A

Surname ____________________

Forename(s) ____________________

Centre Number ____________________

Candidate Number ____________________

Candidate Signature ____________________

I declare this is my own work.

A-level

MATHEMATICS

Paper 1

7357/1

Tuesday 6 June 2023 Afternoon

Time allowed: 2 hours

At the top of the page, write your surname and forename(s), your centre number, your candidate number and add your signature.

[Turn over]

JUN237357101

MATERIALS

For this paper you must have:

- the AQA Formulae for A-level Mathematics booklet
- a graphical or scientific calculator that meets the requirements of the specification.

INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise, marks for method may be lost.

- Do all rough work in this book. Cross through any work you do not want to be marked.

INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 **Find the coefficient of x^7 in the expansion of $(2x - 3)^7$**

Circle your answer. [1 mark]

–2187   –128   2   128

2 **Given that $y = 2x^3$ find $\frac{dy}{dx}$**

Circle your answer. [1 mark]

$\frac{dy}{dx} = 5x^2$ $\frac{dy}{dx} = 6x^2$

$\frac{dy}{dx} = \frac{x^4}{2}$ $\frac{dy}{dx} = 6x^3$

[Turn over]

3 **The curve with equation $y = \ln x$ is transformed by a stretch parallel to the x-axis with scale factor 2**

Find the equation of the transformed curve.

Circle your answer. [1 mark]

$y = \frac{1}{2} \ln x$ $y = 2 \ln x$

$y = \ln \frac{x}{2}$ $y = \ln 2x$

4 **Given that θ is a small angle, find an approximation for $\cos 2\theta$**

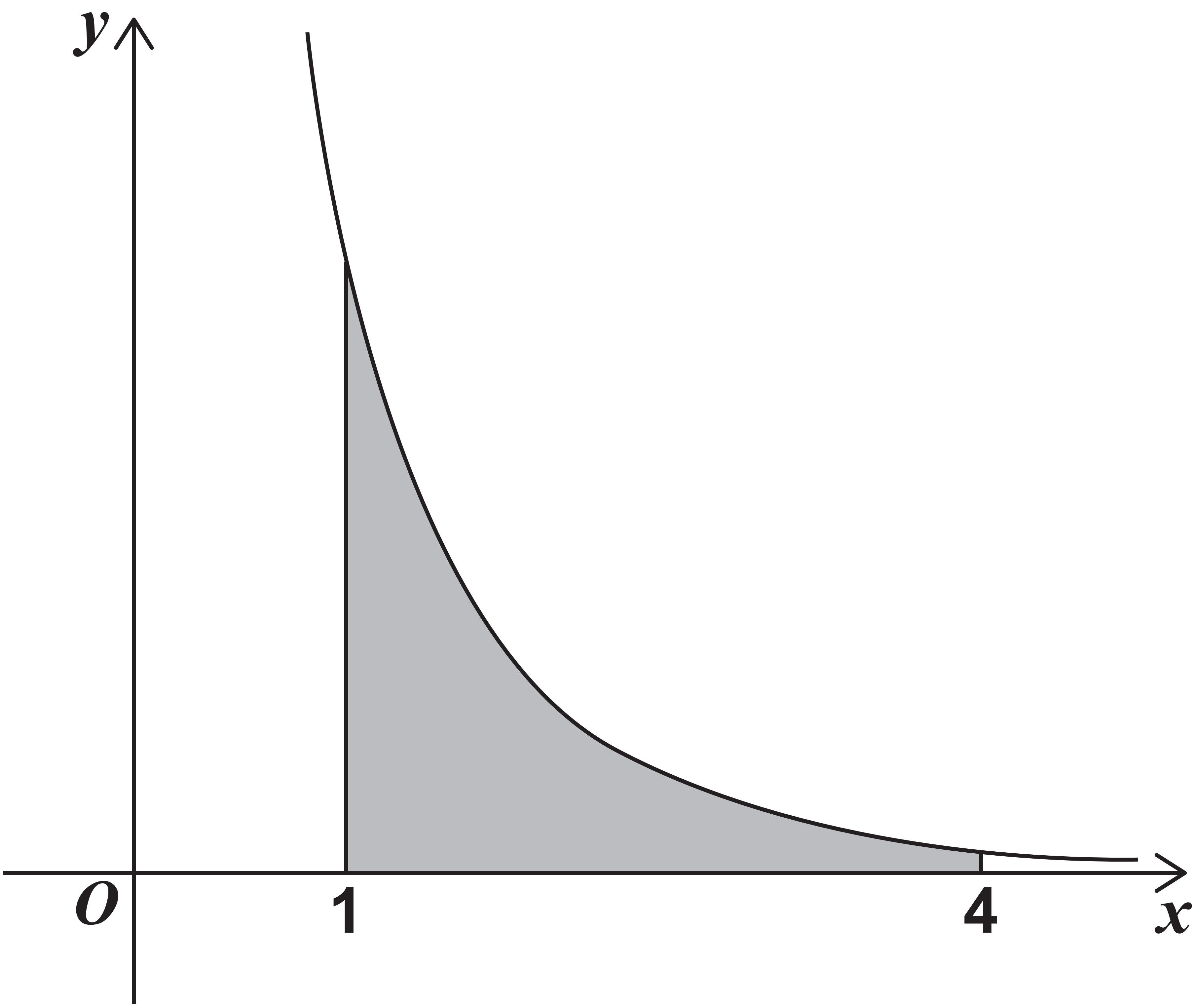
Circle your answer. [1 mark]

$1 - \frac{\theta^2}{2}$ $\qquad$ $2 - 2\theta^2$

$1 - 2\theta^2$ $\qquad$ $1 - \theta^2$

[Turn over]

5 The graph of $y = \dfrac{5}{e^x - 1}$ is shown in the diagram below.



The trapezium rule with 6 ordinates (5 strips) is to be used to find an approximation for the shaded area.

The values required to obtain this approximation are shown in the table below.

x	1	1.6	2.2
y	2.90988	1.26485	0.62305
x	2.8	3.4	4
y	0.32374	0.17263	0.09329

[Turn over]

5 (a) Use the trapezium rule with 6 ordinates (5 strips) to find an approximate value for the shaded area.

Give your answer to four decimal places. [3 marks]

5 (b) Using your answer to part (a), deduce an estimate for $\int_1^4 \frac{20}{e^x - 1} \, dx$

[1 mark]

6 **Show that the equation**

$$2\log_{10} x = \log_{10} 4 + \log_{10} (x + 8)$$

has exactly one solution.

Fully justify your answer.
[5 marks]

7 (a) **Given that n is a positive integer, express**

$$\frac{7}{3 + 5\sqrt{n}} - \frac{7}{5\sqrt{n} - 3}$$

as a single fraction not involving surds. [3 marks]

7 (b) **Hence, deduce that**

$$\frac{7}{3 + 5\sqrt{n}} - \frac{7}{5\sqrt{n} - 3}$$

is a rational number for all positive integer values of n
[1 mark]

8 **Show that**

$$\int_0^{\frac{\pi}{2}} (x \sin 4x)\,\mathrm{d}x = -\frac{\pi}{8}$$ **[6 marks]**

[Turn over]

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[Turn over]

9 **The points *P* and *Q* have coordinates (–6, 15) and (12, 19) respectively.**

9 (a) (i) **Find the coordinates of the midpoint of *PQ* [1 mark]**

9 (a) (ii) Find the equation of the perpendicular bisector of *PQ*

Give your answer in the form $ax + by = c$ where a, b and c are integers. [4 marks]

[Turn over]

9 (b) (i) A circle passes through the points *P* and *Q*

The centre of the circle lies on the line with equation $2x - 5y = -30$

Find the equation of the circle. [3 marks]

[Turn over]

9 (b) (ii) The circle intersects the coordinate axes at n points.

State the value of n [1 mark]

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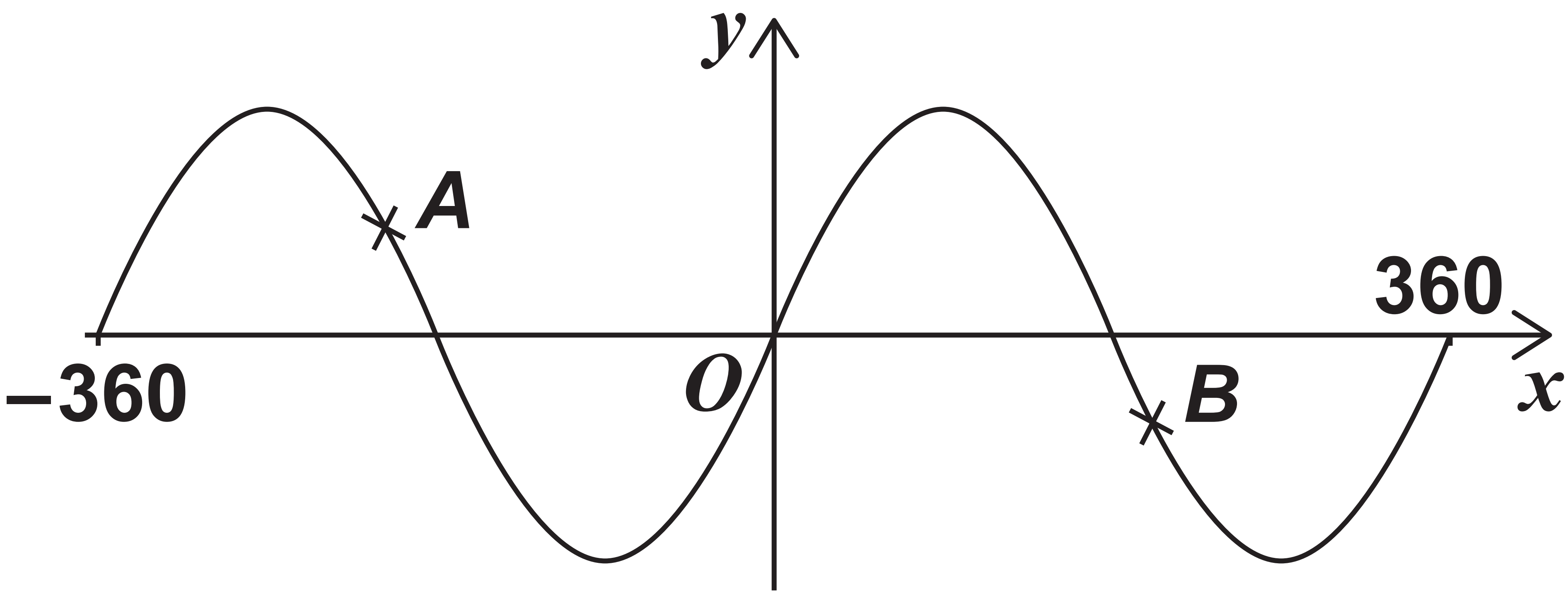
[Turn over]

10 **The curve with equation**

$y = \sin x°$

for $-360 \le x \le 360$ is shown below.



10 (a) **Point *A* on the curve has coordinates (a, 0.5)**

10 (a) (i) **Find the value of a [2 marks]**

[Turn over]

10 (a) (ii) **State the value of sin (180° – a°)** **[1 mark]**

10 (b) **Point *B* on the curve has coordinates** $\left(b, -\frac{3}{7}\right)$

10 (b) (i) **Find the exact value of sin (b° – 180°)** **[2 marks]**

10 (b) (ii) Find the exact value of cos b°
[3 marks]

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[Turn over]

11 The nth term of a sequence is u_n

The sequence is defined by

$u_{n+1} = pu_n + 70$

where $u_1 = 400$ and p is a constant.

11 (a) Find an expression, in terms of p, for u_2 [1 mark]

11 (b) It is given that $u_3 = 382$

11 (b) (i) Show that p satisfies the equation

$$200p^2 + 35p - 156 = 0$$ [3 marks]

11 (b) (ii) It is given that the sequence is a decreasing sequence.

Find the value of u_4 and the value of u_5 [3 marks]

[Turn over]

11 (c) **The limit of u_n as n tends to infinity is L**

11 (c) (i) **Write down an equation for L [1 mark]**

11 (c) (ii) **Find the value of L [1 mark]**

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[Turn over]

12 **One of the rides at a theme park is a room where the floor and ceiling both move up and down for 10π seconds.**

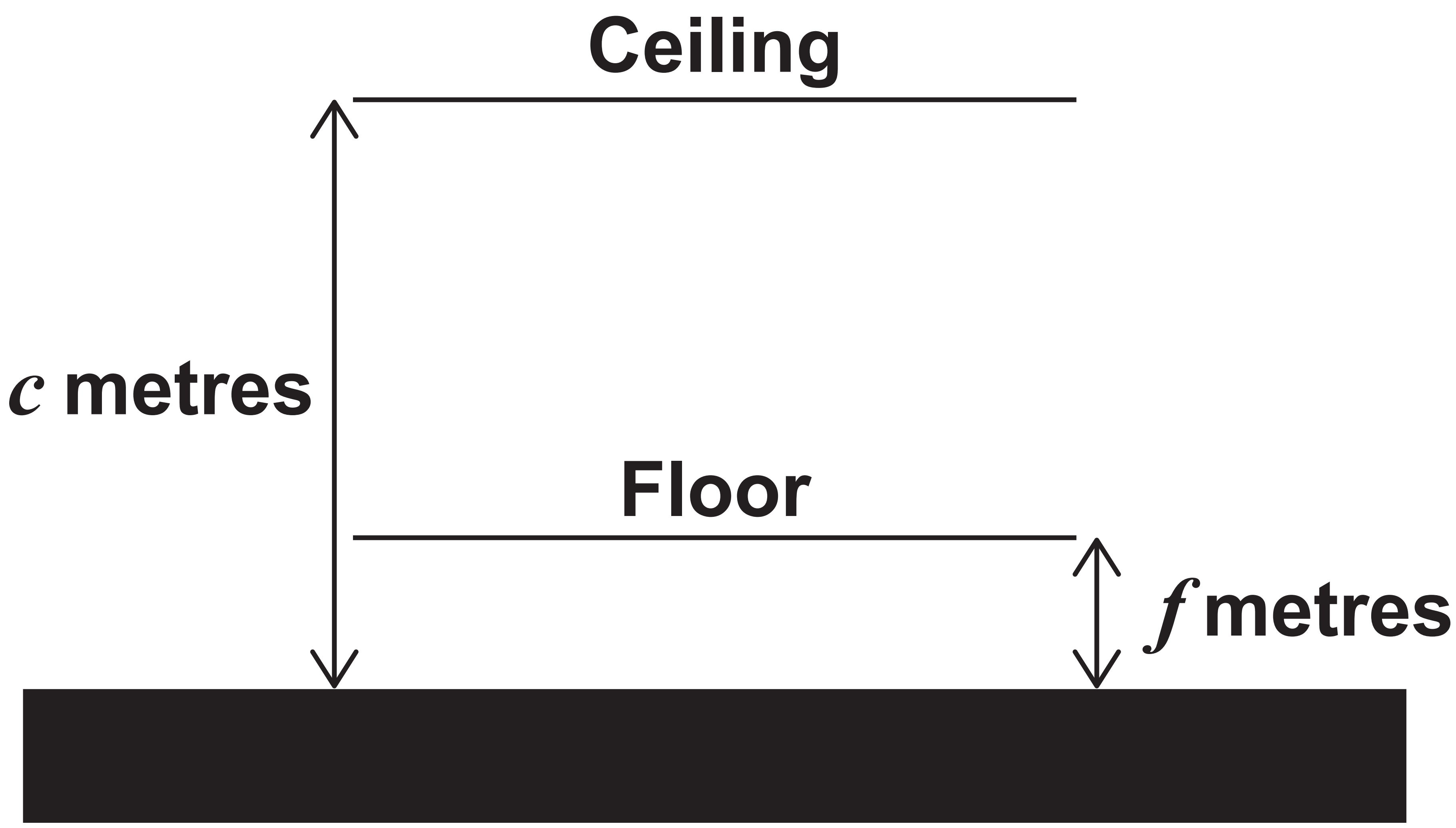
At time t seconds after the ride begins, the distance f metres of the floor above the ground is

$$f = 1 - \cos t$$

At time t seconds after the ride begins, the distance c metres of the ceiling above the ground is

$$c = 8 - 4\sin t$$

The ride is shown in the diagram on the opposite page.



12 (a) Show that the initial distance between the floor and ceiling is 8 metres. [1 mark]

12 (b) **Show that the distance d metres between the floor and ceiling at time t is given by**

$$d = 7 + R\cos(t + \alpha)$$

where R and α are positive constants to be found.
[5 marks]

12 (c) **Hence, find the minimum distance between the ceiling and the floor.**

Give your answer to the nearest centimetre. [2 marks]

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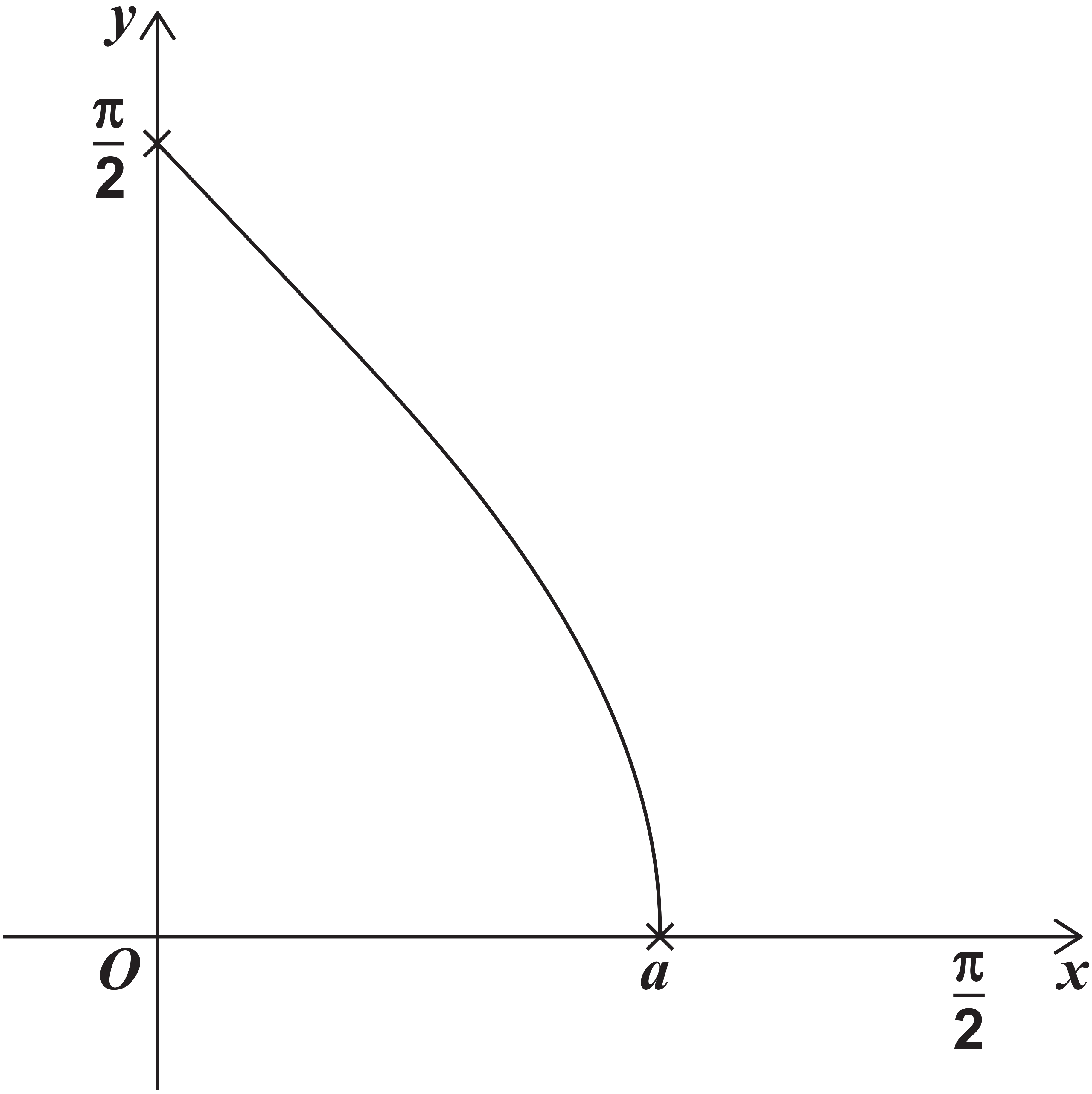
[Turn over]

13 The function f is defined by

$$f(x) = \arccos x \quad \text{for} \quad 0 \le x \le a$$

The curve with equation $y = f(x)$ is shown below.



13 (a) State the value of a [1 mark]

13 (b) (i) On the diagram on the opposite page, sketch the curve with equation

$$y = \cos x \quad \text{for} \quad 0 \leq x \leq \frac{\pi}{2}$$

AND

sketch the line with equation

$$y = x \quad \text{for} \quad 0 \leq x \leq \frac{\pi}{2}$$ [4 marks]

[Turn over]

13 (b) (ii) Explain why the solution to the equation

$$x - \cos x = 0$$

must also be a solution to the equation

$\cos x = \arccos x$ [1 mark]

13 (c) **Use the Newton-Raphson method with $x_0 = 0$ to find an approximate solution, x_3, to the equation**

$$x - \cos x = 0$$

Give your answer to four decimal places. [3 marks]

[Turn over]

14 (a) (i) **Given that**

$y = 2^x$

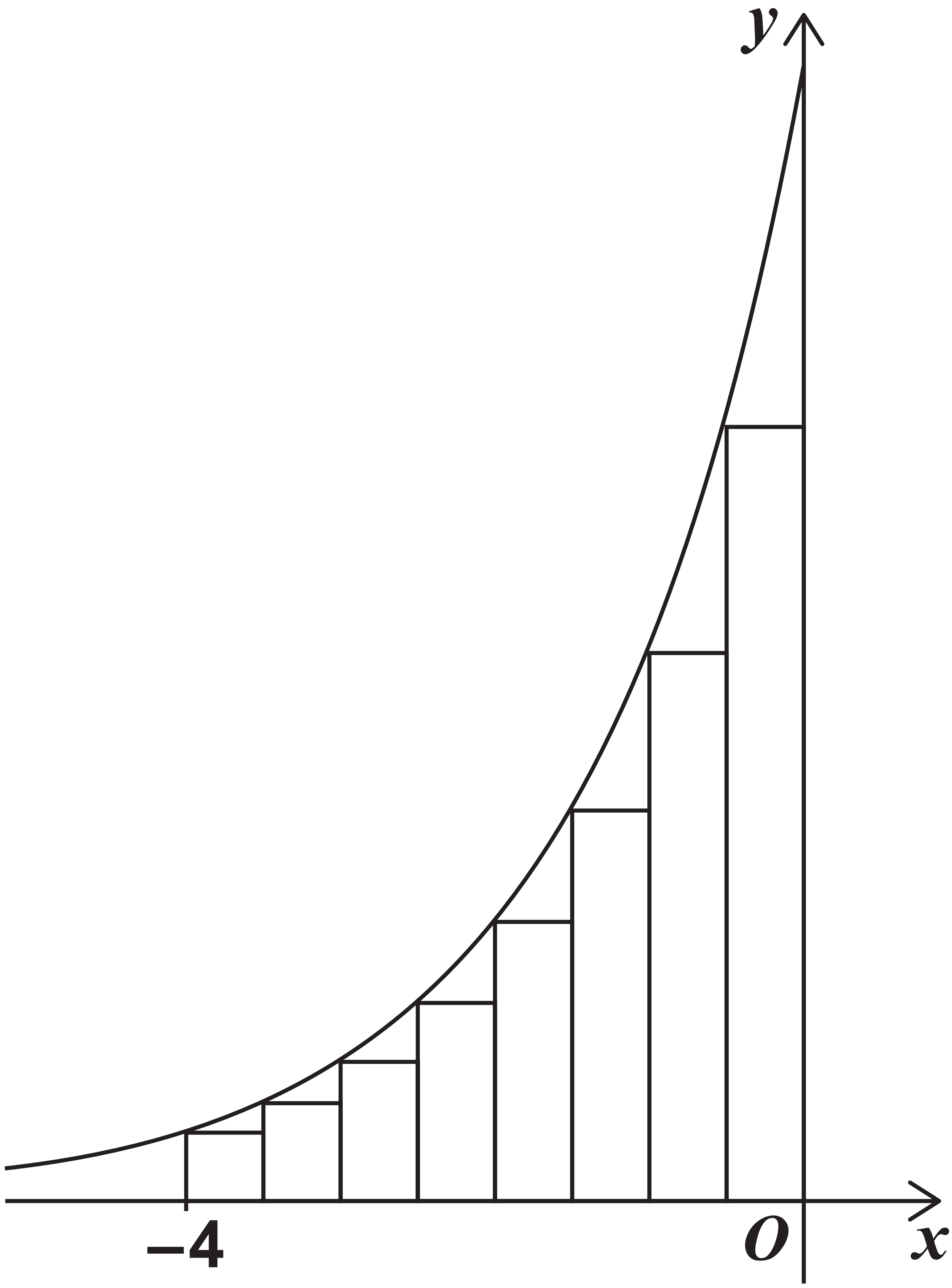
write down $\frac{dy}{dx}$ **[1 mark]**

14 (a) (ii) **Hence find**

$\int 2^x dx$ **[2 marks]**

[Turn over]

14 (b) **The area, A, bounded by the curve with equation $y = 2^x$, the x-axis, the y-axis and the line $x = -4$ is approximated using eight rectangles of equal width as shown in the diagram below.**



14 (b) (i) **Show that the exact area of the largest rectangle is $\frac{\sqrt{2}}{4}$**

[2 marks]

[Turn over]

14 (b) (ii) **The areas of these rectangles form a geometric sequence with common ratio $\frac{\sqrt{2}}{2}$**

Find the exact value of the total area of the eight rectangles.

Give your answer in the form $k(1 + \sqrt{2})$ where k is a rational number. [3 marks]

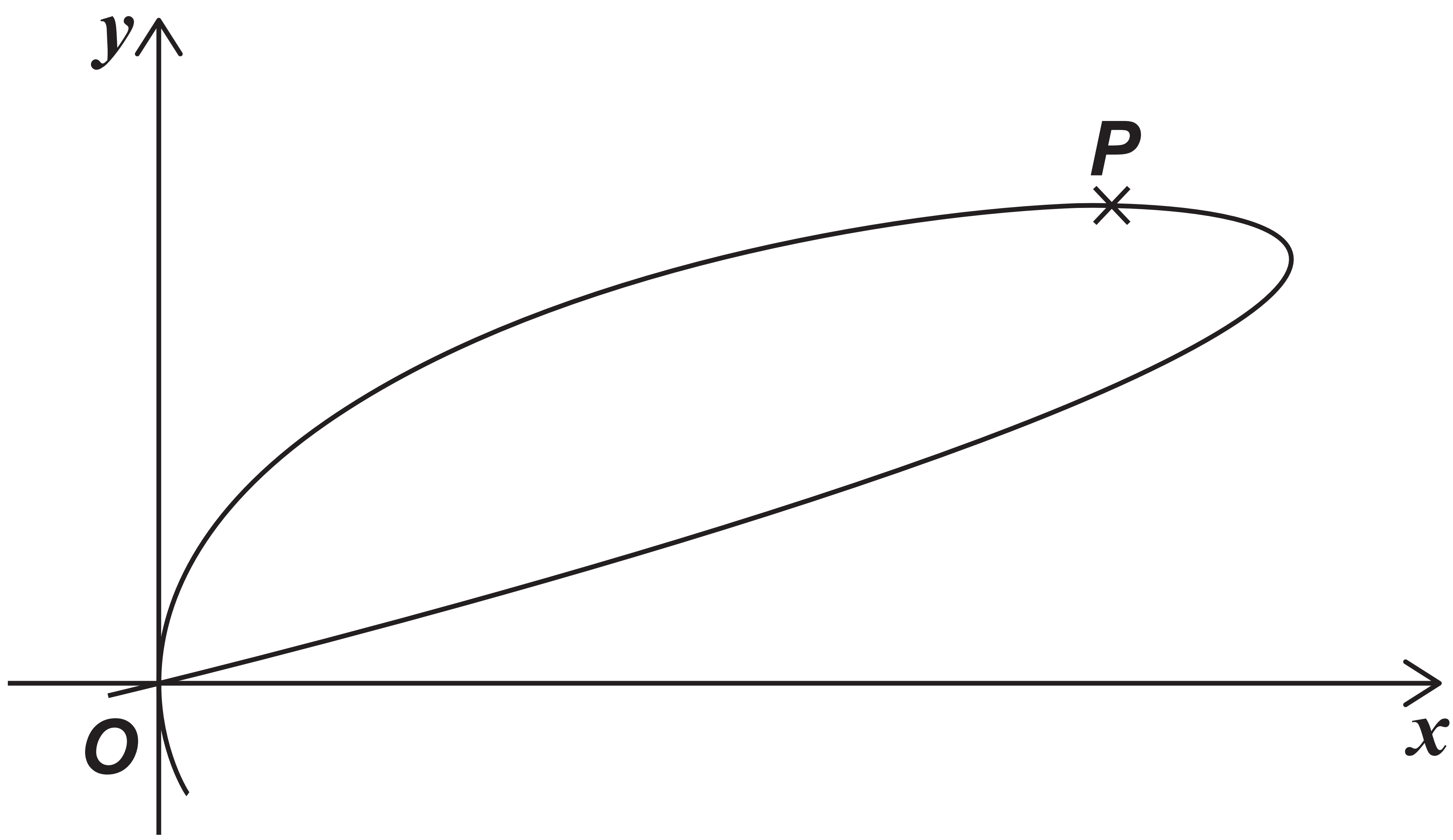
14 (b) (iii) More accurate approximations for A can be found by increasing the number, n, of rectangles used.

Find the exact value of the limit of the approximations for A as $n \to \infty$ [3 marks]

15 The curve with equation

$$x^2 + 2y^3 - 4xy = 0$$

has a single stationary point at P as shown in the diagram below.



15 (a) **Show that the y-coordinate of *P* satisfies the equation**

$y^2(y - 2) = 0$ **[7 marks]**

[Turn over]

[Turn over]

15 (b) **Hence, find the coordinates of *P***
[2 marks]

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[Turn over]

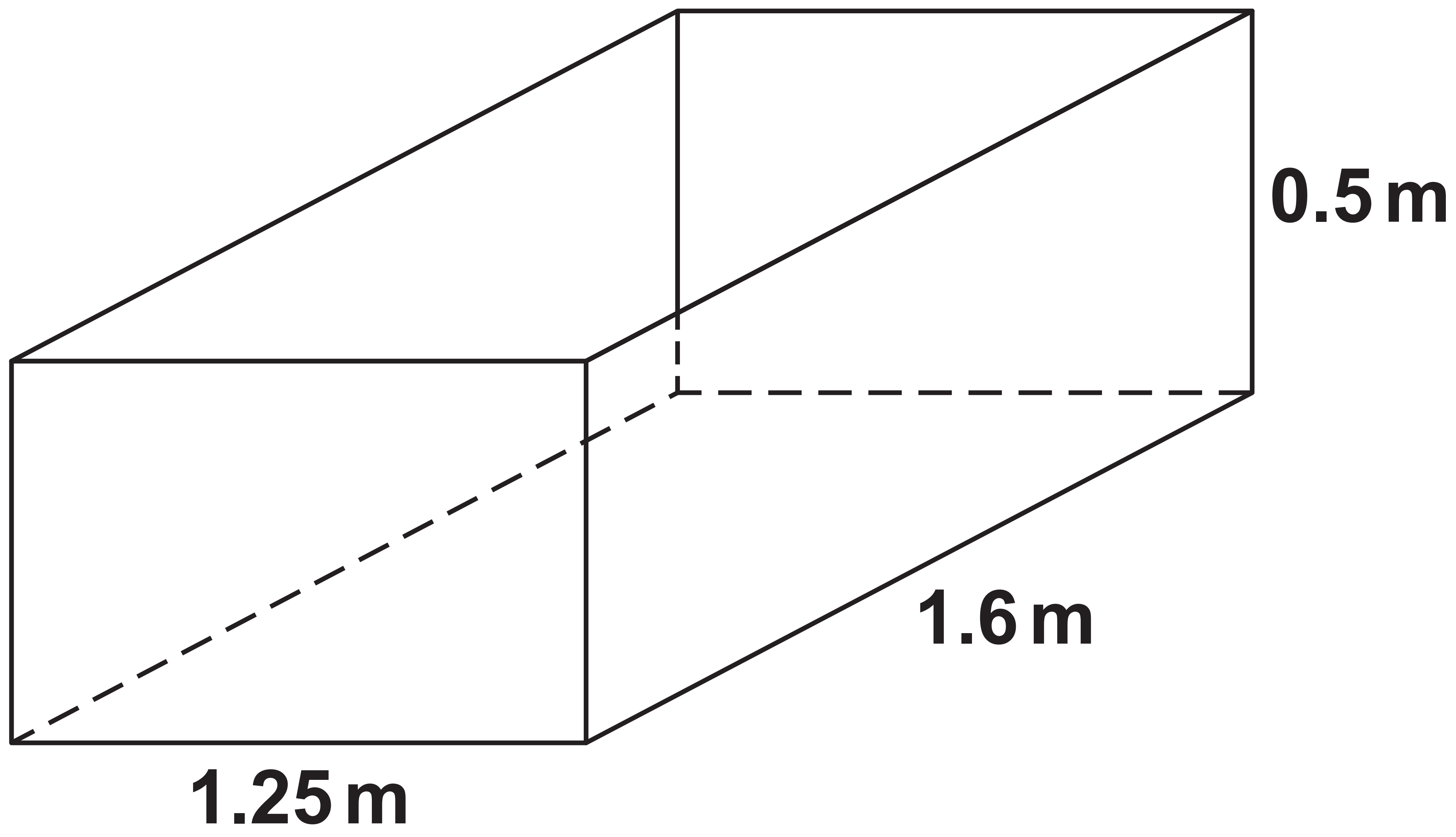
16 (a) **Given that**

$$\frac{1}{16 - 9x^2} \equiv \frac{A}{4 - 3x} + \frac{B}{4 + 3x}$$

find the values of A and B
[3 marks]

[Turn over]

16 (b) **An empty container, in the shape of a cuboid, has length 1.6 metres, width 1.25 metres and depth 0.5 metres, as shown in the diagram below.**



The container has a small hole in the bottom.

Water is poured into the container at a rate of 0.16 cubic metres per minute.

At time t minutes after the container starts to be filled, the depth of water is d metres and water leaks out at a rate of $0.36d^2$ cubic metres per minute.

At time t minutes after the container starts to be filled, the volume of water in the container is V cubic metres.

[Turn over]

16 (b) (i) **Show that**

$$\frac{dV}{dt} = \frac{16 - 9V^2}{100}$$ **[4 marks]**

[Turn over]

16 (b) (ii) Hence, find t in terms of V
[5 marks]

[Turn over]

16 (b) (iii) Determine how long it takes to fill the container with water.

Give your answer to the nearest minute. [2 marks]

END OF QUESTIONS

Additional page, if required. Write the question numbers in the left-hand margin.

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For Examiner's Use	
Question	Mark
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