A

Surname ______________________

Forename(s) ______________________

Centre Number ______________________

Candidate Number ______________________

Candidate Signature ______________________

I declare this is my own work.

A-level

FURTHER MATHEMATICS

Paper 3 Discrete

7367/3D

Wednesday 14 June 2023 Afternoon

Time allowed: 2 hours

At the top of the page, write your surname and forename(s), your centre number, your candidate number and add your signature.

[Turn over]

J U N 2 3 7 3 6 7 3 D 0 1

MATERIALS

For this paper you must have:

- **the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics**
- **a graphical or scientific calculator that meets the requirements of the specification**
- **the other optional Question Paper/Answer Book for which you are entered (EITHER Mechanics OR Statistics). You will have 2 hours to complete BOTH papers.**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided. Do NOT write outside the box around each page or on blank pages.**
- **If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).**
- **Show all necessary working; otherwise marks for method may be lost.**
- **Do all rough work in this book. Cross through any work you do not want to be marked.**

INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

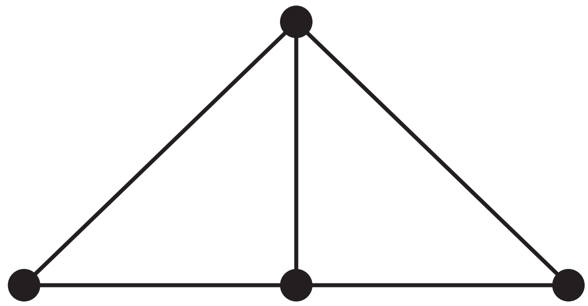
ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 **The simple-connected graph *G* is shown below.**



The graph *G* has n faces.

State the value of n

Circle your answer. [1 mark]

2 **3** **4** **5**

2 Jonathan and Hoshi play a zero-sum game.

The game is represented by the following pay-off matrix for Jonathan.

Hoshi

	STRATEGY	H_1	H_2	H_3
Jonathan	J_1	–2	3	2
	J_2	3	2	0
	J_3	4	–1	3
	J_4	3	1	0

The game does not have a stable solution.

Which strategy should Jonathan NEVER play?

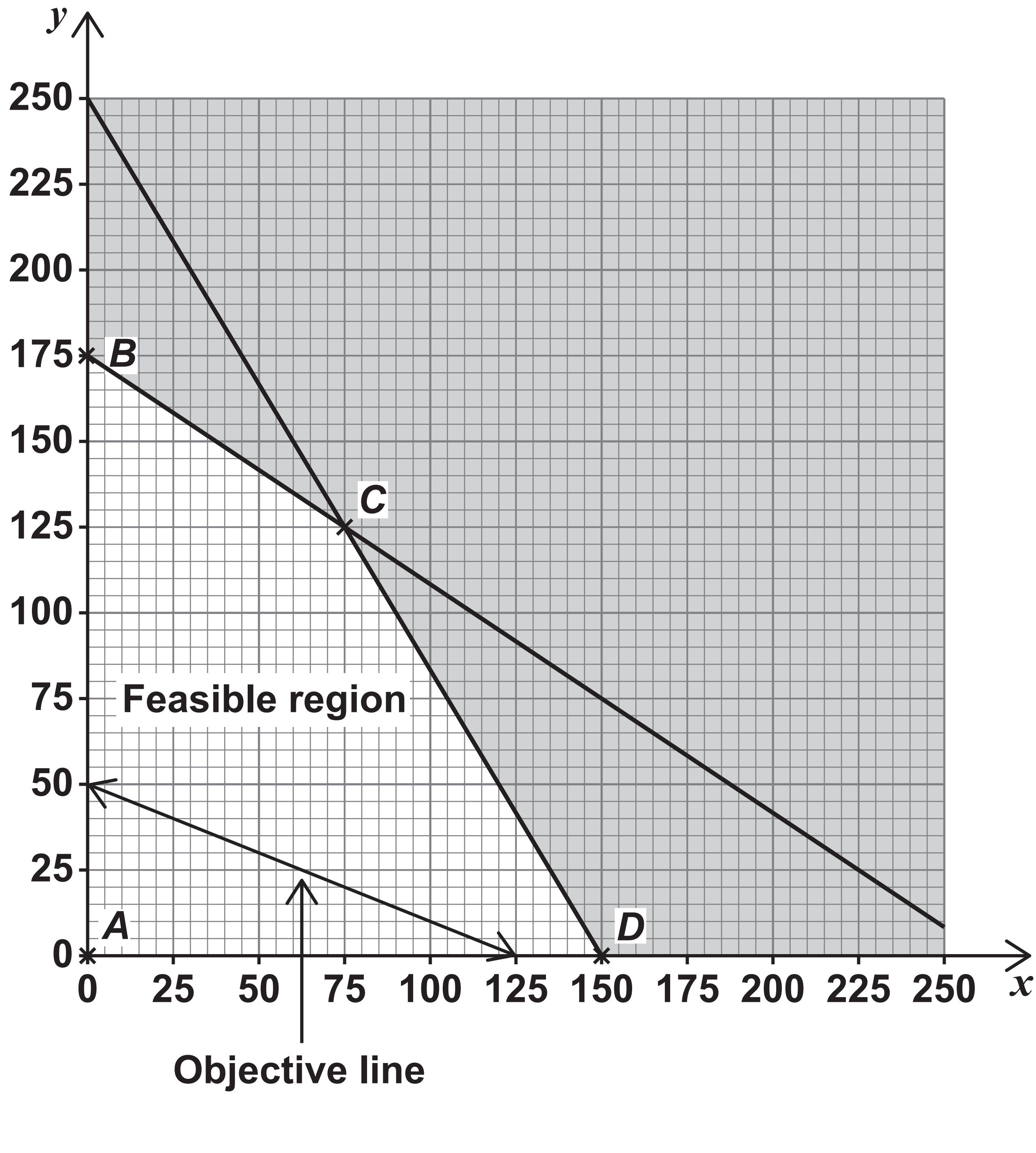
Circle your answer. [1 mark]

J_1 J_2 J_3 J_4

[Turn over]

3 A student is solving a maximising linear programming problem.

The graph below shows the constraints, feasible region and objective line for the student's linear programming problem.



Which vertex is the optimal vertex?

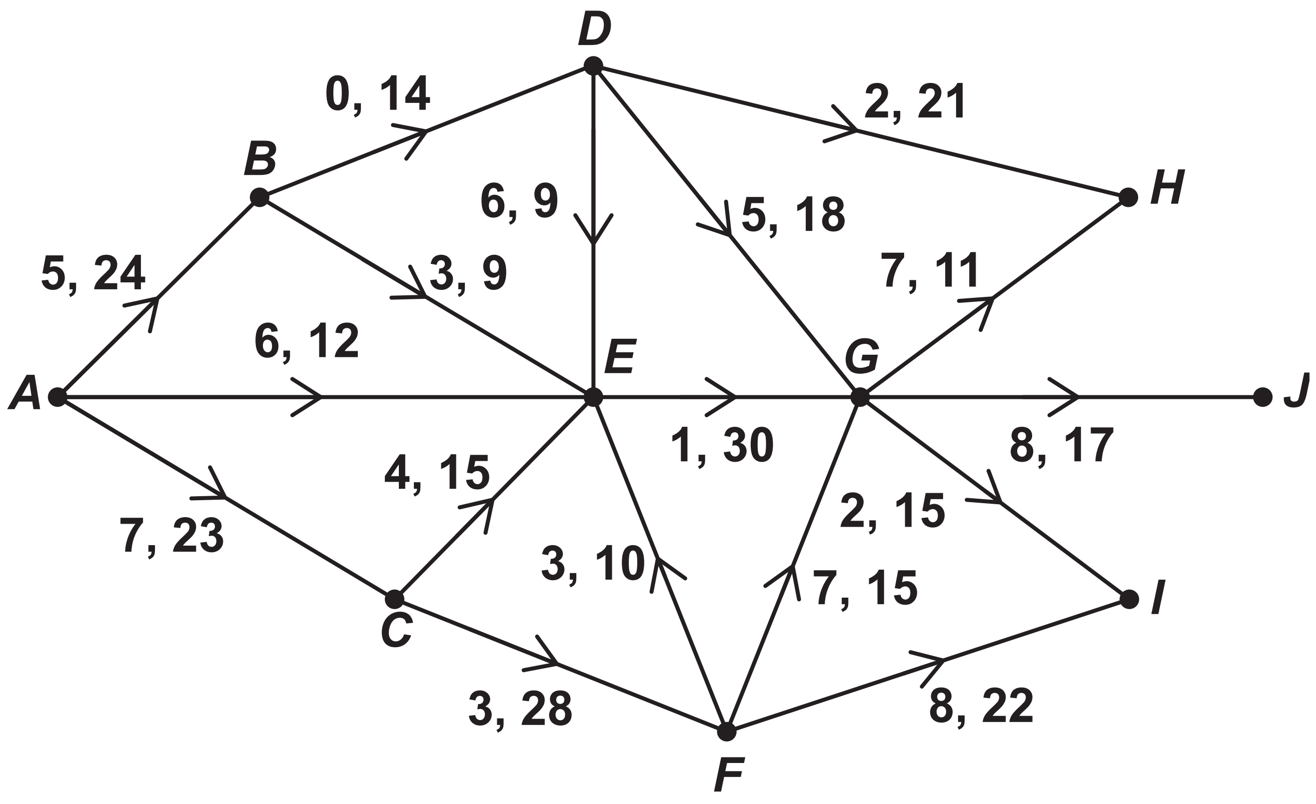
Circle your answer. [1 mark]

A ***B*** ***C*** ***D***

[Turn over]

4 The network below represents a system of water pipes in a geothermal power station.

The numbers on each arc represent the lower and upper capacity for each pipe in gallons per second.



The water is taken from a nearby river at node *A*

The water is then pumped through the system of pipes and passes through one of three treatment facilities at nodes *H*, *I* and *J* before returning to the river.

4 (a) **The senior management at the power station want all of the water to undergo a final quality control check at a new facility before it returns to the river.**

Using the language of networks, explain how the network on the opposite page could be modified to include the new facility. [2 marks]

[Turn over]

4 (b) Find the value of the cut $\{A, B, C, D, E\}$ $\{F, G, H, I, J\}$ [1 mark]

4 (c) Tim, a trainee engineer at the power station, correctly calculates the value of the cut $\{A, B, C, D, E, F\}$ $\{G, H, I, J\}$ to be 106 gallons per second.

Tim then claims that the maximum flow through the network of pipes is 106 gallons per second.

Comment on the validity of Tim's claim. [2 marks]

[Turn over]

5 A student is solving the following linear programming problem.

Minimise $Q = -4x - 3y$

subject to
$$x + y \leq 520$$
$$2x - 3y \leq 570$$

and $x \geq 0, y \geq 0$

5 (a) The student wants to use the simplex algorithm to solve the linear programming problem.

They modify the linear programming problem by introducing the objective function

$P = 4x + 3y$

and the slack variables r and s

State ONE further modification that must be made to the linear programming problem so that it can be solved using the simplex algorithm. [1 mark]

5 (b) (i) **Complete the initial simplex tableau for the modified linear programming problem. [2 marks]**

P	x	y	r	s	VALUE

5 (b) (ii) **Hence, perform ONE iteration of the simplex algorithm. [2 marks]**

P	x	y	r	s	VALUE

5 (c) **The student performs one further iteration of the simplex algorithm, which results in the following correct simplex tableau.**

P	x	y	r	s	VALUE
1	0	0	$\frac{18}{5}$	$\frac{1}{5}$	1986
0	0	1	$\frac{2}{5}$	$-\frac{1}{5}$	94
0	1	0	$\frac{3}{5}$	$\frac{1}{5}$	426

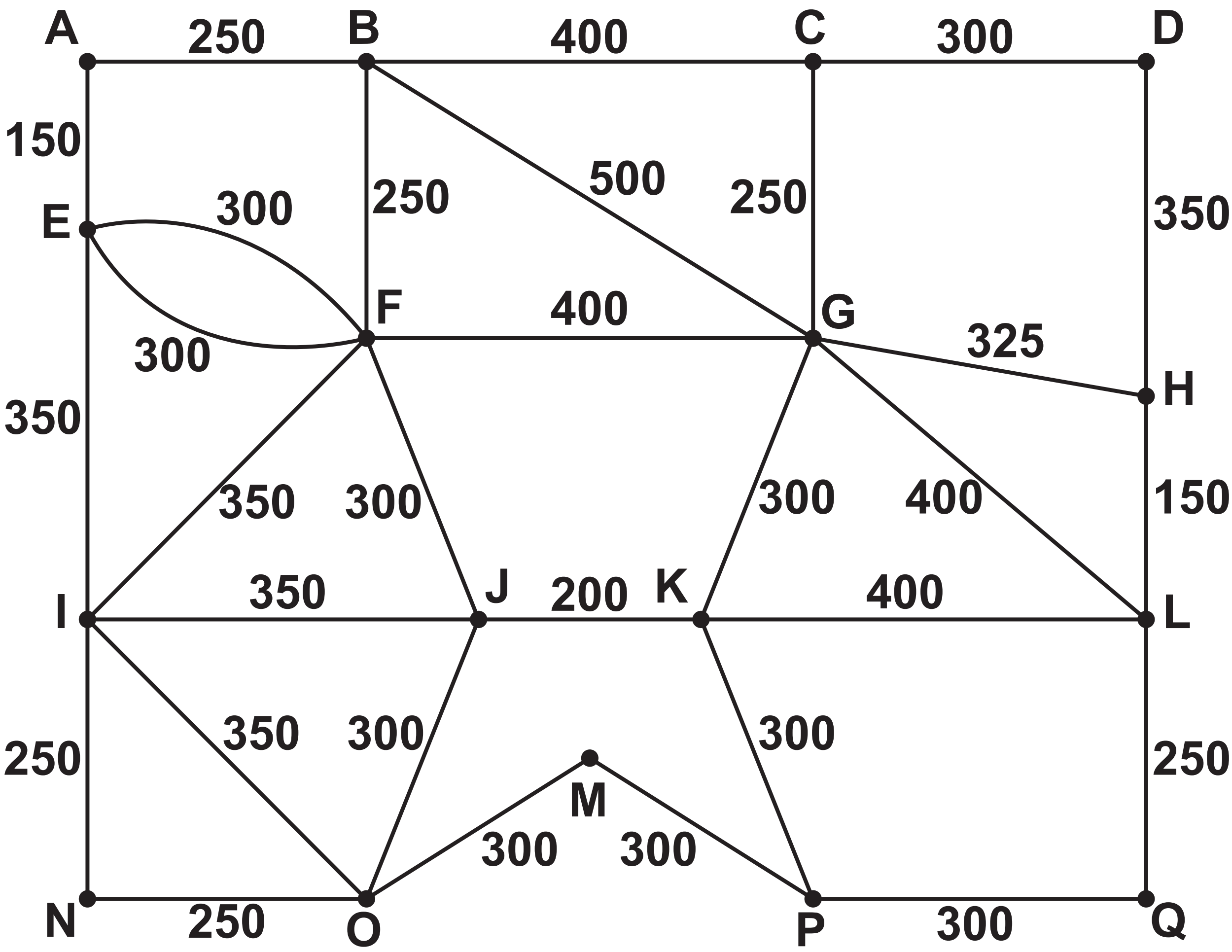
5 (c) (i) **Explain how the student can tell that the optimal solution to the modified linear programming problem can be determined from the above simplex tableau. [1 mark]**

5 (c) (ii) **Find the optimal solution of the ORIGINAL linear programming problem. [2 marks]**

[Turn over]

6 A council wants to grit all of the roads on a housing estate.

The network shows the roads on a housing estate. Each node represents a junction between two or more roads and the weight of each arc represents the length, in metres, of the road.



The total length of all of the roads on the housing estate is 9175 metres.

In order to grit all of the roads, the council requires a gritter truck to travel along each road at least once. The gritter truck starts and finishes at the same junction.

6 (a) The gritter truck starts gritting the roads at 7:00 pm and moves with an average speed of 5 metres per second during its journey.

Find the earliest time for the gritter truck to have gritted each road at least once and arrived back at the junction it started from, giving your answer to the nearest minute.

Fully justify your answer. [6 marks]

6 (b) **Explain how a refinement to the council's requirement, that the gritter truck must start and finish at the same junction, could reduce the time taken to grit all of the roads at least once. [2 marks]**

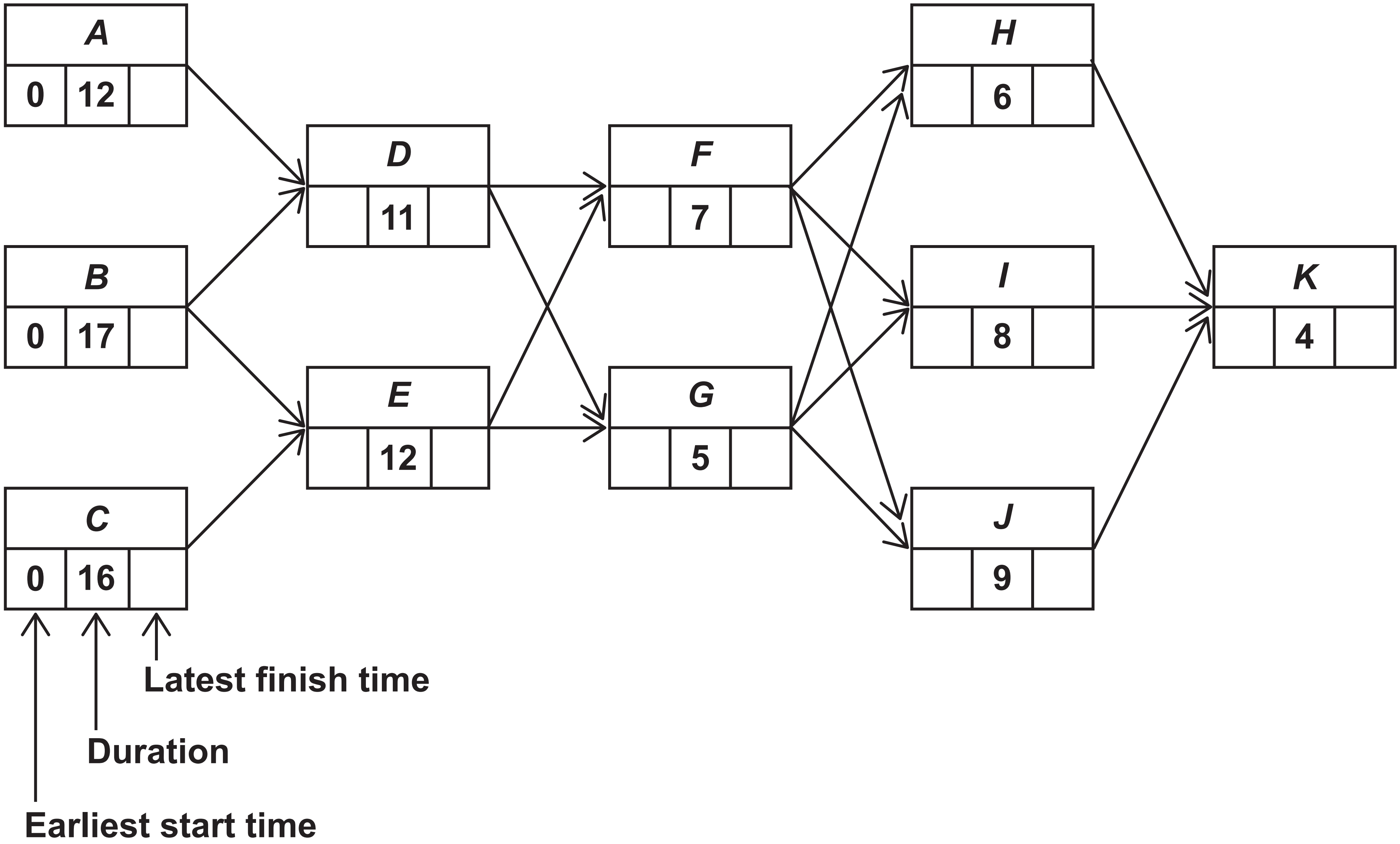
7 **Nova Merit Construction are planning a building project.**

The planning involves producing an activity network for the project, which is shown in FIGURE 1, on the opposite page. The duration of each activity is given in weeks.

7 (a) (i) **Find the earliest start time and the latest finish time for each activity and write these values on the activity network in FIGURE 1.**
[2 marks]

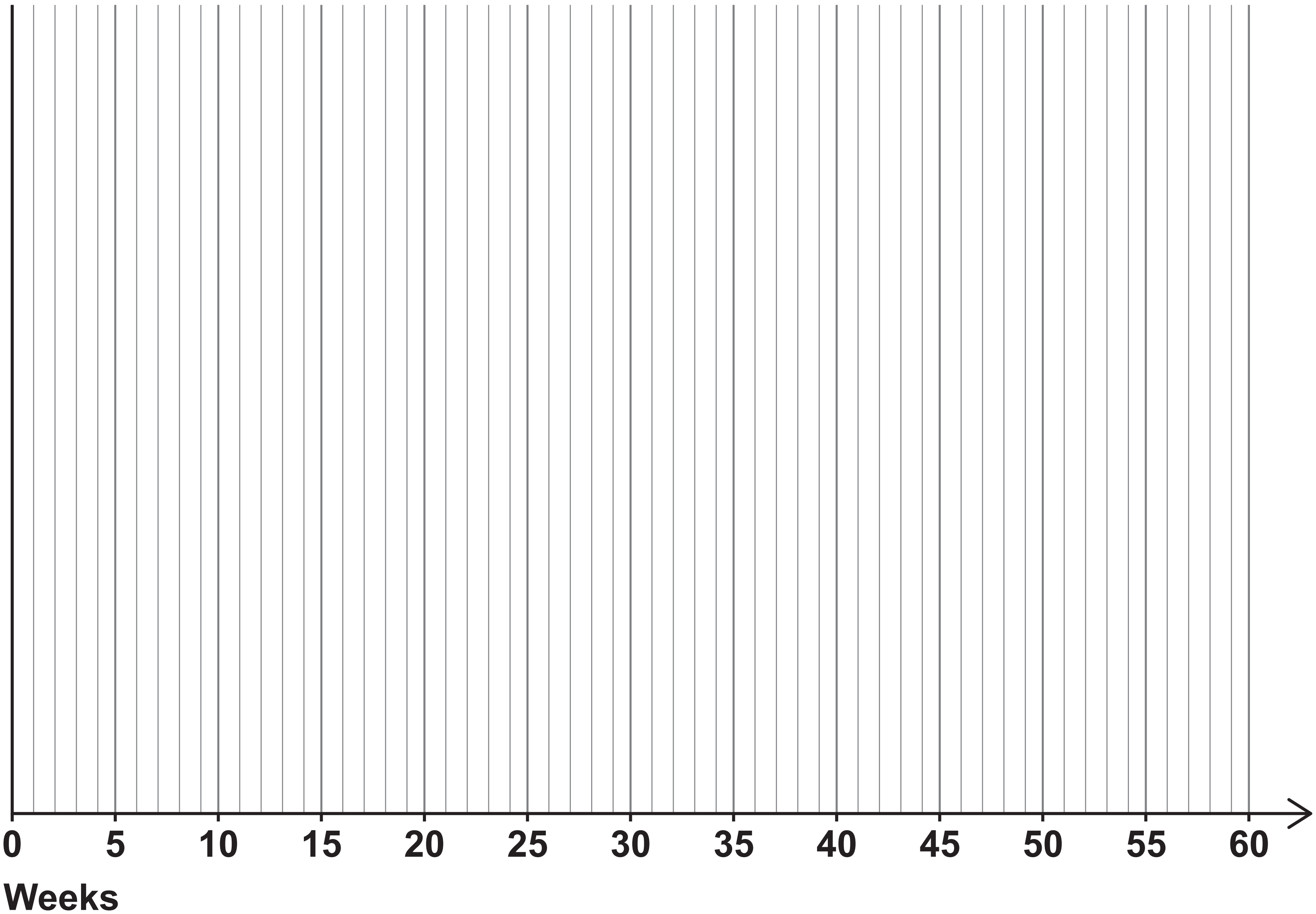
7 (a) (ii) **Write down the critical path. [1 mark]**

FIGURE 1


A
0
12
B
0
17
C
0
16
D
11
E
12
F
7
G
5
H
6
I
8
J
9
K
4
Latest finish time
Duration
Earliest start time

7 (b) **On FIGURE 2, on the opposite page, draw a cascade diagram (Gantt chart) for the planned building project, assuming that each activity starts as early as possible. [3 marks]**

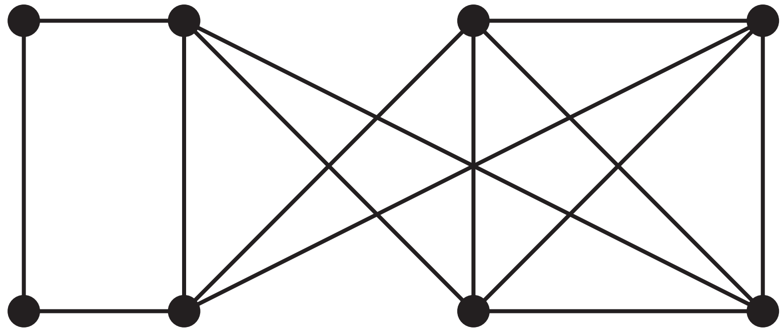
FIGURE 2



7 (c) During further planning of the building project, Nova Merit Construction find that activity *F* is not necessary and they remove it from the project.

Explain the effect removing activity *F* has on the minimum completion time of the project.
[2 marks]

8 **The graph *G* is shown below.**



8 (a) (i) **State, with a reason, whether or not *G* is simple. [2 marks]**

[Turn over]

8 (a) (ii) **A student states that *G* is Eulerian.**

Explain why the student is correct. [2 marks]

8 (b) **The graph *H* has 8 vertices with degrees 2, 2, 4, 4, 4, 4, 4 and 4**

Comment on whether *H* is isomorphic to *G*
[2 marks]

8 (c) **The formula $v - e + f = 2$, where**

v = number of vertices

e = number of edges

f = number of faces

can be used with graphs which satisfy certain conditions.

Prove that *G* does not satisfy the conditions for the above formula to apply. [3 marks]

9 **The group (C, $+_4$) contains the elements 0, 1, 2 and 3**

9 (a) (i) **Show that C is a cyclic group. [2 marks]**

[Turn over]

9 (a) (ii) **State the group of symmetries of a regular polygon that is isomorphic to C [1 mark]**

9 (b) **The group (V, ⊗) contains the elements (1, 1), (1, –1), (–1, 1) and (–1, –1)**

The binary operation ⊗ between elements of V is defined by

$$(a, b) \otimes (c, d) = (a \times c, b \times d)$$

9 (b) (i) Find the element in V that is the inverse of (–1, 1)

Fully justify your answer. [2 marks]

9 (b) (ii) Determine, with a reason, whether or not $C \cong V$ [2 marks]

9 (c) The group G has order 16

Rachel claims that as 1, 2, 4, 8 and 16 are the only factors of 16 then, by Lagrange's theorem, the group G will have exactly 5 distinct subgroups, including the trivial subgroup and G itself.

Comment on the validity of Rachel's claim.
[2 marks]

END OF QUESTIONS

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Additional page, if required.
Write the question numbers in the left-hand margin.

Additional page, if required.
Write the question numbers in the left-hand margin.

Additional page, if required.
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For Examiner's Use	
Question	Mark
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