A-LEVEL
FURTHER MATHEMATICS

7367/2
Report on the Examination

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General

Students performed better than they did in both 2019 and 2022. The mean mark was about 6 marks higher than last year. There was a wide range of marks from 1 to 99.

As in previous years, most students attempted all or nearly all question parts. In some questions, for example questions 3, 7, 9(b), 13(b), 14(a) and 16(b), many students did not find the most direct method. However, most gained marks using a different correct method, and in some cases they obtained a completely correct solution.

Some parts of the paper were very well answered, in particular questions 11 (on vectors), 12 (proof by induction) and 13 (roots of a quadratic). The only parts with very low average marks were questions 8(a) and 15(c), both of which were difficult, and unfamiliar to many students.

Nearly all students attempted at least the first two parts of question 16, showing that shortage of time was not an issue for many students.

Question 1

Over 95% answered this correctly.

Question 2

Around three-quarters of students answered this correctly.

Question 3

Around 70% answered this correctly. Some students evaluated each determinant, rather than using the properties of equal determinants.

Question 4

Over 80% of students answered this question correctly.

Question 5

In the past, questions where students have to explain why a statement is correct have caused some problems. However, this question was answered well, with half gaining full marks.

Most, but not all, students recognised that the transformation represented by **M** was a reflection in the line $y = x$.
A few students lost marks by stating the equations of asymptotes, but not making it clear which of the curves they were asymptotes of. Others missed the final mark by not stating their conclusion.

Question 6

Nearly all students gained a mark in part (a) for finding the modulus, but only about three-quarters of students gained both marks. A few left out "i" from their answer.

In part (b), most students realised that the moduli of the two other complex numbers would be the same as in part (a). However, fewer saw that they would need to add multiples of $\frac{2\pi}{3}$ to the argument from part (a).

There was some incorrect notation seen, for example $5\sqrt{2}e^{i-\frac{3\pi}{4}}$

Question 7

Over four-fifths of students expressed the sum as the difference of two series. Of these, most used the correct limits for each series and were able to gain the second mark.

Not many students saw that the required expression could be obtained by using the difference of two squares. However, around a quarter of all students gained full marks. Many of these used their calculator to solve a quartic equation, then used the two real solutions to obtain one of the required quadratic factors.

Question 8

Part (a) was a non-standard question that caused some difficulties, with under 40% gaining one or more marks. Those who saw that they needed to replace the matrix **B** by $\mathbf{A}^{-1}$ in the given result scored at least one mark. Some used the notation $\mathbf{A}^{-T}$ in their solution. Although this is correct notation, it was not appropriate to use it here, where they are being asked to prove that $\mathbf{A}^{-T}$ is well-defined.

Less than 60% of students gained full marks in part (b)(i), with many students forming the inverse matrix by an incorrect rearrangement of the elements of **A**.

90% obtained a correct answer to part (b)(ii).

Question 9

About two-thirds of students gained full marks on part (a). Most knew to multiply the numerator and denominator by the complex conjugate of the denominator. Some then made errors such as writing the denominator as $1+k$ instead of $1+k^2$.

In part (b), many students correctly found the modulus of z. However, few used a completely correct method to find the argument. Some used a calculator to find the inverse tan of $\frac{\text{Im}(z)}{\text{Re}(z)}$, but then used $\frac{7\pi}{12}$ instead of $-\frac{5\pi}{12}$ with no convincing explanation for the change. When solving exam questions where the result is given, it is important to justify every step.

Question 10

Part (a) was done well, with over two-thirds of students gaining full marks.

In part (b), most students found a correct method, but some made calculation errors. Several different correct approaches were seen, for example the use of the cosine rule or polar coordinates.

Question 11

Around 40% answered part (a) correctly. Of the remainder, nearly all obtained a correct direction vector. Some found the first part of the Cartesian equation but omitted “$z = 7$” or expressed it incorrectly.

Part (b)(i) was done well. Some students stated the explanation but did not perform the calculation, and so could not gain both marks.

About two-thirds of students gained full marks on part (b)(ii). Some used the method shown on the mark scheme. An equal number of students used the fact that the two lines intersect to set up and solve simultaneous equations.

Question 12

This question was a bit more difficult than some of the questions on proof by mathematical induction which have been set recently. Despite this, students performed well, with 90% scoring three or more marks.

Many students were not able to gain the final mark, as all the elements listed in the marking instructions must be present. For example, if students wrote “True for $n \geq 1$” rather than “$\mathrm{f}(n)$ is divisible by 19 for $n \geq 1$” they would not be awarded this mark.

Question 13

Students understand this topic well. Nearly all gained both marks in part (a).

In part (b), most students expanded $(\alpha + \beta)^4$ rather than using the method shown on the mark scheme. This was done successfully, although it was interesting to see some students not using a binomial expansion, but working the expansion out from first principles.

Most students had a good idea what to do on part (c), though there were some errors. Some students lost the final mark by writing a quadratic expression and not an equation.

Question 14

In part (a), some students did not give enough detail in their solution, bearing in mind that they were given a result to work towards. For example, having obtained $y = \frac{1}{3}$, they stated "There is a stationary point at $\left(2, \frac{1}{3}\right)$" with no calculations showing why $x = 2$.

Part (b) was done well, with around two-thirds of students gaining three or more marks. Those who did not take out a factor of $\frac{1}{4}$ before integrating sometimes obtained an answer that was out by a factor of 2.

Many students used correct limiting notation in part (c) and so gained one mark. It was impressive to see that a quarter of students obtained the correct limit of $\tan^{-1}$ and gained both marks.

Question 15

To gain both marks in part (a), students needed to state every step of the proof, and not assume that $z^{-n} = \cos\theta - \mathrm{i}\sin\theta$

Most students were not able to make any progress in part (b).

Part (c) was demanding, and few students gained marks. Some scored the first two marks by obtaining a correct expression for the sum of each geometric series. To continue further, they usually needed to see how to express the denominator in terms of $\sin\theta$. In this type of proof, it can be useful to look ahead to the result.

Question 16

The given answer in question 16ai featured an error, students were given a mathematical expression which had three closing brackets, but only one opening bracket instead of the correct three opening brackets. Student responses were closely monitored during live marking and examiners did not find any evidence of students having difficulty interpreting what was intended by the question. It is likely that the standard form of the equation meant that the equation was still recognisable to most students. The mark scheme was applied in the usual way except for instances of misuse of brackets in solutions, where examiners applied marking leniency in light of the question error. Statistical analysis of the question showed that the question performed as intended and differentiated well between students of higher and lower ability.

Most students made some progress in part (a)(i), with around 60% scoring three or more marks, and around a quarter gaining full marks.

Some students initially made sign errors in their equation of motion. Those who saw that this did not lead to the required solution sometimes then went back and corrected the errors. Others incorrectly wrote down the required solution, even though it was not consistent with their equation of motion.

There was a wide range of marks in part (a)(ii). Over a quarter of students scored full marks, and most gained the first two marks. Most made it easier for themselves by substituting the numerical values at an early stage.

In part (b), not many students appreciated that they could use the correct version of the equation given in part (a)(i) with R equal to zero. Most set up and solved an equation of motion using the new conditions. A few wrongly assumed that the values of the constants A and B would be the same as in part (a).

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.