7}\right)$

10 (b) (i) Find the exact value of $\sin(b^\circ - 180^\circ)$ [2 marks]



10 (b) (ii) Find the exact value of $\cos b^\circ$ [3 marks]

[illegible]

[Turn over]



11 The n th term of a sequence is u_n

The sequence is defined by

$$u_{n+1} = pu_n + 70$$

where $u_1 = 400$ and p is a constant.

11 (a) Find an expression, in terms of p , for u_2
[1 mark]

11 (b) It is given that $u_3 = 382$

11 (b) (i) Show that p satisfies the equation

$$200p^2 + 35p - 156 = 0 \quad [3 \text{ marks}]$$



[Turn over]



11 (b) (ii) It is given that the sequence is a decreasing sequence.

Find the value of u_4 and the value of u_5
[3 marks]

[illegible]

11 (c) The limit of u_n as n tends to infinity is L

11 (c) (i) Write down an equation for L [1 mark]

11 (c) (ii) Find the value of L [1 mark]

[Turn over]



12

One of the rides at a theme park is a room where the floor and ceiling both move up and down for 10π seconds.

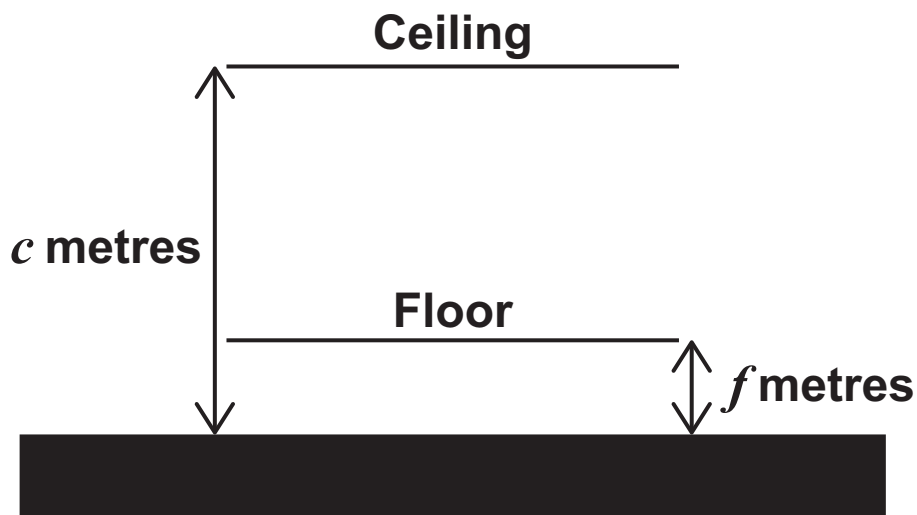
At time t seconds after the ride begins, the distance f metres of the floor above the ground is

$$f = 1 - \cos t$$

At time t seconds after the ride begins, the distance c metres of the ceiling above the ground is

$$c = 8 - 4 \sin t$$

The ride is shown in the diagram below.



- 12 (a)** Show that the initial distance between the floor and ceiling is 8 metres. [1 mark]

- 12 (b)** Show that the distance d metres between the floor and ceiling at time t is given by

$$d = 7 + R \cos(t + \alpha)$$

where R and α are positive constants to be found. [5 marks]

[Turn over]



[illegible]

- 12(c)** Hence, find the minimum distance between the ceiling and the floor.

**Give your answer to the nearest centimetre.
[2 marks]**

[Turn over]

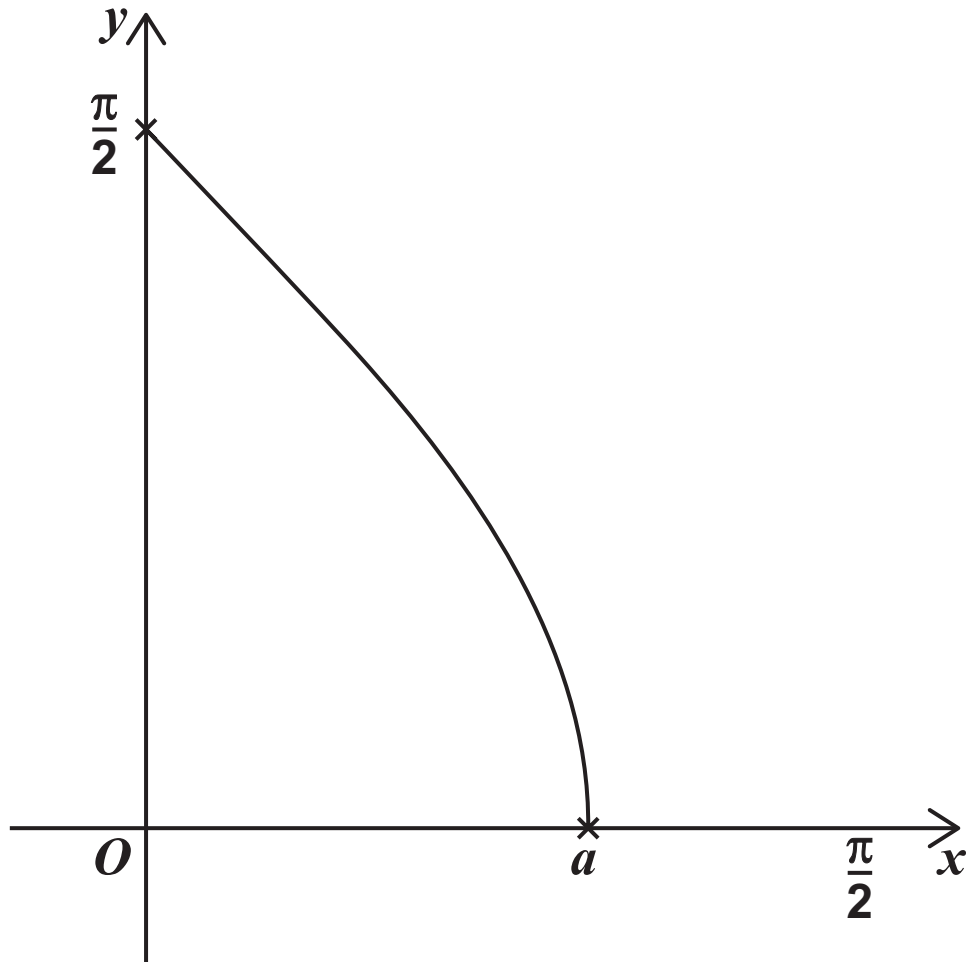


13

The function f is defined by

$$f(x) = \arccos x \quad \text{for } 0 \leq x \leq a$$

The curve with equation $y = f(x)$ is shown below.



13 (a) State the value of a [1 mark]

13 (b) (i) On the diagram on the opposite page, sketch the curve with equation

$$y = \cos x \quad \text{for} \quad 0 \leq x \leq \frac{\pi}{2}$$

AND

sketch the line with equation

$$y = x \quad \text{for} \quad 0 \leq x \leq \frac{\pi}{2} \quad \text{[4 marks]}$$

[Turn over]



13 (b) (ii) Explain why the solution to the equation

$$x - \cos x = 0$$

must also be a solution to the equation

$$\cos x = \arccos x \quad [1 \text{ mark}]$$

13 (c) Use the Newton-Raphson method with $x_0 = 0$ to find an approximate solution, x_3 , to the equation

$$x - \cos x = 0$$

**Give your answer to four decimal places.
[3 marks]**





[illegible]

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14 (a) (i) Given that

$$y = 2^x$$

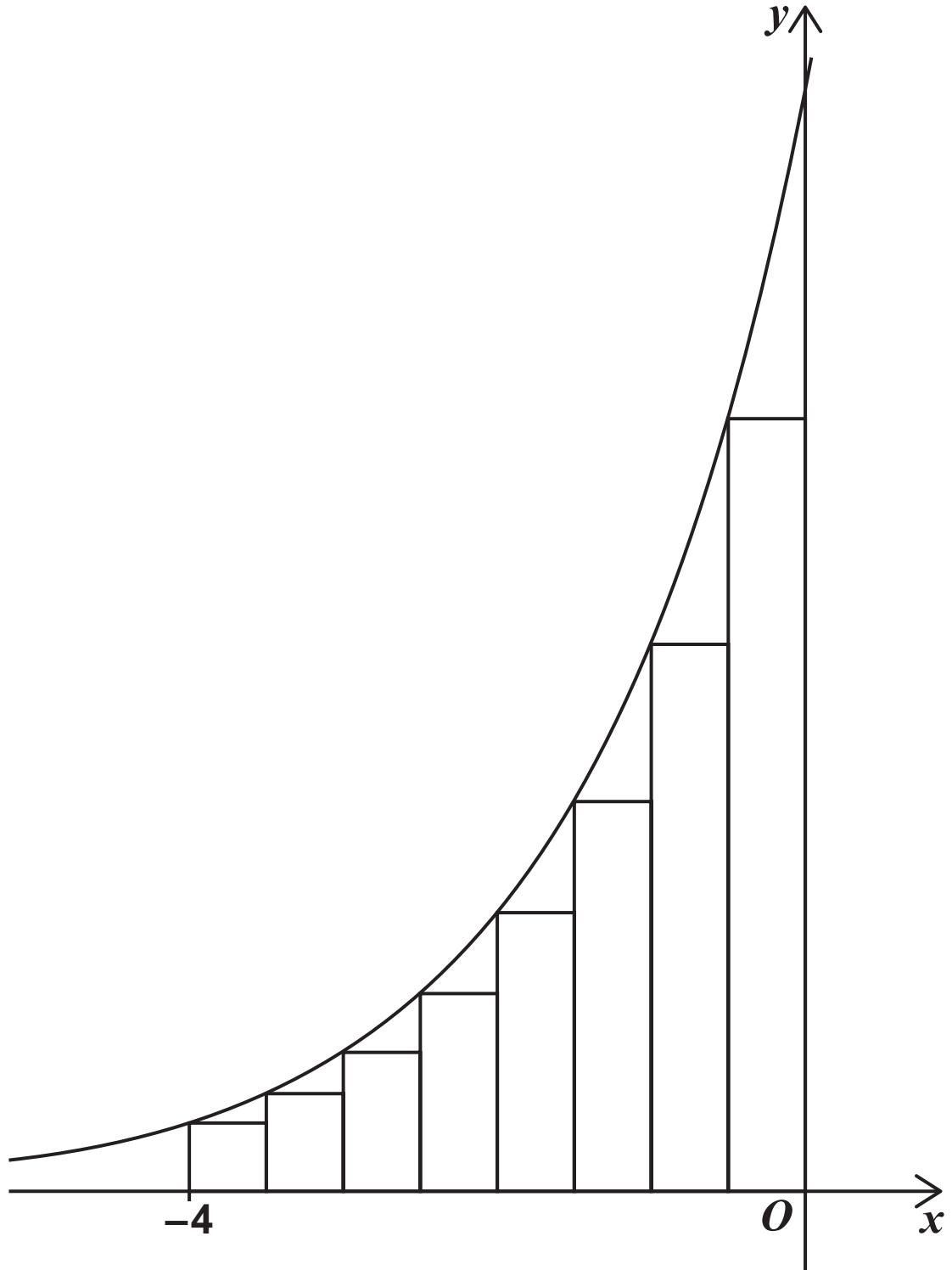
write down $\frac{dy}{dx}$ [1 mark]

14 (a) (ii) Hence find

$$\int 2^x dx \text{ [2 marks]}$$



- 14(b) The area, A , bounded by the curve with equation $y = 2^x$, the x -axis, the y -axis and the line $x = -4$ is approximated using eight rectangles of equal width as shown in the diagram below.



[Turn over]



14 (b) (i) Show that the exact area of the largest rectangle is $\frac{\sqrt{2}}{4}$ [2 marks]



14 (b) (ii) The areas of these rectangles form a geometric sequence with common ratio $\frac{\sqrt{2}}{2}$

Find the exact value of the total area of the eight rectangles.

Give your answer in the form $k(1 + \sqrt{2})$ where k is a rational number. [3 marks]

[Turn over]





14 (b) (iii) More accurate approximations for A can be found by increasing the number, n , of rectangles used.

Find the exact value of the limit of the approximations for A as $n \longrightarrow \infty$ [3 marks]

[illegible]

[Turn over]



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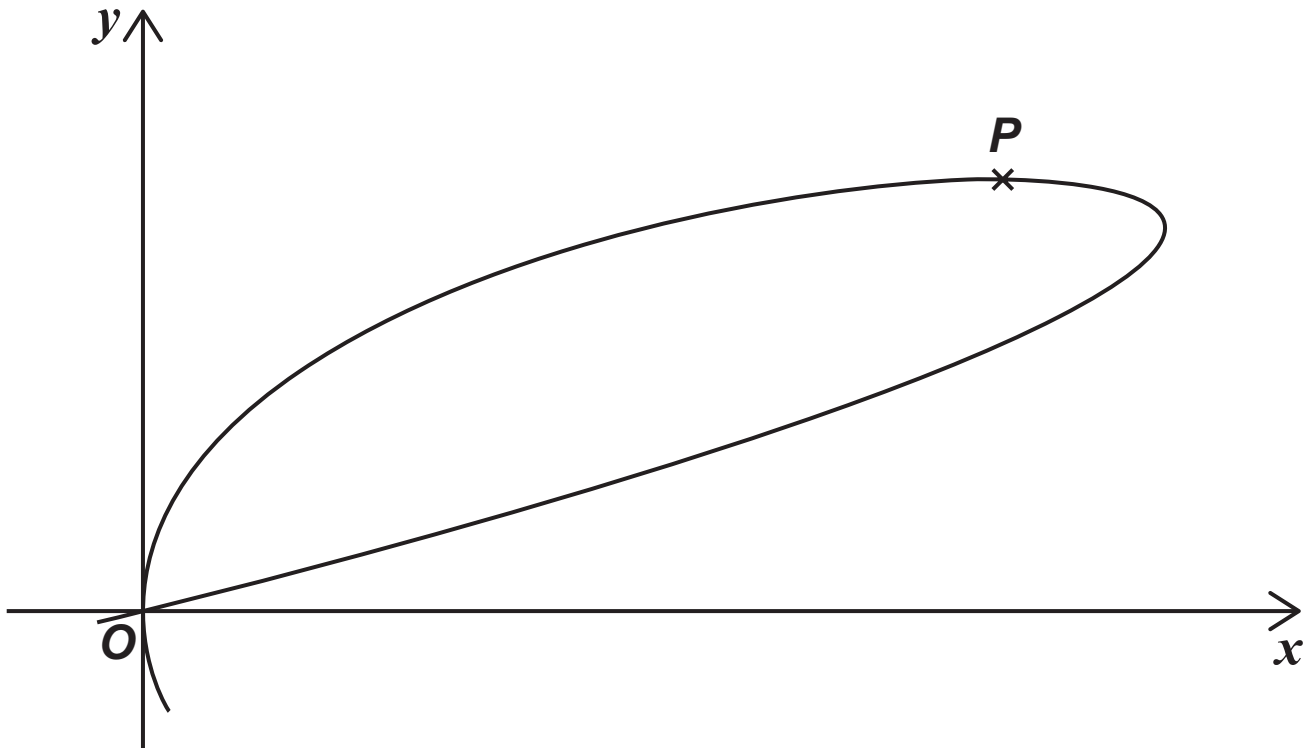
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15 The curve with equation

$$x^2 + 2y^3 - 4xy = 0$$

has a single stationary point at P as shown in the diagram below.



15(a) Show that the y -coordinate of P satisfies the equation

$$y^2(y - 2) = 0 \quad [7 \text{ marks}]$$





15(b) Hence, find the coordinates of P [2 marks]

[Turn over]



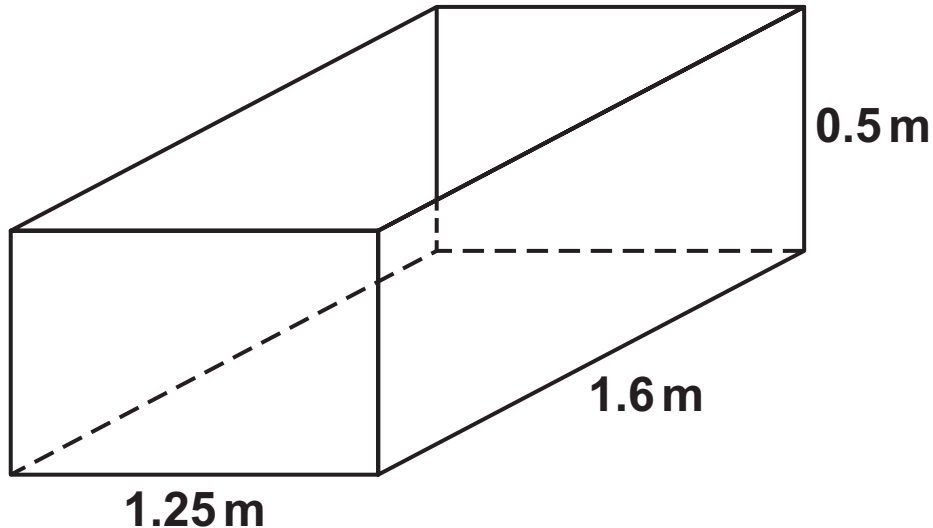
16 (a) Given that

$$\frac{1}{16 - 9x^2} \equiv \frac{A}{4 - 3x} + \frac{B}{4 + 3x}$$

find the values of A and B [3 marks]

[illegible]

- 16(b) An empty container, in the shape of a cuboid, has length 1.6 metres, width 1.25 metres and depth 0.5 metres, as shown in the diagram below.



The container has a small hole in the bottom.

Water is poured into the container at a rate of 0.16 cubic metres per minute.

At time t minutes after the container starts to be filled, the depth of water is d metres and water leaks out at a rate of $0.36d^2$ cubic metres per minute.

At time t minutes after the container starts to be filled, the volume of water in the container is V cubic metres.

[Turn over]



16 (b) (i) Show that

$$\frac{dV}{dt} = \frac{16 - 9V^2}{100} \quad [4 \text{ marks}]$$

[illegible]

[illegible]



16 (b) (iii) Determine how long it takes to fill the container with water.

Give your answer to the nearest minute.

[2 marks]

END OF QUESTIONS



Additional page, if required.

Write the question numbers in the left-hand margin.

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| Question | Mark |
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| TOTAL | |

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