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I declare this is my own work.

A-level

FURTHER MATHEMATICS

Paper 1

7367/1

Thursday 25 May 2023 Afternoon

Time allowed: 2 hours

At the top of the page, write your surname and forename(s), your centre number, your candidate number and add your signature.

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MATERIALS

For this paper you must have:

- **the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics**
- **a graphical or scientific calculator that meets the requirements of the specification.**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question. Do NOT write outside the box around each page or on blank pages.**
- **If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).**
- **Show all necessary working; otherwise marks for method may be lost.**
- **Do all rough work in this book. Cross through any work that you do not want to be marked.**



INFORMATION

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 100.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO



Answer ALL questions in the spaces provided.

- 1 Find the number of solutions of the equation $\tanh x = \cosh x$**

Circle your answer. [1 mark]

0

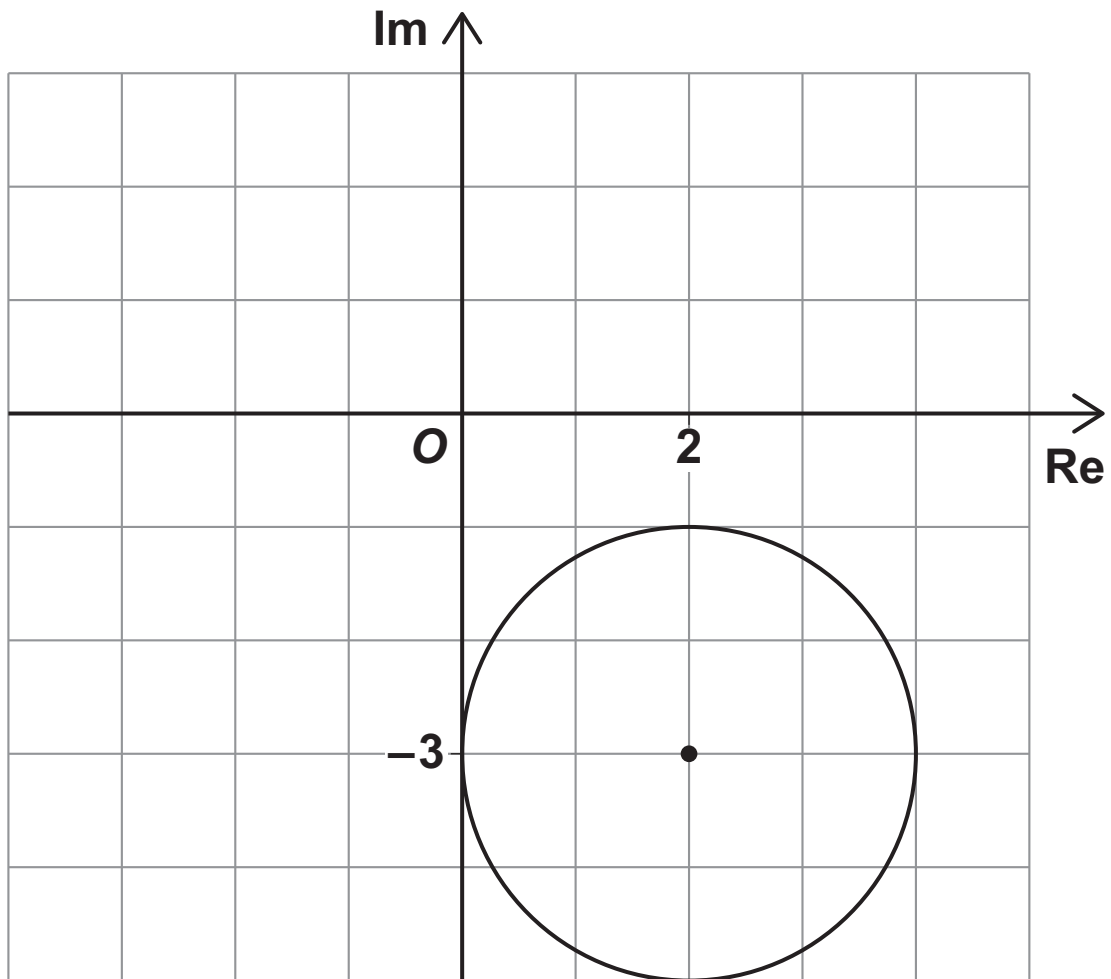
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3



- 2 The diagram below shows a locus on an Argand diagram.



Which of the equations below represents the locus shown above?

Circle your answer. [1 mark]

$$|z - 2 + 3i| = 2$$

$$|z + 2 - 3i| = 2$$

$$|z - 2 + 3i| = 4$$

$$|z + 2 - 3i| = 4$$

[Turn over]



- 3 The matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ represents a transformation.

Which one of the points below is an invariant point under this transformation?

Circle your answer. [1 mark]

(1, 1) (0, 2) (3, 0) (2, 1)

- 4 The solution of a second order differential equation is $f(t)$

The differential equation models heavy damping.

Which one of the statements below could be true?

Tick (✓) ONE box. [1 mark]

$$f(t) = 2e^{-t} \cos(3t) + 5e^{-t} \sin(3t)$$

$$f(t) = 3e^{-t} + 4te^{-t}$$

$$f(t) = 7e^{-t} + 2e^{-2t}$$

$$f(t) = 8e^{-t} \cos(3t - 0.1)$$



[Turn over]



6 The matrix M is given by

$$M = \frac{1}{10} \begin{bmatrix} a & a & -6 \\ 0 & 10 & 0 \\ 9 & 14 & -13 \end{bmatrix}$$

where a is a real number.

The vectors v_1 , v_2 , and v_3 are eigenvectors of M

The corresponding eigenvalues are λ_1 , λ_2 , and λ_3 respectively.

It is given that $\lambda_2 = 1$ and $v_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$,

$v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $v_3 = \begin{bmatrix} c \\ 0 \\ 1 \end{bmatrix}$, where c is

an integer.

6(a) (i) Find the value of λ_1 [2 marks]



6(a) (ii) Find the value of a [2 marks]

[Turn over]



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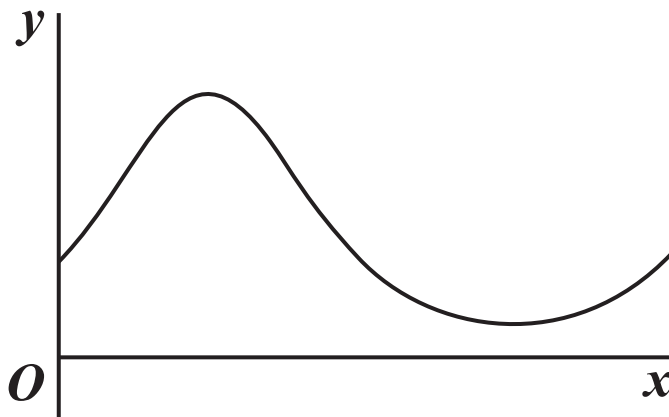
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8 The function g is defined by

$$g(x) = e^{\sin x} \quad (0 \leq x \leq 2\pi)$$

The diagram below shows the graph of $y = g(x)$



8(a) Find the x -coordinate of each of the stationary points of the graph of $y = g(x)$, giving your answers in exact form. [1 mark]



8(b) Use Simpson's rule with 3 ordinates to estimate

$$\int_0^{\pi} g(x) dx$$

giving your answer to two decimal places.
[3 marks]

[Turn over]



- 8(c) Explain how Simpson's rule could be used to find a more accurate estimate of the integral in part (b). [1 mark]



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9(b) The points A , B and C all lie in the plane Π

Find an equation of the plane Π , in the form $\mathbf{r} \cdot \mathbf{n} = d$ [2 marks]



10 The matrix M is defined as

$$M = \begin{bmatrix} 2 & -1 & 1 \\ -1 & -1 & -2 \\ 1 & 2 & c \end{bmatrix}$$

where c is a real number.

10(a) The linear transformation T is represented by the matrix M

Show that, for one particular value of c , the image under T of every point lies in the plane

$$x + 5y + 3z = 0$$

State the value of c for which this occurs.
[3 marks]



10(b) It is given that M is a non-singular matrix.

10(b) (i) State any restrictions on the value of c
[2 marks]

[Turn over]



10(b) (iii) Using your answer from part (b)(ii), solve

$$2x - y + z = -3$$

$$-x - y - 2z = -6$$

$$x + 2y + 4z = 13$$

[3 marks]

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11 The function f is defined by

$$f(x) = 4x^3 - 8x^2 - 51x - 45 \quad (x \in \mathbb{R})$$

11 (a) (i) Fully factorise $f(x)$ [2 marks]



11 (a) (ii) Hence, solve the inequality $f(x) < 0$ [2 marks]

[Turn over]



12(a) Starting from the identities for $\sinh 2x$ and $\cosh 2x$, prove the identity

$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$

[2 marks]

[Turn over]



12(b) (i) The function f is defined by

$$f(x) = \tanh x \quad (x > 0)$$

State the range of f [1 mark]



[Turn over]



14 The curve C has polar equation

$$r = \frac{4}{5 + 3 \cos \theta} \quad (-\pi < \theta \leq \pi)$$

14(a) Show that r takes values in the range $\frac{1}{k} \leq r \leq k$, where k is an integer. [2 marks]



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