A

Surname ______________________________

Forename(s) ______________________________

Centre Number ______________________________

Candidate Number ______________________________

Candidate Signature ______________________________

I declare this is my own work.

AS

FURTHER MATHEMATICS

Paper 1

7366/1

Monday 15 May 2023 Afternoon

Time allowed: 1 hour 30 minutes

At the top of the page, write your surname and forename(s), your centre number, your candidate number and add your signature.

[Turn over]

JUN237366101

MATERIALS

- **You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.**
- **You should have a graphical or scientific calculator that meets the requirements of the specification.**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).**
- **Do NOT write on blank pages.**

- **Show all necessary working; otherwise marks for method may be lost.**

- **Do all rough work in this book. Cross through any work that you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**

- **The maximum mark for this paper is 80.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**

- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 **Which expression below is equivalent to tanh x ?**

Circle your answer. [1 mark]

$\sinh x \cosh x$

$\dfrac{\sinh x}{\cosh x}$

$\dfrac{\cosh x}{\sinh x}$

$\sinh x + \cosh x$

2 The two vectors a and b are such that a.b = 0

State the angle between the vectors a and b

Circle your answer. [1 mark]

0°	45°
90°	180°

[Turn over]

3 **The matrices A and B are given by**

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 4 \\ 7 & 1 \end{bmatrix}$$

Calculate AB

Circle your answer. [1 mark]

$$\begin{bmatrix} 3 & 5 \\ 7 & 6 \end{bmatrix} \qquad \begin{bmatrix} 0 & 20 \\ 21 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4 \\ 0 & 5 \end{bmatrix} \qquad \begin{bmatrix} 7 & 13 \\ 35 & 5 \end{bmatrix}$$

4 The roots of the equation

$$5x^3 + 2x^2 - 3x + p = 0$$

are α, β and γ

Given that p is a constant, state the value of $\alpha\beta + \beta\gamma + \gamma\alpha$

Circle your answer. [1 mark]

$-\frac{3}{5}$ $-\frac{2}{5}$ $\frac{2}{5}$ $\frac{3}{5}$

[Turn over]

5 **The function f is defined by**

$$f(x) = 3x^2 \qquad 1 \le x \le 5$$

5 (a) **Find the mean value of f**
[2 marks]

[Turn over]

5 (b) The function g is defined by

$$g(x) = f(x) + c \qquad 1 \leq x \leq 5$$

The mean value of g is 40

Calculate the value of the constant c [2 marks]

6 (a) **Find and simplify the first five terms in the Maclaurin series for e^{2x} [2 marks]**

[Turn over]

6 (b) **Hence, or otherwise, write down the first five terms in the Maclaurin series for e^{-2x}**
[1 mark]

BLANK PAGE

[Turn over]

6 (c) **Hence, or otherwise, show that the Maclaurin series for $\cosh(2x)$ is**

$$a + bx^2 + cx^4 + \dots$$

where a, b and c are rational numbers to be determined.
[3 marks]

[Turn over]

7 (a) **Show that, for all integers r,**

$$\frac{1}{2r-1} - \frac{1}{2r+1} = \frac{2}{(2r-1)(2r+1)}$$

[1 mark]

7 (b) **Hence, using the method of differences, show that**

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{an}{bn+c}$$

where a, b and c are integers to be determined. [4 marks]

[Turn over]

7 (c) **Hence, or otherwise, evaluate**

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \ldots + \frac{1}{99 \times 101}$$

[2 marks]

8 **Abdoallah wants to write the complex number $-1 + i\sqrt{3}$ in the form $r(\cos\theta + i\sin\theta)$**

where $r \geq 0$ and $-\pi < \theta \leq \pi$

Here is his method:

$$r = \sqrt{(-1)^2 + (\sqrt{3})^2}$$
$$= \sqrt{1} + 3$$
$$= \sqrt{4}$$
$$= 2$$

$$\tan\theta = \frac{\sqrt{3}}{-1}$$
$$\Rightarrow \tan\theta = -\sqrt{3}$$
$$\Rightarrow \theta = \tan^{-1}\left(-\sqrt{3}\right)$$
$$\Rightarrow \theta = -\frac{\pi}{3}$$

$$-1 + i\sqrt{3} = 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

There is an error in Abdoallah's method.

8 (a) **Show that Abdoallah's answer is wrong by writing**

$$2\left(\cos\left(-\frac{\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)$$

in the form $x + \mathrm{i}y$

Simplify your answer. [1 mark]

[Turn over]

8 (b) Explain the error in Abdoallah's method. [1 mark]

[Turn over]

8 (c) **Express $-1 + \mathrm{i}\sqrt{3}$ in the form $r(\cos\theta + \mathrm{i}\sin\theta)$ [1 mark]**

8 (d) Write down the complex conjugate of $-1 + i\sqrt{3}$ [1 mark]

[Turn over]

9 The matrix M represents the transformation T and is given by

$$\mathbf{M} = \begin{bmatrix} 3p + 1 & 12 \\ p + 2 & p^2 - 3 \end{bmatrix}$$

9 (a) In the case when $p = 0$ show that the image of the point (4, 5) under T is the point (64, −7) [2 marks]

9 (b) **In the case when $p = -2$ find the gradient of the line of INVARIANT POINTS under T**
[3 marks]

9 (c) **Show that $p = 3$ is the only real value of p for which M is singular. [6 marks]**

[Turn over]

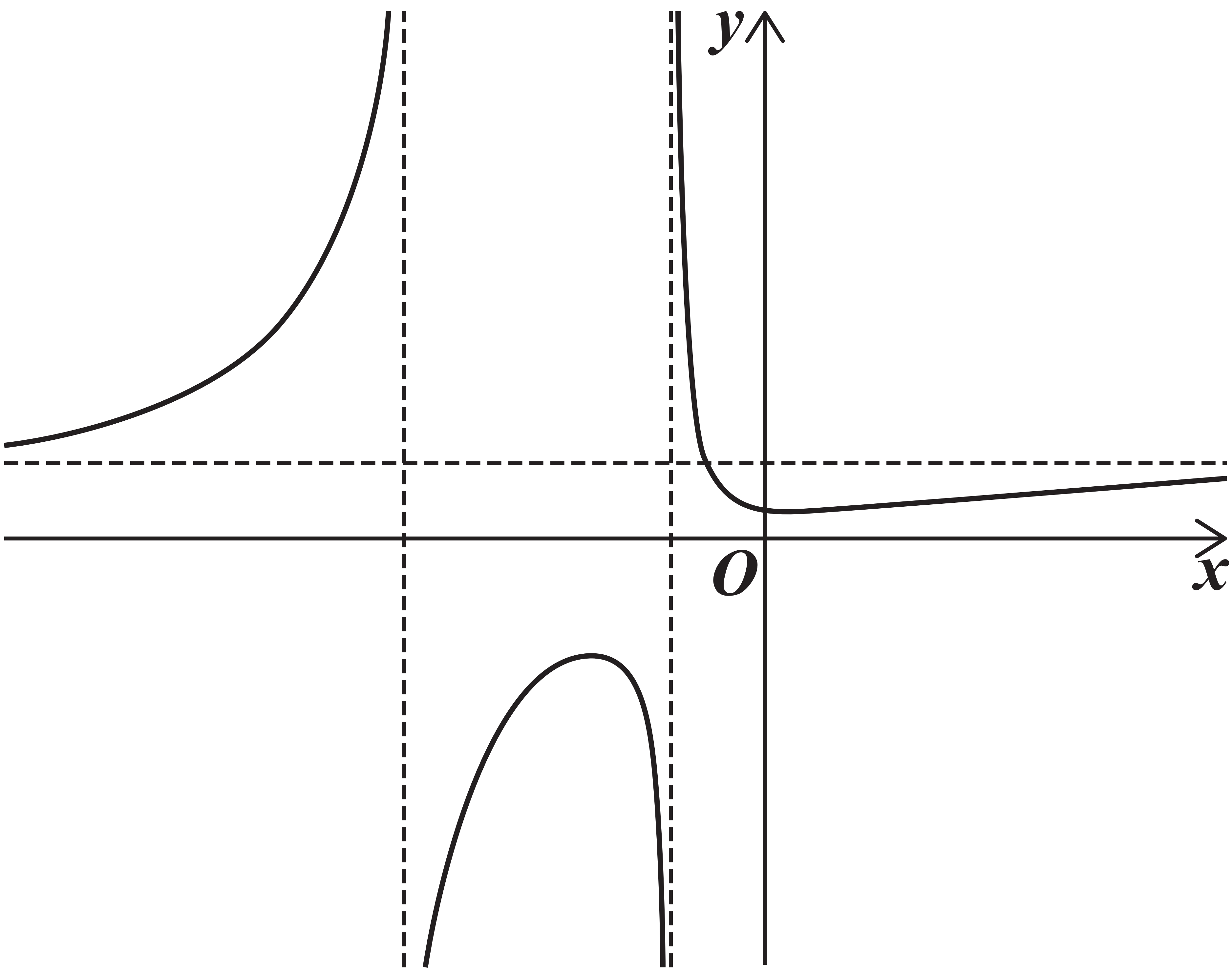
10 The curve C has equation

$$y = \frac{3x^2 + mx + p}{x^2 + px + m}$$

where m and p are integers.

The vertical asymptotes of C are $x = -4$ and $x = -1$

The curve C is shown in the diagram below.



10 (a) **Write down the equation of the horizontal asymptote of *C***
[1 mark]

10 (b) **Find the value of m and the value of p [2 marks]**

10 (c) **Hence, or otherwise, write down the coordinates of the y-intercept of C [1 mark]**

10 (d) **Without using calculus, show that the line $y = -1$ does not intersect *C* [5 marks]**

[Turn over]

[Turn over]

11 A point has Cartesian coordinates (x, y) and polar coordinates (r, θ) where $r \geq 0$ and $-\pi < \theta \leq \pi$

11 (a) Express r in terms of x and y
[1 mark]

11 (b) Express x in terms of r and θ
[1 mark]

11 (c) The curve C_1 has the polar equation

$$r(2 + \cos\theta) = 1 \qquad -\pi < \theta \leq \pi$$

11 (c) (i) Show that the Cartesian equation of C_1 can be written as

$$ay^2 = (1 + bx)(1 + x)$$

where a and b are integers to be determined. [4 marks]

11 (c) (ii) The curve C_2 has the Cartesian equation

$$ax^2 = (1 + by)(1 + y)$$

where a and b take the same values as in part (c)(i).

Describe fully a single transformation that maps the curve C_1 onto the curve C_2
[2 marks]

12 (a) **Show that** $(1 + i)^4 = -4$
[3 marks]

12 (b) **The function f is defined by**

$$f(z) = z^4 + 3z^2 - 6z + 10 \qquad z \in \mathbb{C}$$

12 (b) (i) **Show that $(1 + i)$ is a root of $f(z) = 0$ [2 marks]**

12 (b) (ii) Hence write down another root of $f(z) = 0$ [1 mark]

[Turn over]

12 (b) (iii) One of the linear factors of $f(z)$ is

$$\left(z - (1 + i)\right)$$

Write down another linear factor and hence, or otherwise, find a quadratic factor of $f(z)$ with real coefficients.
[3 marks]

12 (b) (iv) Find another quadratic factor of $f(z)$ with real coefficients.
[2 marks]

[Turn over]

12 (b) (v) Hence explain why the graph of $y = \mathrm{f}(x)$ does not intersect the x-axis. [2 marks]

13 (a) **Prove by induction that, for all integers $n \geq 1$,**

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

[4 marks]

[Turn over]

13 (b) **Hence, or otherwise, write down a factorised expression for the sum of the first $2n$ squares**

$$1^2 + 2^2 + 3^2 + \ldots + (2n)^2$$

[1 mark]

13 (c) **Use the formula in part (a) to write down a factorised expression for the sum of the first n even squares**

$$2^2 + 4^2 + 6^2 + \ldots + (2n)^2$$

[2 marks]

[Turn over]

13 (d) Hence, or otherwise, show that the sum of the first n odd squares is

$$an(bn - 1)(bn + 1)$$

where a and b are rational numbers to be determined.
[3 marks]

[Turn over]

14 **The inequality**

$$(x^2 - 5x - 24)(x^2 + 7x + a) < 0$$

has the solution set

$$\{x : -9 < x < -3\} \cup \{x : 2 < x < b\}$$

Find the values of integers a and b [4 marks]

END OF QUESTIONS

Additional page, if required. Write the question numbers in the left-hand margin.

Additional page, if required. Write the question numbers in the left-hand margin.

Additional page, if required. Write the question numbers in the left-hand margin.

BLANK PAGE

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
TOTAL	

G/TI/Jun23/7366/1/E2

236A7366/1