A

Surname ____________________

Forename(s) ____________________

Centre Number ____________________

Candidate Number ____________________

Candidate Signature ____________________

I declare this is my own work.

A-level

FURTHER MATHEMATICS

Paper 1

7367/1

Thursday 25 May 2023 Afternoon

Time allowed: 2 hours

At the top of the page, write your surname and forename(s), your centre number, your candidate number and add your signature.

[Turn over]

JUN237367101

MATERIALS

For this paper you must have:

- the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics
- a graphical or scientific calculator that meets the requirements of the specification.

INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided. Do NOT write on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).

- **Show all necessary working; otherwise, marks for method may be lost.**
- **Do all rough work in this book. Cross through any work you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 100.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 **Find the number of solutions of the equation tanh x = cosh x**

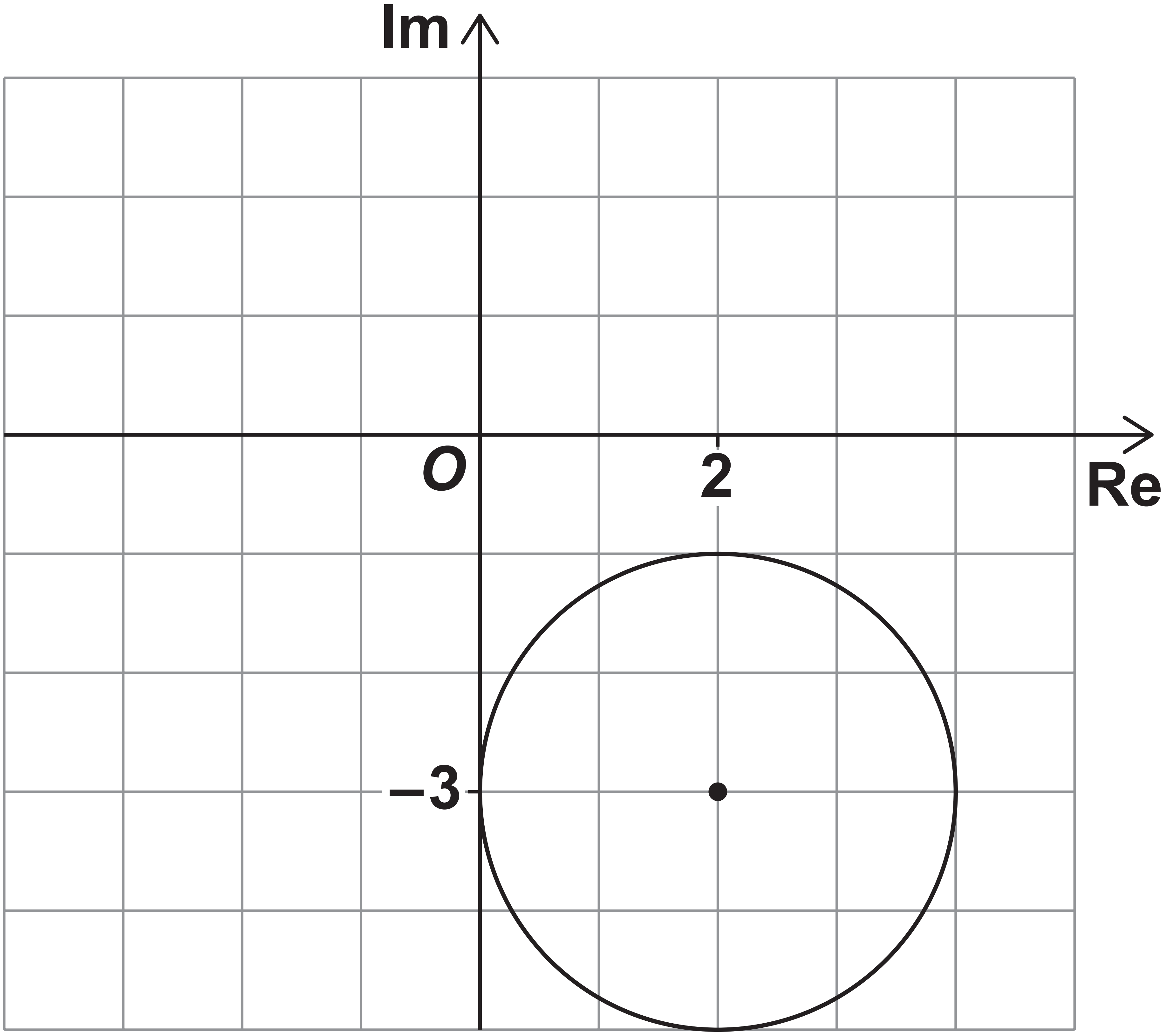
Circle your answer. [1 mark]

0 1 2 3

BLANK PAGE

[Turn over]

2 **The diagram below shows a locus on an Argand diagram.**



Which of the equations below represents the locus shown on the opposite page?

Circle your answer. [1 mark]

$|z - 2 + 3\mathrm{i}| = 2$

$|z + 2 - 3\mathrm{i}| = 2$

$|z - 2 + 3\mathrm{i}| = 4$

$|z + 2 - 3\mathrm{i}| = 4$

[Turn over]

3 The matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

represents a transformation.

Which one of the points below is an invariant point under this transformation?

Circle your answer. [1 mark]

(1, 1)

(0, 2)

(3, 0)

(2, 1)

4 **The solution of a second order differential equation is $\mathrm{f}(t)$**

The differential equation models heavy damping.

Which one of the statements below could be true?

Tick (✓) ONE box. [1 mark]

☐ $\mathrm{f}(t) = 2\mathrm{e}^{-t} \cos(3t) + 5\mathrm{e}^{-t} \sin(3t)$

☐ $\mathrm{f}(t) = 3\mathrm{e}^{-t} + 4t\mathrm{e}^{-t}$

☐ $\mathrm{f}(t) = 7\mathrm{e}^{-t} + 2\mathrm{e}^{-2t}$

☐ $\mathrm{f}(t) = 8\mathrm{e}^{-t} \cos(3t - 0.1)$

5 **The function f is defined by**

$$f(r) = 2^r(r - 2) \qquad (r \in \mathbb{Z})$$

5 (a) **Show that**

$$f(r + 1) - f(r) = r2^r$$ **[2 marks]**

5 (b) Use the method of differences to show that

$$\sum_{r=1}^{n} r2^r = 2^{n+1}(n-1) + 2$$

[4 marks]

6 The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \frac{1}{10}\begin{bmatrix} a & a & -6 \\ 0 & 10 & 0 \\ 9 & 14 & -13 \end{bmatrix}$$

where a is a real number.

The vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are eigenvectors of $\mathbf{M}$

The corresponding eigenvalues are λ_1, λ_2, and λ_3 respectively.

It is given that $\lambda_2 = 1$

and $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

and $\mathbf{v}_3 = \begin{bmatrix} c \\ 0 \\ 1 \end{bmatrix}$, where c is an integer.

6 (a) (i) Find the value of λ_1 [2 marks]

6 (a) (ii) Find the value of a [2 marks]

[Turn over]

6 (b) **Find the integer c and the value of λ_3 [4 marks]**

[Turn over]

6 (c) **Find matrices U, D and U^{-1}, such that D is diagonal and $M = UDU^{-1}$ [3 marks]**

[Turn over]

7 The function f is defined by

$$f(x) = \left| \sin x + \frac{1}{2} \right| \qquad (0 \le x \le 2\pi)$$

Find the set of values of x for which

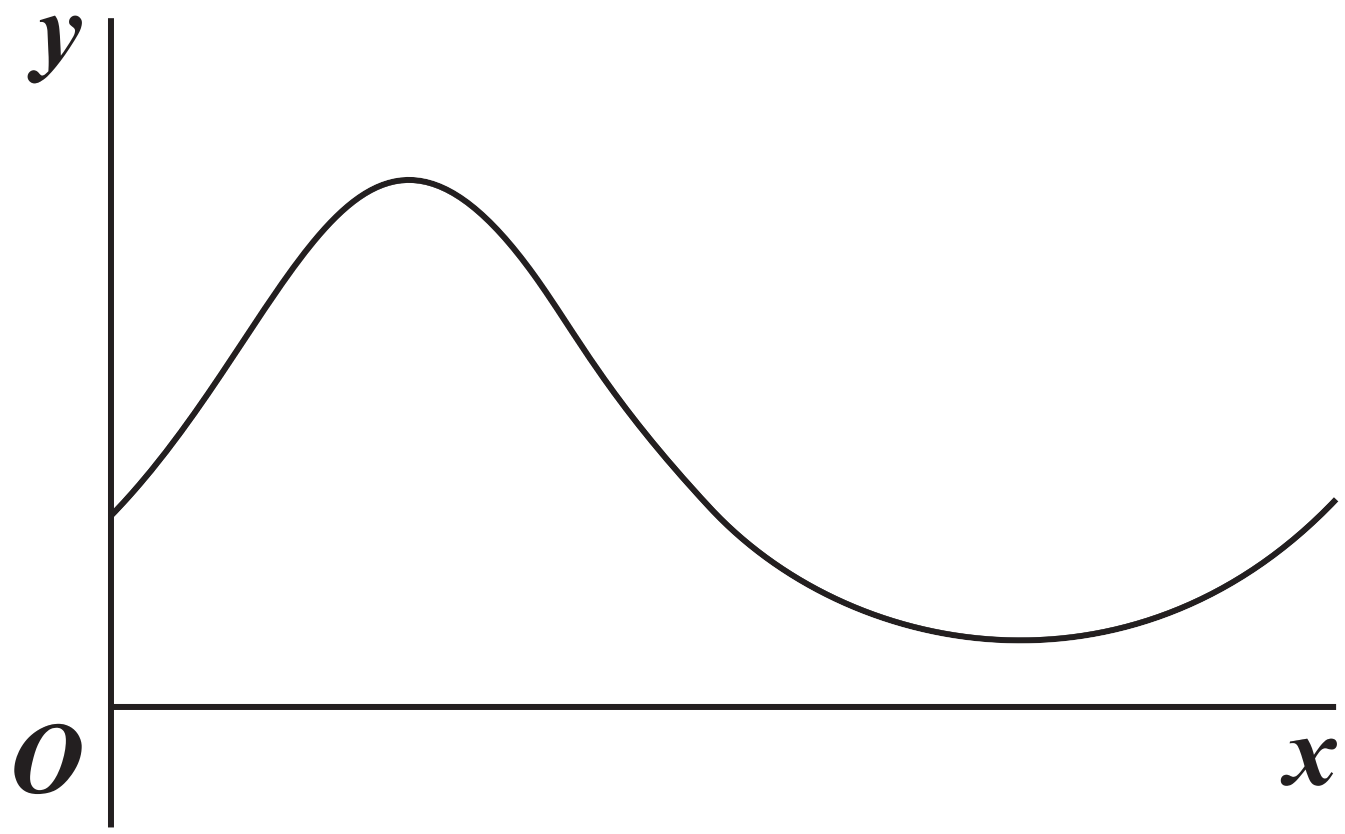
$$f(x) \ge \frac{1}{2}$$

Give your answer in set notation. [5 marks]

8 **The function g is defined by**

$$g(x) = e^{\sin x} \qquad (0 \le x \le 2\pi)$$

The diagram below shows the graph of $y = g(x)$



8 (a) **Find the x-coordinate of each of the stationary points of the graph of $y = \mathrm{g}(x)$, giving your answers in exact form. [1 mark]**

[Turn over]

8 (b) Use Simpson's rule with 3 ordinates to estimate

$$\int_0^{\pi} g(x)\,dx$$

giving your answer to two decimal places. [3 marks]

8 (c) **Explain how Simpson's rule could be used to find a more accurate estimate of the integral in part (b). [1 mark]**

____

____

____

[Turn over]

9 **The position vectors of the points *A*, *B* and *C* are**

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$$

$$\mathbf{b} = -\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{c} = -2\mathbf{j}$$

respectively.

9 (a) **Find the area of the triangle *ABC* [4 marks]**

9 (b) **The points A, B and C all lie in the plane Π**

Find an equation of the plane Π, in the form $\mathbf{r} \bullet \mathbf{n} = d$ [2 marks]

9 (c) **The point *P* has position vector $\mathbf{p} = \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$**

Find the exact distance of *P* from Π [3 marks]

[Turn over]

10 The matrix M is defined as

$$\mathbf{M} = \begin{bmatrix} 2 & -1 & 1 \\ -1 & -1 & -2 \\ 1 & 2 & c \end{bmatrix}$$

where c is a real number.

10 (a) The linear transformation T is represented by the matrix M

Show that, for one particular value of c, the image under T of every point lies in the plane

$$x + 5y + 3z = 0$$

State the value of c for which this occurs. [3 marks]

10 (b) It is given that M is a non-singular matrix.

10 (b) (i) State any restrictions on the value of c [2 marks]

[Turn over]

10 (b) (ii) Find M^{-1} in terms of c

[4 marks]

[Turn over]

10 (b) (iii) Using your answer from part (b)(ii), solve

$$2x - y + z = -3$$

$$-x - y - 2z = -6$$

$$x + 2y + 4z = 13$$ **[3 marks]**

[Turn over]

11 **The function f is defined by**

$$\mathrm{f}(x) = 4x^3 - 8x^2 - 51x - 45 \qquad (x \in \mathbb{R})$$

11 (a) (i) Fully factorise $\mathrm{f}(x)$ [2 marks]

11 (a) (ii) Hence, solve the inequality $f(x) < 0$ [2 marks]

[Turn over]

11 (b) The graph of $y = \mathrm{f}(x)$ is translated by

the vector $\begin{bmatrix} 7 \\ 0 \end{bmatrix}$

The new graph is then reflected in the x-axis, to give the graph of $y = \mathrm{g}(x)$

Solve the inequality $\mathrm{g}(x) \leq 0$
[3 marks]

[Turn over]

12 (a) **Starting from the identities for sinh $2x$ and cosh $2x$, prove the identity**

$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$ **[2 marks]**

12 (b) (i) The function f is defined by

$$\mathrm{f}(x) = \tanh x \qquad (x > 0)$$

State the range of f [1 mark]

[Turn over]

12 (b) (ii) **Use part (a) and part (b)(i) to prove that $\tanh 2x > \tanh x$ if $x > 0$ [3 marks]**

[Turn over]

13 **Use l'Hôpital's rule to prove that**

$$\lim_{x \to \pi}\left(\frac{x\sin 2x}{\cos\left(\frac{x}{2}\right)}\right) = -4\pi$$

[5 marks]

14 **The curve C has polar equation**

$$r = \frac{4}{5 + 3\cos\theta} \qquad (-\pi < 0 \leq \pi)$$

14 (a) **Show that r takes values in the range $\frac{1}{k} \leq r \leq k$, where k is an integer. [2 marks]**

14 (b) **Find the Cartesian equation of *C* in the form $y^2 = f(x)$**
[4 marks]

14 (c) **The ellipse *E* has equation**

$$y^2 + \frac{16x^2}{25} = 1$$

Find the transformation that maps the graph of *E* onto *C*
[4 marks]

15 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = \cos 2x + 5x$$

[9 marks]

[Turn over]

16 (a) Show that

$$\int_{0.5}^{4} \frac{1}{t} \ln t \, dt = a(\ln 2)^2$$

where a is a rational number to be found. [4 marks]

16 (b) A curve C is defined parametrically for $t > 0$ by

$$x = 2t \qquad\qquad y = \frac{1}{2}t^2 - \ln t$$

The arc formed by the graph of C from $t = 0.5$ to $t = 4$ is rotated through 2π radians about the x-axis to generate a surface with area S

Find the exact value of S, giving your answer in the form

$$S = \pi\left(b + c\ln 2 + d(\ln 2)^2\right)$$

where b, c and d are rational numbers to be found. [7 marks]

___

___

___

___

END OF QUESTIONS

BLANK PAGE

Additional page, if required. Write the question numbers in the left-hand margin.

Additional page, if required. Write the question numbers in the left-hand margin.

Additional page, if required. Write the question numbers in the left-hand margin.

BLANK PAGE

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
TOTAL	