AQA

AS
FURTHER MATHEMATICS
7366/1

Paper 1

Mark scheme

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236A7366/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles the correct answer	1.2	B1	$\dfrac{\sinh x}{\cosh x}$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles the correct answer	1.2	B1	90°
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles the correct answer	1.1b	B1	$\begin{bmatrix} 7 & 13 \\ 35 & 5 \end{bmatrix}$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4	Circles the correct answer	1.1b	B1	$-\dfrac{3}{5}$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Evaluates the definite integral of f between 1 and 5	1.1a	M1	$\frac{1}{5-1}\int_1^5 3x^2\,dx$
	Obtains 31	1.1b	A1	$=\frac{1}{4}\times 124$ $=31$
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Sets up a correct equation to find c Follow through their part **(a)** answer.	3.1a	M1	$31 + c = 40$ $c = 9$
	Obtains the correct result. Follow through their part **(a)** answer.	1.1b	A1F	
	Subtotal		**2**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Obtains a correct unsimplified Maclaurin series for e^{2x} eg substitutes $2x$ into the Maclaurin series for e^x (condone missing brackets)	1.1a	M1	$e^{2x}=1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\frac{(2x)^4}{4!}+\ldots$ $=1+2x+\frac{4x^2}{2}+\frac{8x^3}{6}+\frac{16x^4}{24}+\ldots$ $=1+2x+2x^2+\frac{4}{3}x^3+\frac{2}{3}x^4+\ldots$
	Obtains correct series, evaluating powers of 2 and factorials. Ignore any higher power terms. ISW	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Rewrites their part **(a)** of the form $a+bx+cx^2+dx^3+ex^4$ as $a-bx+cx^2-dx^3+ex^4$ where a,b,c,d,e are non-zero. Ignore any higher power terms. **or** Obtains a correct series for e^{-2x} evaluating powers of 2 and factorials.	1.1b	B1F	$e^{-2x}=1-2x+2x^2-\frac{4}{3}x^3+\frac{2}{3}x^4+\ldots$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)	States the definition of cosh(2*x*) Or finds the correct first four derivatives of cosh(2*x*)	1.1b	B1	$\cosh(2x) = \frac{1}{2}(e^{2x} + e^{-2x})$ $= \frac{1}{2}\left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + 1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4\right) + \ldots$ $= \frac{1}{2}\left(2 + 4x^2 + \frac{4}{3}x^4\right) + \ldots$ $= 1 + 2x^2 + \frac{2}{3}x^4 + \ldots$
	Adds their polynomial expansions from parts **(a)** and **(b)**. Condone subtraction. Or uses the general Maclaurin series to find the first five terms.	3.1a	M1	
	Obtains the correct simplified series with any equivalent rational coefficients. Must come from correct **(a)** and **(b)** Ignore any higher power terms.	2.1	R1	
	Subtotal		**3**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Completes a rigorous argument to prove the required result. Must include the LHS, at least one intermediate step, and the RHS.	2.1	B1	$\frac{1}{2r-1}-\frac{1}{2r+1}=\frac{2r+1-(2r-1)}{(2r-1)(2r+1)}$ $=\frac{2}{(2r-1)(2r+1)}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Writes at least two pairs of subtracting fractions. Condone any consistent multiple of the correct fractions.	1.1a	M1	$\sum_{r=1}^{n}\frac{2}{(2r-1)(2r+1)}$ $=\sum_{r=1}^{n}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)$ $=\frac{1}{1}-\frac{1}{3}$ $+\frac{1}{3}-\frac{1}{5}$ $+\ldots\ldots\ldots$ $+\frac{1}{2n-3}-\frac{1}{2n-1}$ $+\frac{1}{2n-1}-\frac{1}{2n+1}$ $=\frac{1}{1}-\frac{1}{2n+1}$ $=\frac{2n+1-1}{2n+1}$ $=\frac{2n}{2n+1}$ So $\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}=\frac{n}{2n+1}$
	Writes at least one pair of cancelling fractions. Condone any consistent multiple of the correct fractions.	1.1a	M1	
	Correctly reduces the required expression to two terms. **PI** Condone any consistent multiple of the correct fractions.	1.1b	A1	
	Completes a reasoned argument using the method of differences to reach the required result. This mark is only available if at least the first two pairs of fractions and the last pair are shown. Accept an unsimplified fraction eg $\frac{2n}{4n+2}$	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	Uses $n=50$ Condone 49 or 51	3.1a	M1	$\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\ldots+\frac{1}{99\times101}$ $=\sum_{r=1}^{50}\frac{1}{(2r-1)(2r+1)}$ $=\frac{50}{101}$
	Obtains the correct exact value. oe eg $0.\dot{4}95\dot{0}$ FT their $\frac{50a}{50b+c}$	1.1b	A1F	
	Subtotal		**2**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Obtains the correct simplified answer $1 - \mathrm{i}\sqrt{3}$ Accept $-(-1 + \mathrm{i}\sqrt{3})$ Condone no conclusion	1.1b	B1	$2\left(\cos\left(-\frac{\pi}{3}\right)+\mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)$ $=2\left(\frac{1}{2}+\mathrm{i}\left(-\frac{\sqrt{3}}{2}\right)\right)$ $=1-\mathrm{i}\sqrt{3}$ $\therefore$ Abdoallah's answer must be wrong
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Explains that there is another solution to $\tan\theta = -\sqrt{3}$ Accept any indication that there is another solution to $\tan\theta = -\sqrt{3}$	2.3	B1	There are two solutions to $\tan\theta = -\sqrt{3}$ in the interval $-\pi < \theta \le \pi$ Abdoallah has chosen the wrong one. $-1+\mathrm{i}\sqrt{3}$ is in the 2nd quadrant of an Argand diagram, so θ should be obtuse.
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)	Obtains the correct answer.	1.1b	B1	$\theta = -\frac{\pi}{3}+\pi = \frac{2\pi}{3}$ $-1+\mathrm{i}\sqrt{3} = 2\left(\cos\left(\frac{2\pi}{3}\right)+\mathrm{i}\sin\left(\frac{2\pi}{3}\right)\right)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(d)	Writes the correct answer in any form.	1.1b	B1	$-1-\mathrm{i}\sqrt{3}$
	Subtotal		**1**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Forms the product $\mathbf{M}\begin{bmatrix}4\\5\end{bmatrix}$ or $\mathbf{M}^{-1}\begin{bmatrix}64\\-7\end{bmatrix}$ May use the letter **M**, or **M** in terms of p, or with $p=0$	1.1a	M1	When $p = 0$, then $\mathbf{M}\begin{bmatrix}4\\5\end{bmatrix}=\begin{bmatrix}1 & 12\\2 & -3\end{bmatrix}\begin{bmatrix}4\\5\end{bmatrix}$ $=\begin{bmatrix}64\\-7\end{bmatrix}$ $\therefore$ the image of (4, 5) is (64, –7)
	Completes a reasoned argument to prove the required result. Condone no conclusion.	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Multiplies **M** (or $\mathbf{M}^{-1}$) by $\begin{bmatrix}x\\y\end{bmatrix}$ and equates to $\begin{bmatrix}x\\y\end{bmatrix}$ **PI** Accept y replaced with mx or $mx+c$	1.1a	M1	$\begin{bmatrix}-5 & 12\\0 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$ $\therefore\ -5x+12y=x$ and $0x+y=y$ $12y=6x$ $y=\frac{1}{2}x$ The gradient is $\frac{1}{2}$
	Multiplying the top row of **M** by their $\begin{bmatrix}x\\y\end{bmatrix}$	1.1a	M1	
	Obtains correct gradient	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	Uses or states det **M** $= 0$	3.1a	B1	det **M** $= 0$ det **M** = $(3p + 1)(p^2 – 3) – 12(p + 2)$ $= 3p^3 + p^2 – 9p – 3 – 12p – 24$ $= 3p^3 + p^2 – 21p – 27$ $\therefore$ **M** is singular when $3p^3 + p^2 – 21p – 27 = 0$ $\Rightarrow$ $p = 3$ or $p = \frac{-5 \pm i\sqrt{2}}{3}$ $\therefore$ $p = 3$ is the only real value of p for which **M** is singular
	Forms an expression for det M in p **or** Substitutes $p = 3$ and evaluates det **M** Condone $ad + bc$	1.1a	M1	
	Obtains a correct expression for det **M** in terms of p	1.1b	A1	
	Obtains a correct simplified equation for det **M** $= 0$ in terms of p	1.1b	A1	
	Uses a correct method to deduce that det **M** $= 0$ has exactly one real root	2.2a	M1	
	Completes a reasoned argument that $p = 3$ is the only real value of p for which **M** is singular.	2.1	R1	
	Subtotal		**6**	

	Question total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	States $y = 3$	1.1b	B1	$y = 3$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Identifies the correct factors of the denominator **PI**	1.1a	M1	The denominator is $(x + 4)(x + 1)$ $= x^2 + 5x + 4$ $\therefore\ m = 4$ and $p = 5$
	Obtains the correct values.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)	Obtains the correct y-coordinate of the intercept. Follow through their $\dfrac{p}{m}$	1.1b	B1F	When $x = 0$, then $y = \dfrac{p}{m} = \dfrac{5}{4}$ $\therefore\ y$-intercept is $\left(0, \dfrac{5}{4}\right)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(d)	Forms an equation to find the intersection point(s) if they exist. Could equate to a letter, eg k instead of -1	1.1a	M1	$y = -1$ intersects C when $\frac{3x^2+4x+5}{x^2+5x+4} = -1$ $\Rightarrow\ 3x^2 + 4x + 5 = -(x^2 + 5x + 4)$ $\Rightarrow\ 4x^2 + 9x + 9 = 0$ $b^2 - 4ac = 9^2 - 4 \times 4 \times 9 = -63 < 0$ $\therefore$ there are no real roots $\therefore\ y = -1$ does not intersect C
	Rearranges into a three-term quadratic equation. Allow one arithmetic error. Could be in terms of k	1.1a	M1	
	Obtains a correct quadratic equation. Could be in terms of k $(k-3)x^2 + (5k-4)x + 4k - 5 = 0$	1.1b	A1	
	Uses a correct method to deduce that their quadratic equation has no real roots. **or** Considers the sign of the discriminant in terms of k $\Delta = 9k^2 + 28k - 44$	1.1a	M1	
	Completes a reasoned argument to conclude that the line $y = -1$ does not intersect C	2.1	R1	
	Subtotal		**5**	

	Question total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Writes a correct expression for r	1.2	B1	$r = \sqrt{x^2 + y^2}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)	Writes a correct expression for x	1.2	B1	$x = r\cos\theta$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
11(c)(i)	Substitutes their **(a)** and **(b)** to form an equation in x and y only.	3.1a	M1	$r(2+\cos\theta) = 1$ $\Rightarrow 2r + r\cos\theta = 1$ $\Rightarrow 2\sqrt{x^2+y^2} + x = 1$ $\Rightarrow 2\sqrt{x^2+y^2} = 1 - x$ $\Rightarrow 4(x^2+y^2) = (1-x)^2$ $\Rightarrow 4x^2 + 4y^2 = 1 - 2x + x^2$ $\Rightarrow 4y^2 = 1 - 2x - 3x^2$ $\Rightarrow 4y^2 = (1-3x)(1+x)$
	Correctly removes the square root from their equation. Must be an equation in terms of x and y only.	1.1a	M1	
	Obtains a correct equation without roots.	1.1b	A1	
	Obtains the correct equation in the required form.	3.2a	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
11(c)(ii)	Identifies a reflection. Must also specify a line of reflection (which could be wrong for this mark). **or** Identifies a rotation about (0, 0)	1.1a	M1	y is replaced with x, and vice versa Reflection in $y = x$
	Fully describes a correct transformation. Accept 90° rotation about (0, 0) If a rotation direction is included, it must be anticlockwise.	1.1b	A1	
	Subtotal		**2**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
12(a)	Applies the binomial expansion to $(1 + i)^4$ or $(1 + i)^3$ Allow one incorrect term. Or $(1 + i)^2 = 1 + 2i + i^2$ (or just 2i)	1.1a	M1	$(1 + i)^4$ $= 1^4 + 4.1^3.i + 6.1^2.i^2 + 4.1.i^3 + i^4$ $= 1 + 4i - 6 - 4i + 1$ $= -4$
	Replaces i^2 with –1, or i^3 with $-i$, or i^4 with 1	1.2	B1	
	Completes a reasoned argument to reach the required result. Must include the LHS, at least two intermediate steps, and the RHS. Accept $(1 + i)^2$ replaced with 2i without explanation.	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)(i)	Substitutes (1 + i) into f	1.1a	M1	$f(1 + i) = (1 + i)^4 + 3(1 + i)^2 - 6(1 + i) + 10$ $= 0$ $\therefore$ (1 + i) is a root of $f(z) = 0$
	Equates to 0 and concludes that (1 + i) is a root.	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)(ii)	Identifies $1 - i$ as a root. Accept $-1 + 2i$ or $-1 - 2i$	1.2	B1	$1 - i$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)(iii)	Identifies a correct linear factor. Accept $(z-(-1+2\text{i}))$ or $(z-(-1-2\text{i}))$ Follow through their part **(b)(ii)**.	1.1b	B1F	2nd linear factor is $(z-(1-\text{i}))$ Quadratic factor is $(z-(1+\text{i}))(z-(1-\text{i}))$ $= z^2-(1-\text{i})z-(1+\text{i})z+(1+\text{i})(1-\text{i})$ $= z^2-(1-\text{i}+1+\text{i})z+1-\text{i}^2$ $= z^2-2z+2$
	Forms the product $(z-w)(z-w^*)$ for any non-real w	3.1a	M1	
	Obtains z^2-2z+2 Accept z^2+2z+5	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)(iv)	Obtains a second quadratic factor of $\text{f}(z)$ with at least two correct terms.	3.1a	M1	$z^4+3z^2-6z+10$ $=(z^2-2z+2)(z^2+2z+5)$ 2nd quadratic factor is z^2+2z+5
	Obtains a correct second quadratic factor. Accept z^2-2z+2 if z^2+2z+5 is the answer to their part **(b)(iii)**	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)(v)	Explains that $\text{f}(z)=0$ has no real roots. Condone "no real roots" with an incorrect or no other statement.	2.4	M1	$\text{f}(z)=0$ has no real roots. Hence $y=\text{f}(x)$ does not intersect the x-axis.
	Completes a reasoned argument to conclude that $y=\text{f}(x)$ does not intersect the x-axis.	2.1	R1	
	Subtotal		**2**	

	Question total		**13**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Substitutes $n = 1$ into LHS and RHS of $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$	2.2a	B1	When $n = 1$: $\sum_{r=1}^{1} r^2 = 1^2 = 1$ and $\frac{1}{6}\times 1\times 2\times 3 = 1$ $\therefore$ the rule is true for $n = 1$
	Uses $\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$ and considers $\sum_{r=1}^{k} r^2 + (k+1)^2$ Condone use of n in place of k	2.4	M1	Assume it is true for $n = k$ $\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$ $\Rightarrow \sum_{r=1}^{k} r^2 + (k+1)^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$ $\Rightarrow \sum_{r=1}^{k+1} r^2 = \frac{1}{6}(k+1)(k(2k+1) + 6(k+1))$
	Completes working to show $\frac{1}{6}k(k+1)(2k+1) + (k+1)^2$ is equivalent to $\frac{1}{6}(k+1)(k+1+1)(2(k+1)+1)$ Accept $\frac{1}{6}(k+1)(k+2)(2k+3)$	2.2a	A1	$= \frac{1}{6}(k+1)(2k^2 + 7k + 6)$ $= \frac{1}{6}(k+1)(k+2)(2k+3)$ $= \frac{1}{6}(k+1)(k+1+1)(2(k+1)+1)$ $\therefore$ the rule is also true for $n = k + 1$ So, by induction, $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ is true for all integers $n \geq 1$

	Completes a reasoned argument by stating that the rule is true for $n = 1$ **and** if the rule is true for $n = k$ then it is also true for $n = k + 1$ **and** concludes that by induction $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ is true for all integers $n \geq 1$ This mark is dependent on all previous marks. The algebra must use an alternative letter to n Condone reference to ‘rule’/ ‘statement’ / ‘it’ in the concluding statement.	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Writes the correct expression. Accept partial factorisation, eg $\frac{n}{3}(8n^2+6n+1)$ ISW	1.1b	B1	$\frac{1}{6}\times 2n(2n+1)(4n+1)$ $=\frac{1}{3}n(2n+1)(4n+1)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
13(c)	Writes the required sum as a multiple of $\sum_{r=1}^{n} r^2$	3.1a	M1	$\sum_{r=1}^{n}(2r)^2 = 4\sum_{r=1}^{n} r^2$ $= 4\times\frac{1}{6}n(n+1)(2n+1)$ $=\frac{2}{3}n(n+1)(2n+1)$
	Obtains the correct expression. Accept partial factorisation, eg $\frac{2n}{3}(2n^2+3n+1)$ ISW	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(d)	Subtracts their **(c)** from their **(b)** Accept **(c)** – **(b)** **or** Writes a sum of odd squares in terms of $\sum r^2$ and $\sum r$ **PI**	3.1a	M1	$1^2+3^2+5^2+\ldots+(2n-1)^2$ $=\sum_{1}^{2n} r^2-\sum_{1}^{n}(2r)^2$ $=\frac{1}{3}n(2n+1)(4n+1)-\frac{2}{3}n(n+1)(2n+1)$ $=\frac{1}{3}n(2n+1)((4n+1)-2(n+1))$ $=\frac{1}{3}n(2n-1)(2n+1)$
	Obtains correct expression in terms of n in any form	1.1b	A1	
	Completes a reasoned argument to obtain the correct expression.	2.1	R1	
	Subtotal		**3**	

	Question total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
14	Factorises $x^2 - 5x - 24$ to $(x + m)(x + n)$ where $m + n = -5$ or $mn = -24$ and identifies their $m, n > 2$ as b Or uses the coefficients of a quartic to form an equation in a and/or b eg $-(-5 + 7) = -9 + (-3) + 2 + b$ eg $-24a = -9 \times -3 \times 2 \times b$ Or multiplies two or more of the factors $(x + 9)$, $(x + 3)$, $(x - 2)$ and $(x - b)$	3.1a	M1	$x^2 - 5x - 24 = (x - 8)(x + 3)$ $\therefore$ the critical values include –3 and 8 $\therefore$ $b = 8$ $(x + 9)(x - 2) = (x^2 + 7x - 18)$ $\therefore$ $a = -18$
	Obtains $a = -18$ **or** $b = 8$	1.1b	A1	
	Expands $(x + 9)(x - 2)$ and identifies the constant term as a Or correctly forms two equations in a and b Or divides the expanded quartic by a quadratic or a cubic formed by multiplying two or three of $(x + 9)$, $(x + 3)$, $(x - 2)$ and $(x - b)$ Or compares coefficients in the expansions of $(x^2 - 5x - 24)(x^2 + 7x + a)$ and $(x + 9)(x + 3)(x - 2)(x - b)$	3.1a	M1	
	Obtains $a = -18$ **and** $b = 8$	1.1b	A1	
	Question total		**4**	

	Question Paper total		**80**	