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A-level

FURTHER MATHEMATICS

Paper 1

7367/1

Thursday 25 May 2023 Afternoon

Time allowed: 2 hours

At the top of the page, write your surname and forename(s), your centre number, your candidate number and add your signature.



MATERIALS

For this paper you must have:

- the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics
- a graphical or scientific calculator that meets the requirements of the specification.

INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question. Do NOT write outside the box around each page or on blank pages.
- If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.



INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO



Answer ALL questions in the spaces provided.

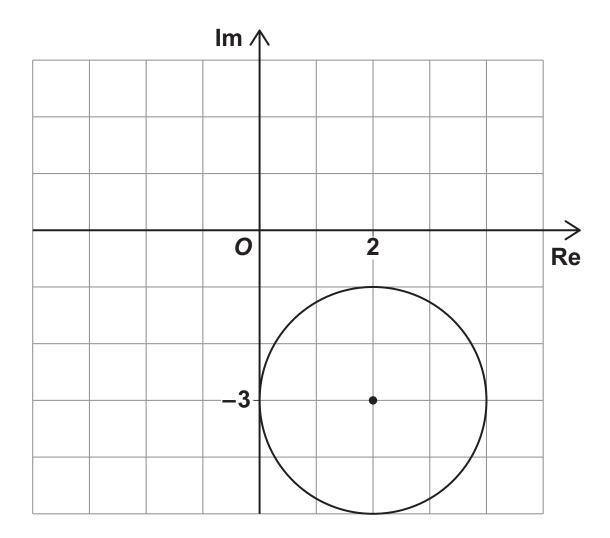
Find the number of solutions of the equation tanh x = cosh x

Circle your answer. [1 mark]

0 1 2 3



The diagram below shows a locus on an 2 Argand diagram.



Which of the equations below represents the locus shown above?

Circle your answer. [1 mark]

$$|z-2+3i|=2$$

$$|z-2+3i|=2$$
 $|z+2-3i|=2$

$$|z-2+3i|=4$$
 $|z+2-3i|=4$

$$|z + 2 - 3i| = 4$$



The matrix
$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$
 represents a

transformation.

Which one of the points below is an invariant point under this transformation?

Circle your answer. [1 mark]

$$(1, 1)$$
 $(0, 2)$ $(3, 0)$ $(2, 1)$

4 The solution of a second order differential equation is f(t)

> The differential equation models heavy damping.

Which one of the statements below could be true?

Tick (✓) ONE box. [1 mark]

$$f(t) = 2e^{-t}\cos(3t) + 5e^{-t}\sin(3t)$$

$$f(t) = 3e^{-t} + 4te^{-t}$$

$$f(t) = 7e^{-t} + 2e^{-2t}$$



$$f(t) = 8e^{-t}\cos(3t - 0.1)$$



5 The function f is defined	by
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$$f(r) = 2^r(r-2) \qquad (r \in \mathbb{Z})$$

5(a) Show that

$$f(r + 1) - f(r) = r2^r$$
 [2 marks]



5 (b)	Use the method of differences to show that
	$\sum_{r=1}^{n} r 2^{r} = 2^{n+1}(n-1) + 2 $ [4 marks]





6 The matrix M is given by

$$M = \frac{1}{10} \begin{bmatrix} a & a & -6 \\ 0 & 10 & 0 \\ 9 & 14 & -13 \end{bmatrix}$$

where a is a real number.

The vectors v_1 , v_2 , and v_3 are eigenvectors of \boldsymbol{M}

The corresponding eigenvalues are λ_1 , λ_2 , and λ_3 respectively.

It is given that $\lambda_2 = 1$ and $v_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$,

$$\mathbf{v_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$
 and $\mathbf{v_3} = \begin{bmatrix} c \\ 0 \\ 1 \end{bmatrix}$, where c is

an integer.

6(a) (i) Find the value of λ_1 [2 marks]



6 (a) (ii)	Find the value of <i>a</i> [2 marks]



6(b)	Find the integer c and the value of λ_3 [4 marks]



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6(c)	Find matrices U, D and U ⁻¹ , such that D is diagonal and $M = UDU^{-1}$ [3 marks]



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by

$$f(x) = \left| \sin x + \frac{1}{2} \right| \qquad (0 \le x \le 2\pi)$$

Find the set of values of x for which

$$f(x) \ge \frac{1}{2}$$

Give your answer in set notation. [5 marks]

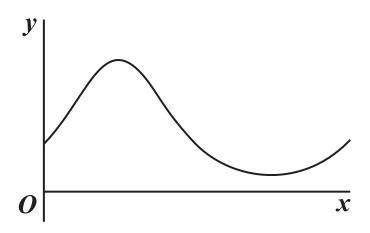




8 The function g is defined by

$$g(x) = e^{\sin x} \qquad (0 \le x \le 2\pi)$$

The diagram below shows the graph of y = g(x)



8(a) Find the x-coordinate of each of the stationary points of the graph of y = g(x), giving your answers in exact form. [1 mark]

8 (b)	Use Simpson's rule with 3 ordinates to estimate
	$\int_0^\pi g(x) dx$
	giving your answer to two decimal places. [3 marks]



8(c)	Explain how Simpson's rule could be used to find a more accurate estimate of the integral in part (b). [1 mark]					



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9	The position vectors of the points A, B and C are
	a = 2i + j + 2k
	b = -i - 8j + 2k
	c = -2j
	respectively.
9 (a)	Find the area of the triangle ABC [4 marks]





9 (b)	The points A , B and C all lie in the plane Π				
	Find an equation of the plane Π , in the form $\mathbf{r} \bullet \mathbf{n} = d$ [2 marks]				



9 (c)	The point P has position vector $p = i + 4j + 2k$
	Find the exact distance of ${\it P}$ from Π [3 marks]



10	The	matrix	M i	s defined	as
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$$\mathbf{M} = \begin{bmatrix} 2 & -1 & 1 \\ -1 & -1 & -2 \\ 1 & 2 & c \end{bmatrix}$$

where c is a real number.

10(a) The linear transformation T is represented by the matrix M

Show that, for one particular value of c, the image under T of every point lies in the plane

$$x + 5y + 3z = 0$$

State the value of c for which this occurs. [3 marks]



10 (b)	It is given that M is a non-singular matrix.
10 (b) (i)	State any restrictions on the value of c [2 marks]



10 (b) (ii)	Find M^{-1} in terms of c [4 marks]



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10(b)(iii) Using your answer from part (b)(ii), solve

$$2x - y + z = -3$$

$$-x - y - 2z = -6$$

$$x + 2y + 4z = 13$$

[3 marks]





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11	The function f is defined by
	$f(x) = 4x^3 - 8x^2 - 51x - 45 \qquad (x \in \mathbb{R})$
11 (a) (i)	Fully factorise f(x) [2 marks]



11 (a) (ii)	Hence, solve the inequality $f(x) < 0$ [2 marks]



11 (b)	The graph of $y = f(x)$ is translated by
	the vector $\begin{bmatrix} 7 \\ 0 \end{bmatrix}$
	The new graph is then reflected in the x -axis, to give the graph of $y = g(x)$
	Solve the inequality $g(x) \le 0$ [3 marks]
	•



12 (a)	Starting from the identities for $\sinh 2x$ and
	$\cosh 2x$, prove the identity

$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$

[2 marks]					



12 (b) (i)	The function f is defined by
	$f(x) = \tanh x \qquad (x > 0)$
	State the range of f [1 mark]



12 (b) (ii)	Use part (a) and part (b)(i) to prove that $\tanh 2x > \tanh x$ if $x > 0$ [3 marks]



13 Use l'Hôpital's rule to prove that

$$\lim_{x \longrightarrow \pi} \left(\frac{x \sin 2x}{\cos \left(\frac{x}{2} \right)} \right) = -4\pi$$

[5 marks]			
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14	The or	150 C	hac	nolar	aguation
14	THE CU	II VE C	11a5	polai	equation

$$r = \frac{4}{5 + 3\cos\theta} \qquad (-\pi < 0 \le \pi)$$

14(a)	Show that r takes values in the range $\frac{1}{k} \le r \le k$,
	where k is an integer. [2 marks]







14(c)	The ellipse <i>E</i> has	equation
	0	

$$y^2 + \frac{16x^2}{25} = 1$$

Find of E	the tra	nsfor [4 m	matio arks]	n that	maps	the gi	raph

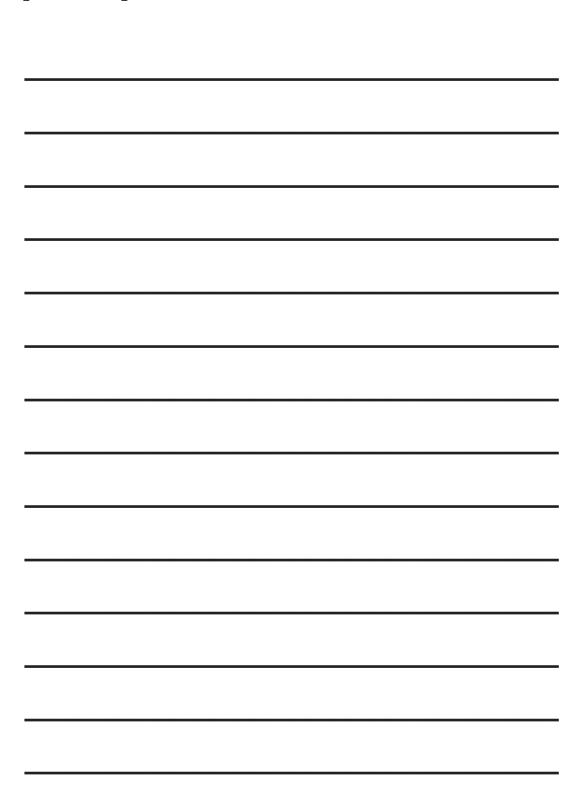




15	Find the general solution of the differential
	equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 3\frac{\mathrm{d}y}{\mathrm{d}x} - 4y = \cos 2x + 5x$$

[9 marks]















6 (a)	Show that
O (O.)	
	$\int_{0.5}^{4} \frac{1}{t} \ln t \mathrm{d}t = a (\ln 2)^2$
	$\int_{0.5} t^{mt} dt \qquad a(m2)$
	where a is a rational number to be found.
	[4 marks]



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16(b)	A curve C is defined parametrically for
	t > 0 by

$$x = 2t y = \frac{1}{2}t^2 - \ln t$$

The arc formed by the graph of C from t=0.5 to t=4 is rotated through 2π radians about the x-axis to generate a surface with area S

Find the exact value of S, giving your answer in the form

$$S = \pi (b + c \ln 2 + d(\ln 2)^2)$$

where b, c and d are rational numbers to be found. [7 marks]



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END OF QUESTIONS



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