A

Surname ______________________________

Forename(s) ______________________________

Centre Number ______________________________

Candidate Number ______________________________

Candidate Signature ______________________________

I declare this is my own work.

A-level

FURTHER MATHEMATICS

Paper 2

7367/2

Monday 5 June 2023 Afternoon

Time allowed: 2 hours

At the top of the page, write your surname and forename(s), your centre number, your candidate number and add your signature.

[Turn over]

MATERIALS

For this paper you must have:

- the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics
- a graphical or scientific calculator that meets the requirements of the specification.

INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided. Do NOT write on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).

- **Show all necessary working; otherwise, marks for method may be lost.**
- **Do all rough work in this book. Cross through any work you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 100.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 Given that $y = \sin x + \sinh x$,

find $\dfrac{d^2y}{dx^2} + y$

Circle your answer. [1 mark]

$2 \sin x$

$-2 \sin x$

$2 \sinh x$

$-2 \sinh x$

2 Which one of the expressions below is NOT equal to zero?

Circle your answer. [1 mark]

$\lim_{x \to \infty} (x^2 e^{-x})$

$\lim_{x \to 0} (x^5 \ln x)$

$\lim_{x \to \infty} \left(\frac{e^x}{x^5}\right)$

$\lim_{x \to 0} (x^3 e^x)$

[Turn over]

3 **The determinant** $A = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 3 & 2 & 1 \end{vmatrix}$

Which one of the determinants below has a value which is NOT equal to the value of A**?**

Tick (✓) ONE box. [1 mark]

☐ $\begin{vmatrix} 3 & 1 & 3 \\ 2 & 0 & 2 \\ 3 & 2 & 1 \end{vmatrix}$

☐ $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \\ 1 & 2 & 1 \end{vmatrix}$

☐ $\begin{vmatrix} 2 & 2 & 2 \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix}$

☐ $\begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 0 & 2 \end{vmatrix}$

[Turn over]

4 **It is given that**

$$f(x) = \cosh^{-1}(x - 3)$$

Which of the sets listed below is the greatest possible domain of the function f ?

Circle your answer. [1 mark]

$\{x : x \geq 4\}$

$\{x : x \geq 3\}$

$\{x : x \geq 1\}$

$\{x : x \geq 0\}$

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[Turn over]

5 Josh and Zoe are solving the following mathematics problem:

The curve C_1 has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

The matrix $\mathbf{M} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ maps C_1 onto C_2

Find the equations of the asymptotes of C_2

Josh says that to solve this problem you MUST first carry out the transformation on C_1 to find C_2, and then find the asymptotes of C_2

Zoe says that you will get the same answer if you first find the asymptotes of C_1, and then carry out the transformation on these asymptotes to obtain the asymptotes of C_2

Show that Zoe is correct.
[5 marks]

6 (a) Express $-5 - 5\mathrm{i}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $-\pi < \theta \leq \pi$
[2 marks]

[Turn over]

6 (b) **The point on an Argand diagram that represents $-5 - 5\mathrm{i}$ is one of the vertices of an equilateral triangle whose centre is at the origin.**

Find the complex numbers represented by the other two vertices of the triangle.

Give your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $-\pi < \theta \leq \pi$
[3 marks]

[Turn over]

7 Show that

$$\sum_{r=11}^{n+1} r^3 = \frac{1}{4}(n^2 + an + b)(n^2 + an + c)$$

where a, b and c are integers to be found. [3 marks]

[Turn over]

8 **A is a non-singular 2×2 matrix and $\mathbf{A}^{\mathrm{T}}$ is the transpose of A**

8 (a) **Using the result**

$(\mathbf{AB})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$

show that

$(\mathbf{A}^{-1})^{\mathrm{T}} = (\mathbf{A}^{\mathrm{T}})^{-1}$ **[3 marks]**

[Turn over]

8 (b) It is given that $A = \begin{bmatrix} 4 & 5 \\ -1 & k \end{bmatrix}$,

where k is a real constant.

8 (b) (i) Find $(A^{-1})^{T}$, giving your answer in terms of k [2 marks]

8 (b) (ii) State the restriction on the possible values of k [1 mark]

[Turn over]

9 The complex number z is such that

$$z = \frac{1 + \mathrm{i}}{1 - k\mathrm{i}}$$

where k is a real number.

9 (a) Find the real part of z and the imaginary part of z, giving your answers in terms of k
[2 marks]

9 (b) **In the case where $k = \sqrt{3}$, use part (a) to show that**

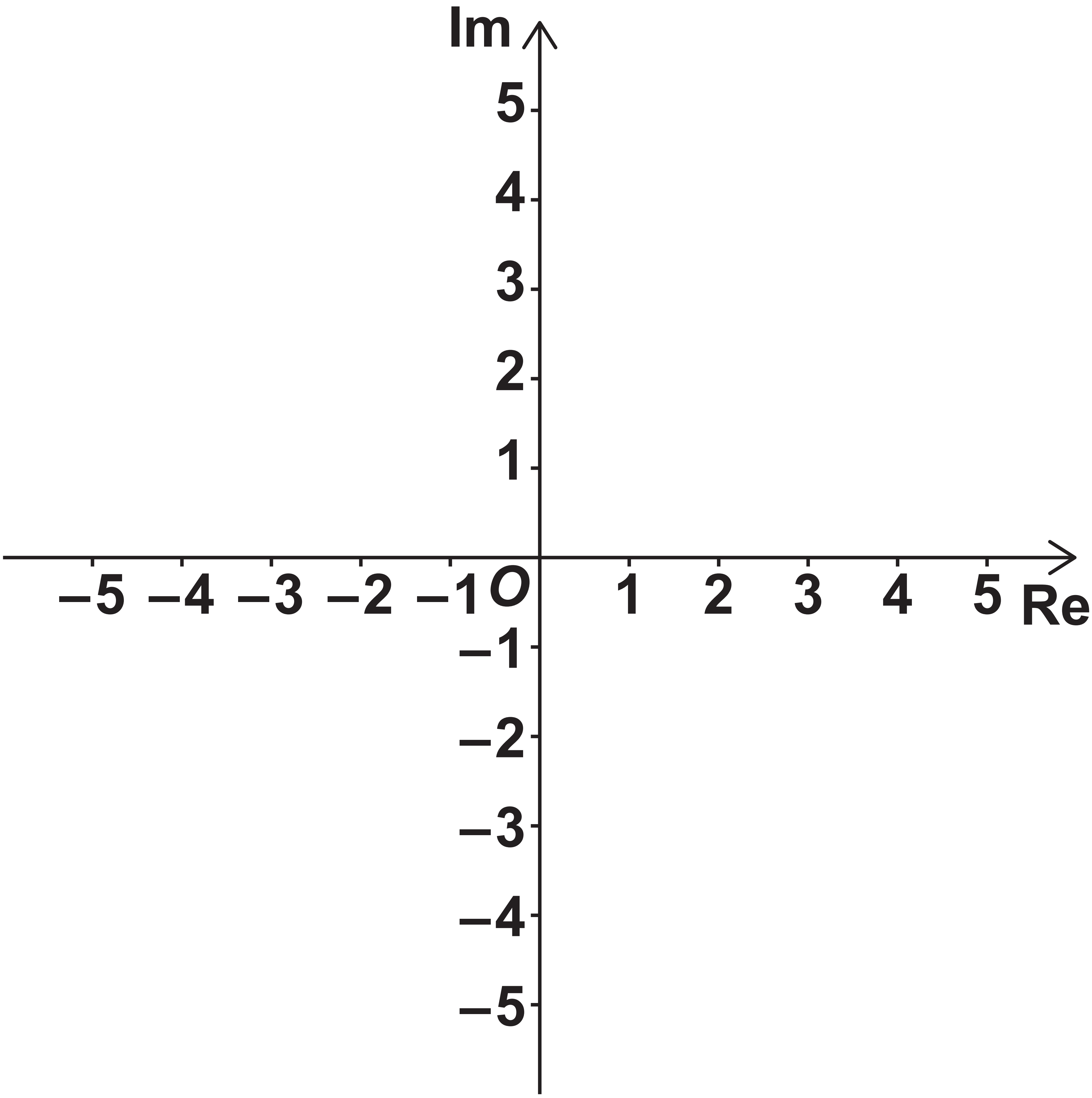
$$\cos\frac{7\pi}{12} = \frac{\sqrt{2} - \sqrt{6}}{4}$$ **[5 marks]**

10 **The region *R* on an Argand diagram satisfies**

both $|z + 2\mathrm{i}| \leq 3$

and $-\dfrac{\pi}{6} \leq \arg(z) \leq \dfrac{\pi}{2}$

10 (a) **Sketch *R* on the Argand diagram below. [3 marks]**



[Turn over]

10 (b) **Find the maximum value of $|z|$ in the region *R*, giving your answer in exact form. [5 marks]**

11 **The line l_1 passes through the points $A(6, 2, 7)$ and $B(4, -3, 7)$**

11 (a) **Find a Cartesian equation of l_1**
[2 marks]

11 (b) The line l_2 has vector equation

$$\mathbf{r} = \begin{bmatrix} 8 \\ 9 \\ c \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$ where c is

a constant.

11 (b) (i) Explain how you know that the lines l_1 and l_2 are not perpendicular. [2 marks]

11 (b) (ii) The lines l_1 and l_2 both lie in the same plane.

Find the value of c [5 marks]

12 The function f is defined by

$$\mathrm{f}(n) = 3^{3n+1} + 2^{3n+4} \qquad (n \in \mathbb{Z}^+)$$

Prove by induction that $\mathrm{f}(n)$ is divisible by 19 for $n \geq 1$
[6 marks]

[Turn over]

13 The quadratic equation $z^2 - 5z + 8 = 0$ has roots α and β

13 (a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$ [2 marks]

13 (b) **Without finding the value of α or the value of β, show that $\alpha^4 + \beta^4 = -47$ [4 marks]**

13 (c) **Find a quadratic equation, with integer coefficients, which has roots $\alpha^3 + \beta$ and $\beta^3 + \alpha$**
[5 marks]

[Turn over]

14 **The function f is defined by**

$$\mathrm{f}(x) = \frac{1}{4x^2 + 16x + 19} \qquad (x \in \mathbb{R})$$

14 (a) **Show, without using calculus, that the graph of $y = \mathrm{f}(x)$ has a stationary point at $\left(-2, \frac{1}{3}\right)$**

[3 marks]

[Turn over]

14 (b) Show that $\int_{-2}^{-\frac{1}{2}} f(x)\,dx = \frac{\pi\sqrt{3}}{18}$

[5 marks]

[Turn over]

14 (c) **Find the value of** $\int_{-2}^{\infty} \mathrm{f}(x)\,\mathrm{d}x$

Fully justify your answer.
[2 marks]

15 (a) **Given that $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that**

$$z^n - z^{-n} = 2\mathrm{i}\sin n\theta$$ **[2 marks]**

[Turn over]

15 (b) **The series S is defined as**

$$S = \sin\theta + \sin 3\theta + \ldots + \sin(2n-1)\theta$$

Use part (a) to express S in the form

$$S = \frac{1}{2\mathrm{i}}(G_1) - \frac{1}{2\mathrm{i}}(G_2)$$

where each of G_1 and G_2 is a geometric series. [3 marks]

[Turn over]

15 (c) **Hence, show that**

$$S = \frac{\sin^2 (n\theta)}{\sin \theta}$$ **[5 marks]**

16 **A bungee jumper of mass m kg is attached to an elastic rope.**

The other end of the rope is attached to a fixed point.

The bungee jumper falls vertically from the fixed point.

At time t seconds after the rope first becomes taut, the extension of the rope is x metres and the speed of the bungee jumper is v m s^{-1}

[Turn over]

16 (a) A model for the motion while the rope remains taut assumes that the forces acting on the bungee jumper are

- the weight of the bungee jumper
- a tension in the rope of magnitude kx newtons
- an air resistance force of magnitude Rv newtons

where k and R are constants such that $4km > R^2$

16 (a) (i) Show that this model gives the result

$$x = e^{-\frac{Rt}{2m}}\left(A\cos\left(\frac{\sqrt{4km - R^2}}{2m}\right)t + B\sin\left(\frac{\sqrt{4km - R^2}}{2m}\right)t\right) + \frac{mg}{k}$$

where A and B are constants, and g m s^{-2} is the acceleration due to gravity.

You do not need to find the value of A or the value of B
[6 marks]

[Turn over]

16 (a) (ii) It is also given that:

$k = 16$

$R = 20$

$m = 62.5$

$g = 9.8\,\text{ms}^{-2}$

and that the speed of the bungee jumper when the rope becomes taut is 14 ms^{-1}

Show that, to the nearest integer, $A = -38$ and $B = 16$
[6 marks]

[Turn over]

16 (b) **A second, simpler model assumes that the air resistance is zero.**

The values of k, m and g remain the same.

Find an expression for x in terms of t according to this simpler model, giving the values of all constants to two significant figures. [4 marks]

END OF QUESTIONS

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