A-LEVEL
FURTHER MATHEMATICS

7367/1 Paper 1
Report on the Examination

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General

Overall, the paper had a variety of questions of a similar length. This allowed students to demonstrate their skills across a wide range of the specification. There were plenty of places for both basic skills to be demonstrated and deeper understanding to be shown. Students continue to show an increasing ability to tackle the vast variety of topics and a number of different techniques were on display for many questions.

Question 1

Students who did a sketch, either in the booklet or on their calculator, or used their calculator to try to find a solution had no problem with this question and as such it was done very well indeed.

Question 2

This was a simple locus on an Argand diagram question, which was very well done with nearly every student remembering that $|z-a|=r$

Question 3

Nearly all students remembered what an invariant point is and got the correct answer.

Question 4

The link between the solutions of a second order differential equation and the type of damping that it models turned out to be a more challenging concept than had been expected. The first and last answers, after rearrangement, have the same structure and so could be ruled out by logic and the second answer relates to critical damping. This leaves the third answer as the correct one.

Question 5

This was a straightforward method of differences question, but throughout it some students demonstrated a poor understanding of basic laws of indices.

In part (a) many students dropped a mark as they forgot to include $\mathrm{f}(r+1)-\mathrm{f}(r)$ when showing that $\mathrm{f}(r+1)-\mathrm{f}(r)=r2^r$

Part (b) was very well done with more students including $\sum_{r=1}^{n} r2^r$ when showing that

$\sum_{r=1}^{n} r2^r = 2^{n+1}(n-1)+2$ than the equivalent in part (a). A few students forgot to include $r=n-1$ in their expansion of the sum, not realising that a fully correct proof needs to include cancellation at the end as well as at the beginning.

Question 6

This question involved the repeated use of the definition of an eigenvector and its associated eigenvalue, $\mathbf{Mv} = \lambda\mathbf{v}$ and most students struggled to complete it successfully.

In part (a) using $\mathbf{Mv}_1 = \lambda_1\mathbf{v}_1$ gives the value of λ_1 using the z component and using this value, the value of a can be obtained using the x-component.
In an alternative approach, the value of a could be obtained by using the x-component of $\mathbf{Mv}_2 = \lambda_2\mathbf{v}_2$ then putting this into $\mathbf{M}$ and using the x-component of $\mathbf{Mv}_1 = \lambda_1\mathbf{v}_1$ the value of λ_1 can be obtained.

In part (b) there are many approaches available, including:

- Using $\mathbf{Mv}_3 = \lambda_3\mathbf{v}_3$ to obtain two equations in λ_3 and c which can be solved simultaneously
- Using the characteristic equation to obtain a cubic equation in λ which can be solved to give λ_3. This can then be used in $\mathbf{Mv}_3 = \lambda_3\mathbf{v}_3$ to obtain c.

Some students used methods which are not on the specification such as the trace of $\mathbf{M}$ being equal to the sum of its eigenvalues and full credit was given when this was done correctly.
Part (c) was done extremely well, with students generally obtaining $\mathbf{U}^{-1}$ from $\mathbf{U}$ using their calculator as expected.

Question 7

Those who drew a sketch of the function generally fared better than those who did not. A number of students dropped the last mark as they forgot that $x = 2\pi$ is also a possible solution.

Question 8

This question was done very well, with most errors occurring in part (b) where some students confused 3 ordinates with 3 strips (4 ordinates). This gained no credit, as Simpson's rule cannot be used with an odd number of strips. Some students used 5 ordinates and could demonstrate a correct use of Simpson's rule for which they were given some credit.

Question 9

Students showed great skill in this question. Parts (a) and (b) were dealt with extremely well, though in part (a) some students made the mistake of using the given position vectors rather than direction vectors corresponding to sides of the triangle *ABC*.

In part (c) the most popular approach was to find the length of the perpendicular from the point *P* to the plane.

Question 10

Parts (a) and (b)(iii) were done very poorly, whilst parts (b)(i) and (b)(ii) were done very well.

In part (a) many students never showed that "the image under T of every point lies in the plane $x + 5y + 3z = 0$". Instead they found the value of c such that $\det(\mathbf{M}) = 0$: not what was asked for!

Others worked with $\mathbf{M}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ rather than $\mathbf{M}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$ which did not allow for a clear argument.

Students were well drilled in the work involved in finding $\mathbf{M}^{-1}$ and the restrictions on c in parts (b)(i) and (b)(ii).

In part (b)(iii) no credit was given for students who simply solved the equations using their calculator. The question was testing an understanding of the solution to the system of equations being given by $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{M}^{-1}\begin{bmatrix} -3 \\ -6 \\ 13 \end{bmatrix}$. The actual computation can, of course, be done on a calculator.

Question 11

Parts (a)(i) and (a)(ii) were both done well, with those drawing a sketch doing better in part (a)(ii) than those who did not.

In part (b), those who tried modifying the original cubic equation to show the effect of the transformations struggled, whilst those who drew a series of sketches for the transformations fared well.

Question 12

Part (a) was a straightforward question that was generally done very well. Some students took a longer route via $\dfrac{2\sinh x\cosh x}{1+2\sinh^2 x}$ but generally got to the result in the end. Those students who used the exponential definition of $\tanh 2x$ did not progress very far.

In part (b)(i) a large number of students missed that the domain was restricted to $x > 0$ and gave an incorrect answer of $-1 < \mathrm{f}(x) < 1$ or $|\mathrm{f}(x)| < 1$.

Part (b)(ii) was done poorly. When asked to "Use part (a) and part (b)(i) to prove that $\tanh 2x > \tanh x$" it is not appropriate to start at $\tanh 2x > \tanh x$ and work backwards to the result in part (b)(i). A few students also attempted to use trial and improvement.

Question 13

This question was generally done well, but many students forgot to explain why l'Hôpital's rule is necessary by evaluating $x\sin 2x$ and $\cos\left(\dfrac{x}{2}\right)$ at $x = \pi$ and showing that the limit is an indeterminate form. In addition, a few students differentiated $x\sin 2x$ incorrectly.

Question 14

In part (a) a few students forgot to write their final answer as $\frac{1}{2} \le r \le 2$, as requested in the question and failed to gain full credit .

The most common error in part (b) was poor squaring of the algebraic equation $5\sqrt{x^2+y^2}+3x=4$ to incorrectly obtain $25\left(x^2+y^2\right)+9x^2=16$.

Part (c) was more challenging, with students often finding it difficult to rearrange their answer to part (b) into the equation of an ellipse to allow comparison with the equation of *E*.

Question 15

This straightforward, but technically demanding question allowed almost all students to demonstrate some knowledge, usually obtaining the correct complementary function. Many showed their understanding of the particular integral, but the main error was not in remembering that $C\cos 2x + D\sin 2x$ was needed due to the presence of the single trigonometric function $\cos 2x$, but that the full linear polynomial $Ex+F$ was needed due to the presence of $5x$.

Question 16

In part (a) it was not appropriate to "Show that $\int_{0.5}^{4} \frac{1}{t}\ln t\,\mathrm{d}t = a\left(\ln 2\right)^2$" by using a calculator to find an approximate value of $\int_{0.5}^{4} \frac{1}{t}\ln t\,\mathrm{d}t$ and then divide it by another approximate value of $\left(\ln 2\right)^2$ to get the exact value of $a=\frac{3}{2}$ and such an approach gained no marks. Successful solutions were seen using integration by parts, substitution of $u=\ln t$, or inspection.

Although part (b) had some challenging integration in it, the first four marks were accessible to more than those who actually scored them. Students were often limited to one or two marks because of errors in basic differentiation and algebra, such as $\frac{\mathrm{d}y}{\mathrm{d}t}=2t-\frac{1}{t}$ or $\left(t-\frac{1}{t}\right)^2=t^2-2-\frac{1}{t^2}$.

To progress beyond the first two marks students needed to recognise that $4+t^2-2+\frac{1}{t^2}=\left(t+\frac{1}{t}\right)^2$

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.