Teacher Support Materials 2008

Maths GCE

Paper Reference MD01

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Question 1b

1 Six people, A, B, C, D, E and F, are to be matched to six tasks, 1, 2, 3, 4, 5 and 6.

The following adjacency matrix shows the possible matching of people to tasks.

	Task 1	**Task 2**	**Task 3**	**Task 4**	**Task 5**	**Task 6**
A	0	0	1	0	1	1
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	1	0	1	0	1	0
F	0	0	0	1	1	0

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task 3, B to task 4, C to task 2 and E to task 5. From this initial matching, use the maximum matching algorithm to obtain a complete matching. List your complete matching. *(5 marks)*

Student Response

Question number

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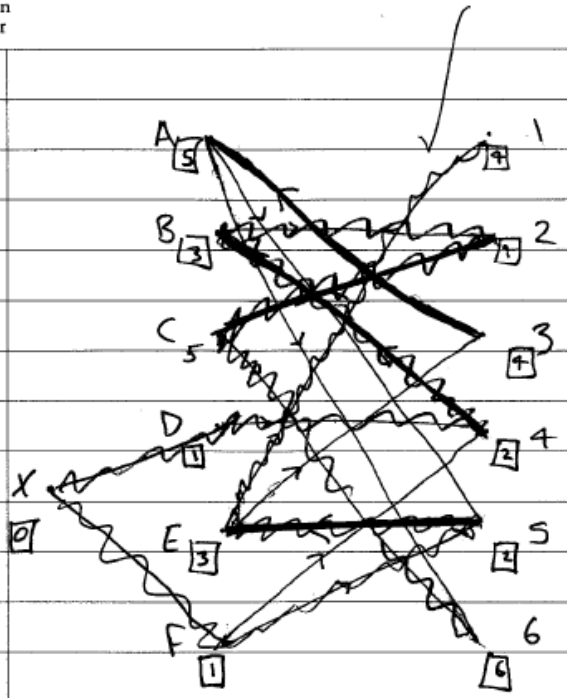
1 b)



original matching

deleted matches which had both a bold line and a route over it!

Keep lines with just one line, making a new match.

NEW MATCHES are

F - 5
E - 1
D - 4
C - 6
B - 2
A - 3

M1 A1

* A complete match is

A - 3
B - 2
C - 6
D - 4
E - 1
F - 5

B1

3

5

Commentary

In all examiners reports it has been highlighted that candidates must clearly show their alternating path. Moreover if they choose to work on their diagram then no more than 1 path should be on a diagram. This' solution' shows a number of arrows on the diagram with **no** clear order shown. The candidate appears to start at vertex1 but it is then unclear how the path follows on. The candidate only scores the mark for the final match

Mark scheme

(b)	$A3, B4, C2, E5$			Initial match
	Start from D, F or 1, 6	M1 M1		1st path } must go beyond 2nd 2nd path } letter/number eg $D-4\,(+)\,B\,/\,F$ If working is **only** on diagram, **the path(s) must be clear**, and only 1 path per diagram can be credited. If 2 paths shown on one diagram, max mark M1A1
	Accept paths in reverse order $D-4\,(+)B-2\,(+)\,C-6$ $F-5\,(+)E-1$ or $F-4(+)B-2(+)C-6$ $D-4(+)F-5(+)E-1$	 A1 A1		 1st correct path 2nd correct path or $F-5(+)E-3(+)A-6$ $D-4(+)B-2(+)C-6(+)A-3(+)E-1$
	Match: $A3, B2, C6, D4, E1, F5$	B1	5	Must be clearly stated or indicated

Question 2

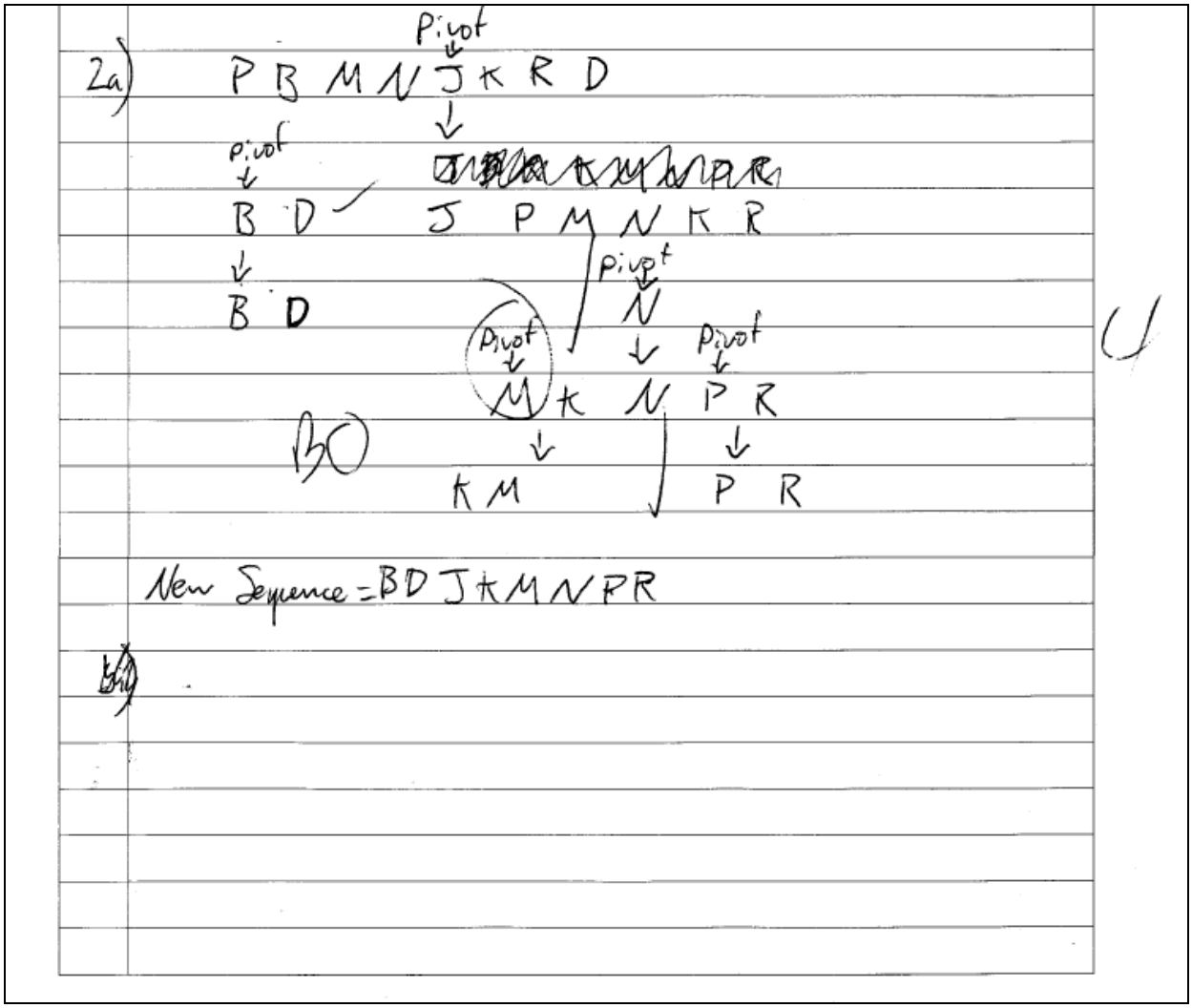
2 (a) Use a quick sort to rearrange the following letters into alphabetical order. You must indicate the pivot that you use at each pass.

P B M N J K R D *(5 marks)*

(b) (i) Find the maximum number of swaps needed to rearrange a list of 8 numbers into ascending order when using a **bubble** sort. *(1 mark)*

(ii) A list of 8 numbers was rearranged into ascending order using a **bubble** sort. The maximum number of swaps was needed. What can be deduced about the original list of numbers? *(1 mark)*

Student response



Question number | Leave blank

5 i) Original (,)

P 5:111	B C111 S
B	M
M	N
N	P
J	J
K	K
B	D
D	B

C = 6 S = 4

b i) 28

ii) The original sequence was in reverse order.

3a i) n - 1 = 10

ii) n - 1

b i)

	A	B	C	D	E	F	G	H	I	J	K
A	–	18	19	–	–	–	–	–	–	–	–
B	18	–	13	15	21	–	–	–	–	–	–
C	19	13	–	–	21	16	–	–	–	–	–
D	–	15	–	–	18	–	17	–	–	–	–
E	–	21	21	18	–	19	–	–	–	–	–
F	–	–	16	–	19	–	–	–	–	17	–
G	–	–	–	17	–	–	–	18	–	–	12
H	–	–	–	–	20	–	18	–	18	–	17
I	–	–	–	–	–	14	–	18	–	18	17
J	–	–	–	–	17	–	–	–	18	–	16
K	–	–	–	–	–	–	12	17	17	16	–

1
1
6
1
1

Commentary

This solution shows a lack of understanding of a quicksort. They have started with a pivot of J, perfectly acceptable – although not the best approach. They think that M is before J in the alphabet! On the next line they have chosen to work with the first sublist only – again acceptable. Next line working with the second subset is **ok** apart from their earlier mistake. However they have then ignored working with the the first subset ie B D and moved onto the second subset.

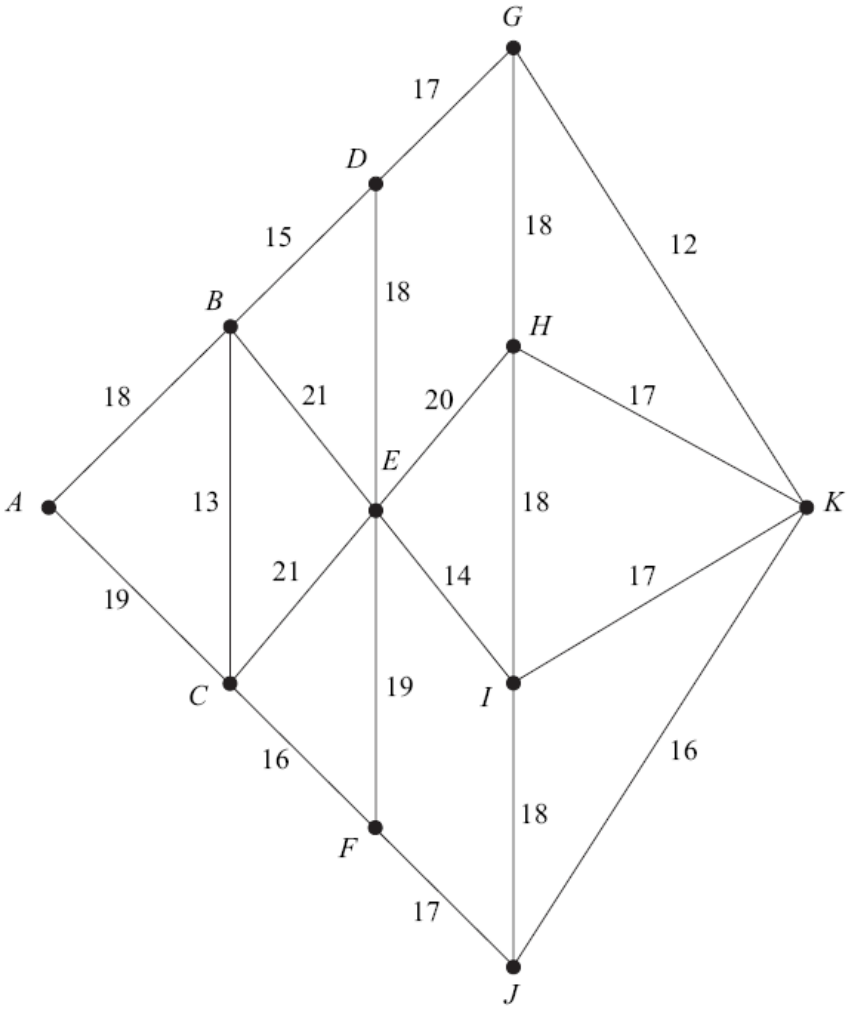
The overall solution has scored the method mark but none of the accuracy marks.
The second accuracy mark was achievable if they had considered B D at the appropriate time

Mark Scheme

2(a)	P B M N J K R D	M1		Using quick sort
	B M N J K D P R	A1		First pass (based on their pivot)
	B M N J K D P R			
	B J K D M N P R	A1		A correct third pass
	B D J K M N P R	A1		All passes correct
		B1	5	Consistent pivots clearly labelled (at least three passes)
(b)(i)	28	B1	1	
(ii)	In reverse order	B1	1	Allow descending
	Total		7	

Question 3b

(b) The following network has 11 vertices, A, B, ..., K. The number on each edge represents the distance, in miles, between a pair of vertices.

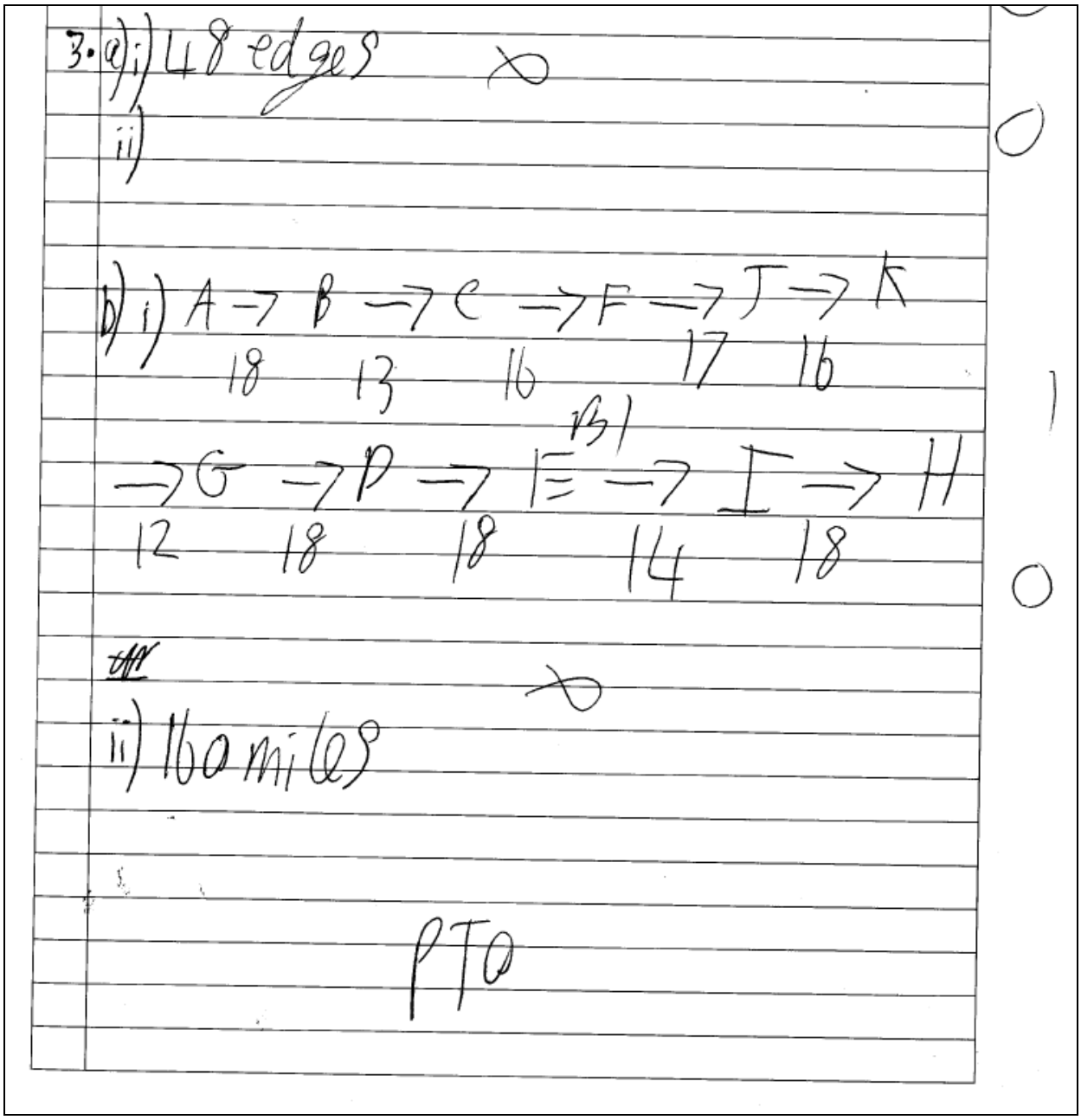


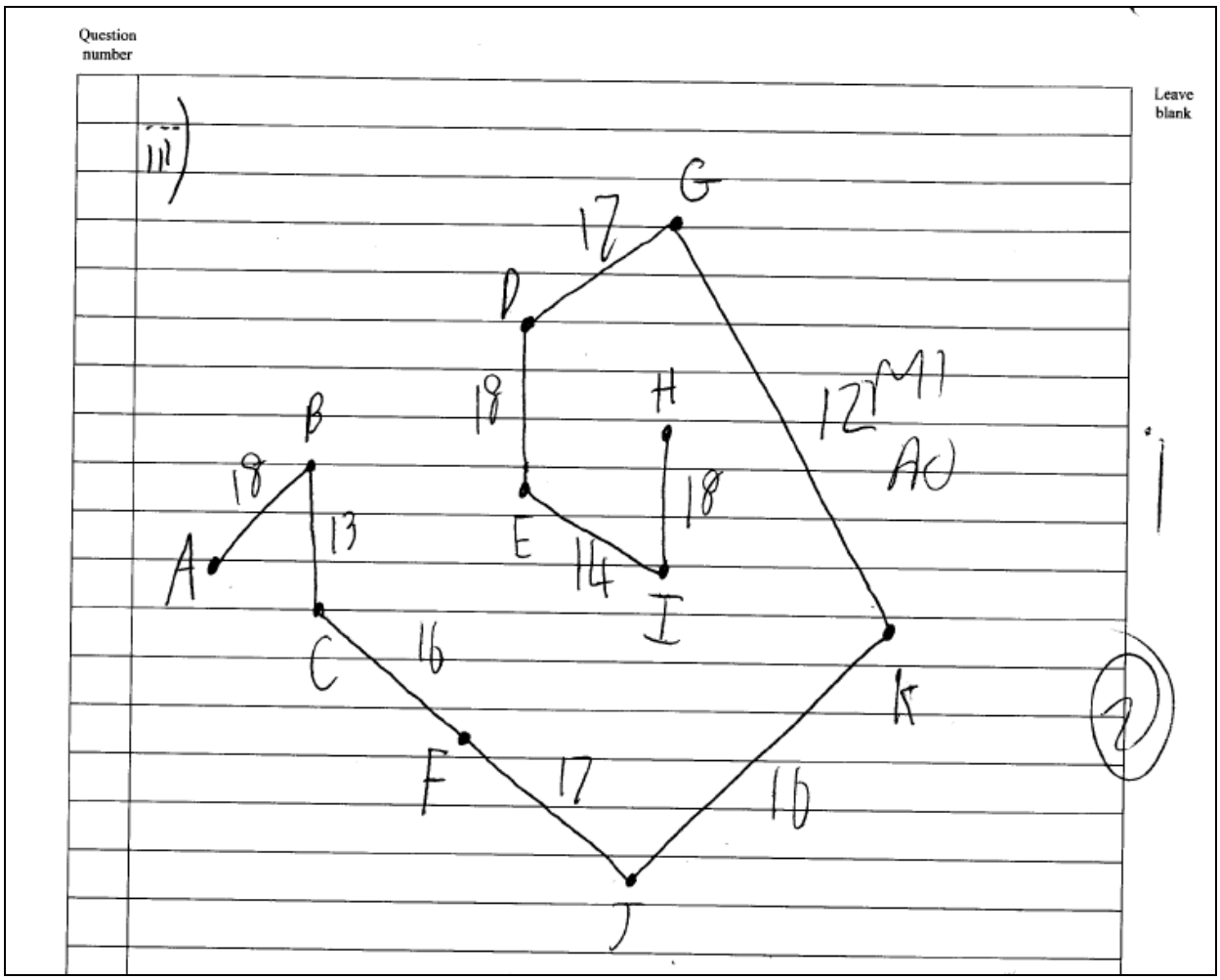
(i) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

(ii) Find the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

Student Response





Commentary

Every year a number of candidates fail to realise the difference between finding a minimum spanning tree and a path through a network. This solution typifies the problem. The candidate has started at A and worked through to H. It is still possible that these candidates gain some reward as their 'path' is still a spanning tree. Candidates must be aware that both Prim's and Kruskal's algorithm are fundamental parts of the course

Mark Scheme

3(a)(i)	10	B1	1	
(ii)	$n-1$	B1	1	
(b)	Condone candidates attempting all of part (b) together / in different order			

Question 4(a)(ii)

4 David, a tourist, wishes to visit five places in Rome: Basilica (B), Coliseum (C), Pantheon (P), Trevi Fountain (T) and Vatican (V). He is to start his tour at one of the places, visit each of the other places, before returning to his starting place.

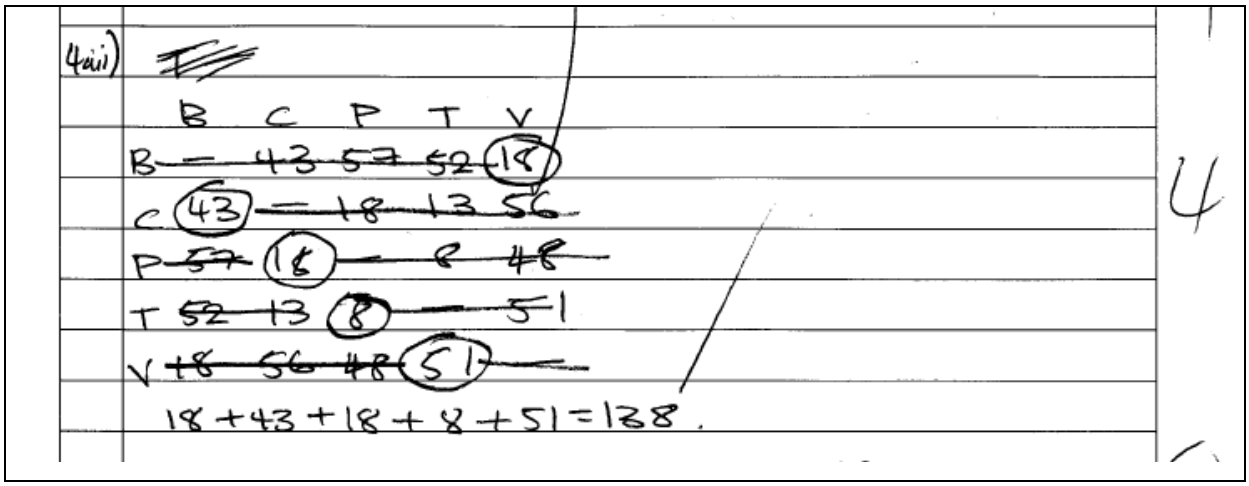
The table shows the times, in minutes, to travel between these places. David wishes to keep his travelling time to a minimum.

	B	***C***	***P***	***T***	***V***
B	–	43	57	52	18
C	43	–	18	13	56
P	57	18	–	8	48
T	52	13	8	–	51
V	18	56	48	51	–

(a) (i) Find the total travelling time for the tour $TPVBCT$. *(1 mark)*

(ii) Find the total travelling time for David's tour using the nearest neighbour algorithm starting from T. *(4 marks)*

Student Response



Commentary

Every year in the examiner's report, it is brought to the attention of centres that the nearest neighbour algorithm finds a Tour. This means that a path **returns** to the start vertex. This solution shows the classic mistake. The candidate still scores 1 of the method marks.

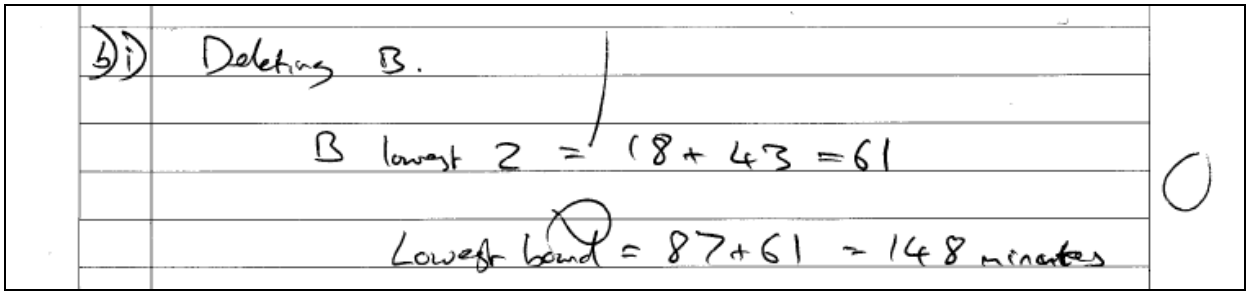
Mark Scheme

4(a)(i)	130	B1	1	$\begin{pmatrix} T & P & V & B & C & T \\ & 8 & 48 & 18 & 43 & 13 \end{pmatrix}$
(ii)	$T \quad P \quad C \quad B \quad V \quad T$ $8 \quad 18 \quad 43 \quad 18 \quad 51$	M1		Tour (vertices or edges) starting from T (Letters not numbers)
		M1		Visits all vertices starting from T
		A1		Correct order
	$= 138$	B1	4	

Question 4(b)(i)

(b) (i) By deleting B, find a lower bound for the total travelling time for the minimum tour. *(5 marks)*

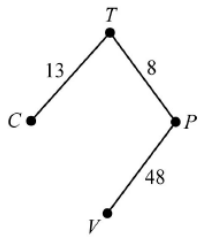
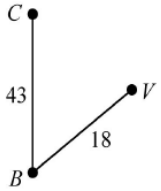
Student Response



Commentary

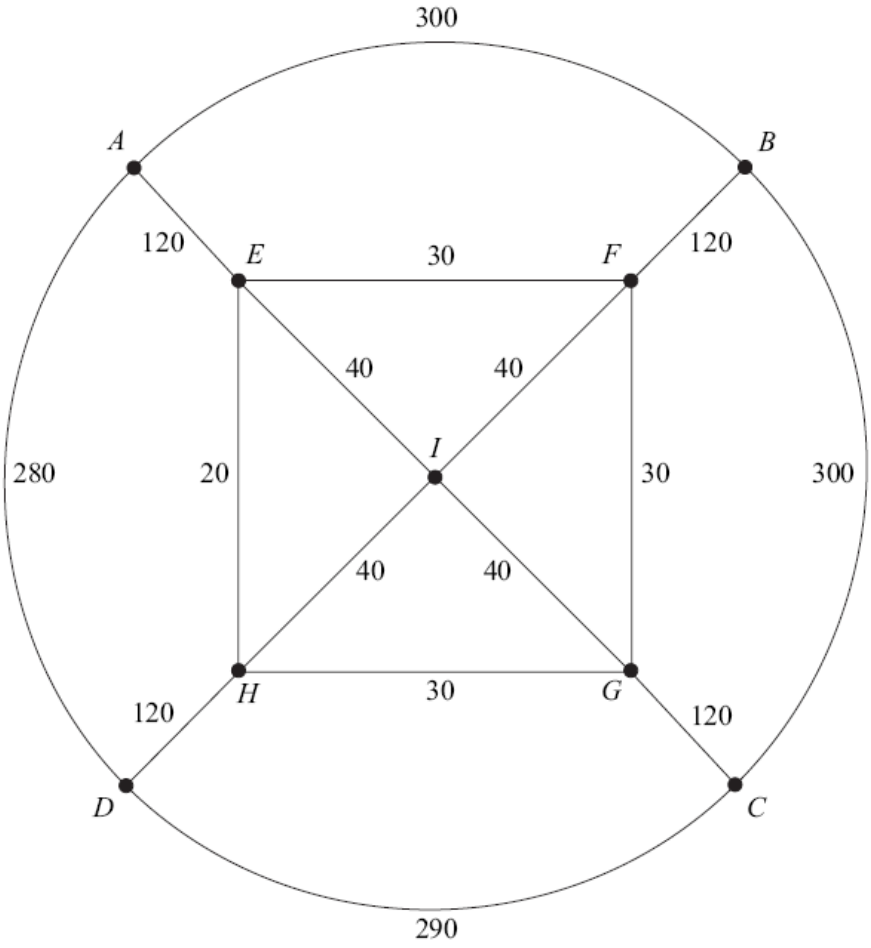
The method of finding lower bounds is still not well understood. Conceptually it is difficult but it is important that centres concentrate on pupils understanding. Having deleted a vertex candidates need to connect the remaining vertices with a minimum spanning tree **not** a tour without the deleted vertex. The solution highlights this error. The candidate has correctly identified the 2 shortest edges from B, but has found a **tour** starting and finishing at T. This makes the idea of adding 2 extra edges bizarre.

Mark Scheme

(b)(i)		M1		Spanning tree with 3 edges
	PT, CT, PV 	A1		Correct
		m1		2 edges from B
	+ 2 shortest from B 	A1		Correct
	(Lower bound =) 130	A1	5	CSO

Question 5(a)

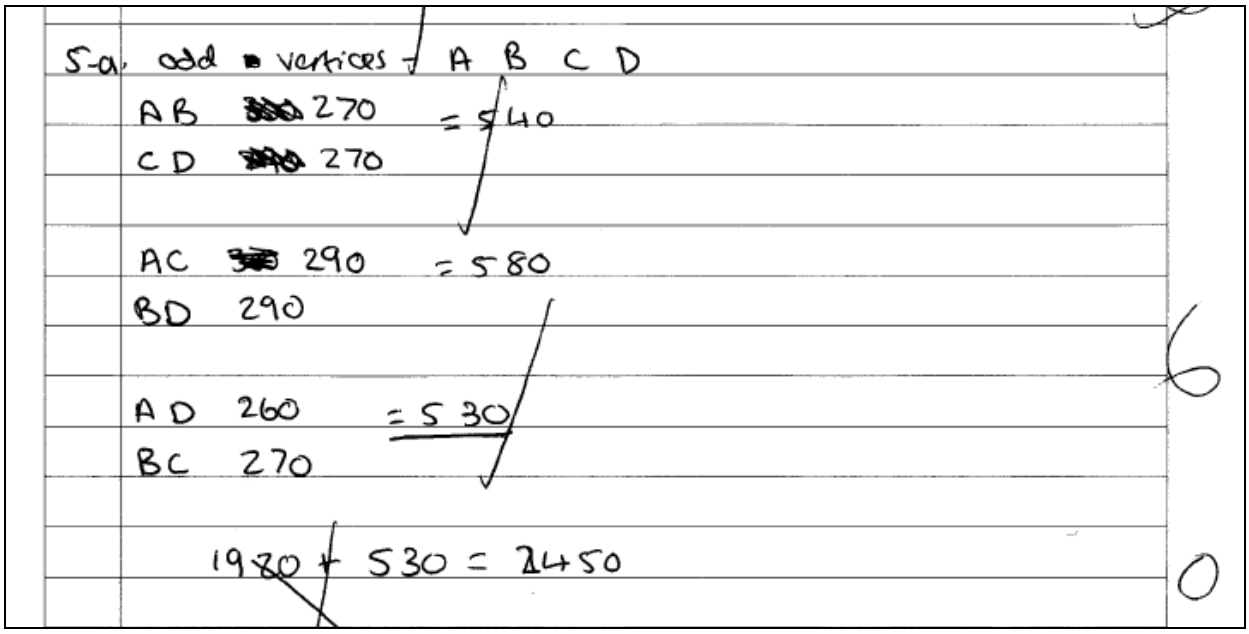
5 The diagram shows a network of sixteen roads on a housing estate. The number on each edge is the length, in metres, of the road. The total length of the sixteen roads is 1920 metres.



Total Length = 1920 metres

(a) Chris, an ice-cream salesman, travels along each road at least once, starting and finishing at the point A. Find the length of an optimal 'Chinese postman' route for Chris. *(6 marks)*

Student Response



Commentary

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When trying to find optimal Chinese postman routes candidates must list the odd vertices, write down possible pairings, evaluate the sums of these pairings and then add the shortest value onto the total of all the edges. This solution is a candidate knowing something about odd vertices but not knowing exactly what to do.

They have found *AB*, *AC* and *AD* without realising that **pairs** of vertices are required.
Again in their explanation they have referred to Eulerian without fully understanding the implications.

Mark Scheme

5(a)	Odds *A*, *B*, *C*, *D*	M1		PI (but *A*, *B*, *C*, *D* must be mentioned)
		m1		Considering 3 sets of pairings of odd vertices, eg *AB* with *CD* etc
	$AB + CD = 270 + 270 = 540$ $AC + BD = 290 + 290 = 580$ $AD + BC = 260 + 270 = 530$	A2,1,0		A1 for 2 correct, A2 for all correct
	Repeat *AD*, *BC*	A1F		Follow through their shortest pairing PI by adding 530 to 1920 Or *AEHD* or *DHEA* **and** *BFGC* or *CGFB* listed in any route
	(Length = 1920 + 530 =) 2450 (metres)	B1	6	

Question 6(a)

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

A factory makes two types of lock, standard and large, on a particular day.

On that day:

the maximum number of standard locks that the factory can make is 100;
the maximum number of large locks that the factory can make is 80;
the factory must make at least 60 locks in total;
the factory must make more large locks than standard locks.

Each standard lock requires 2 screws and each large lock requires 8 screws, and on that day the factory must use at least 320 screws.

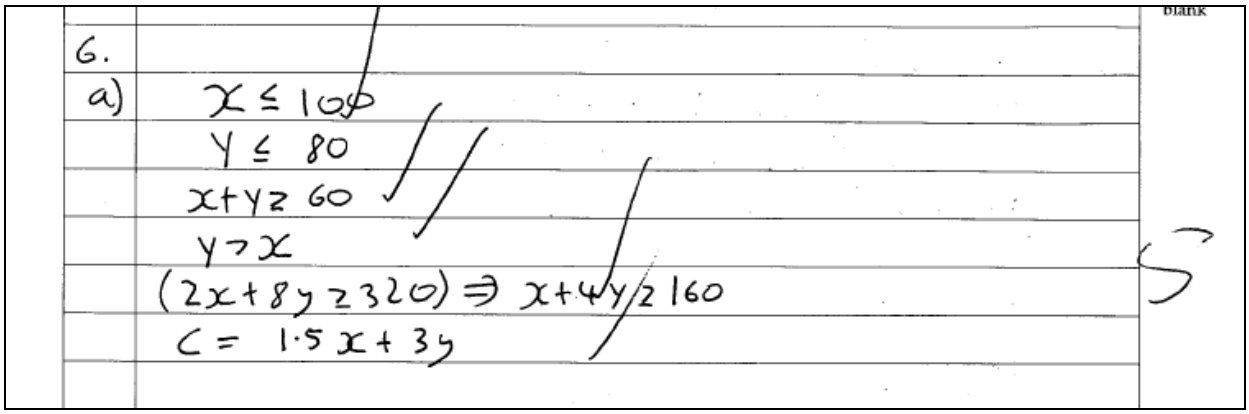
On that day, the factory makes x standard locks and y large locks.

Each standard lock costs £1.50 to make and each large lock costs £3 to make.

The manager of the factory wishes to minimise the cost of making the locks.

(a) Formulate the manager's situation as a linear programming problem. *(5 marks)*

Student Response



Commentary

The question clearly states the variables as x and y. This candidate has chosen to ignore the question and use s and l. This would be acceptable if later these letters were amended to x and y. This candidate was not penalised for notation in the remaining parts of the question Linear programming questions will always be set using x and y as the variables, as the questions will normally require graphical solutions.

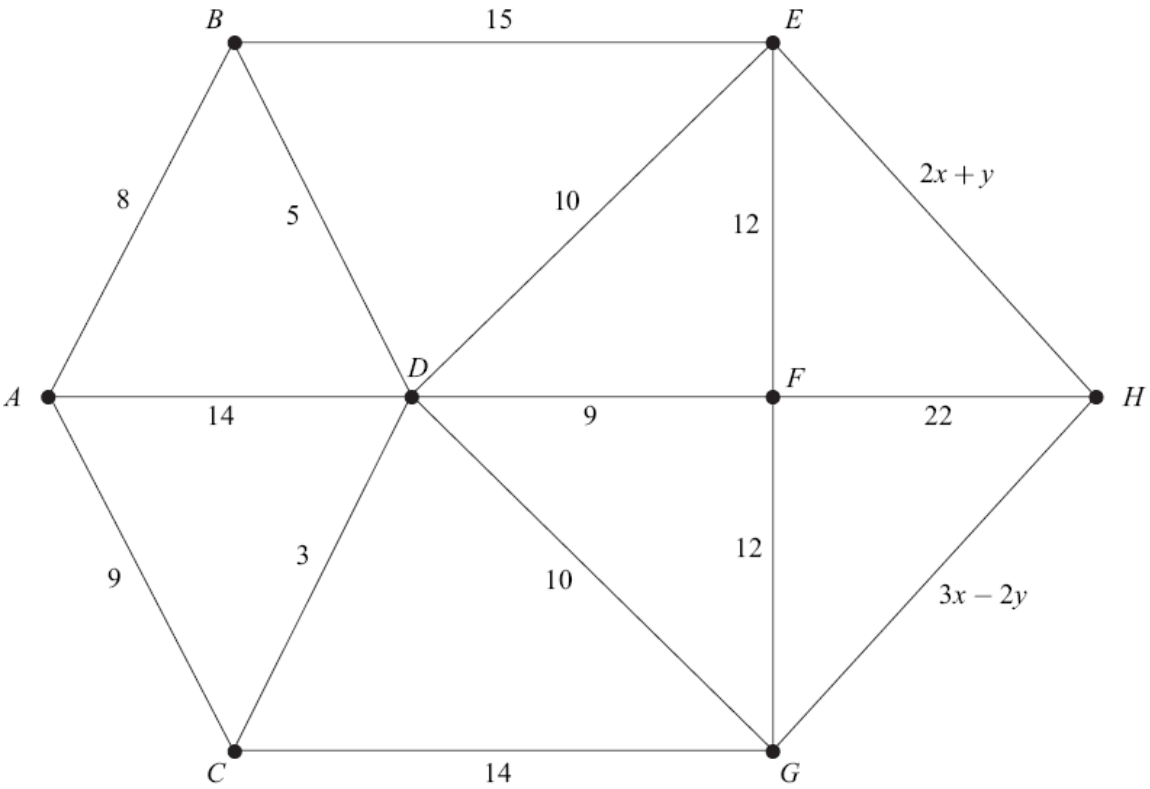
Mark Scheme

6(a)	**All inequalities must be as below**			
	$x \leqslant 100$, $y \leqslant 80$	B1		Both
	$x + y \geqslant 60$	B1		
	$x < y$	B1		
	$2x + 8y \geqslant 320$	B1		OE
	(minimise $C =$) $1.5x + 3y$	B1	5	

Question 7

7 [**Figure 2**, printed on the insert, is provided for use in this question.]

The following network has eight vertices, A, B, ..., H, and edges connecting some pairs of vertices. The number on each edge is its weight. The weights on the edges EH and GH are functions of x and y.

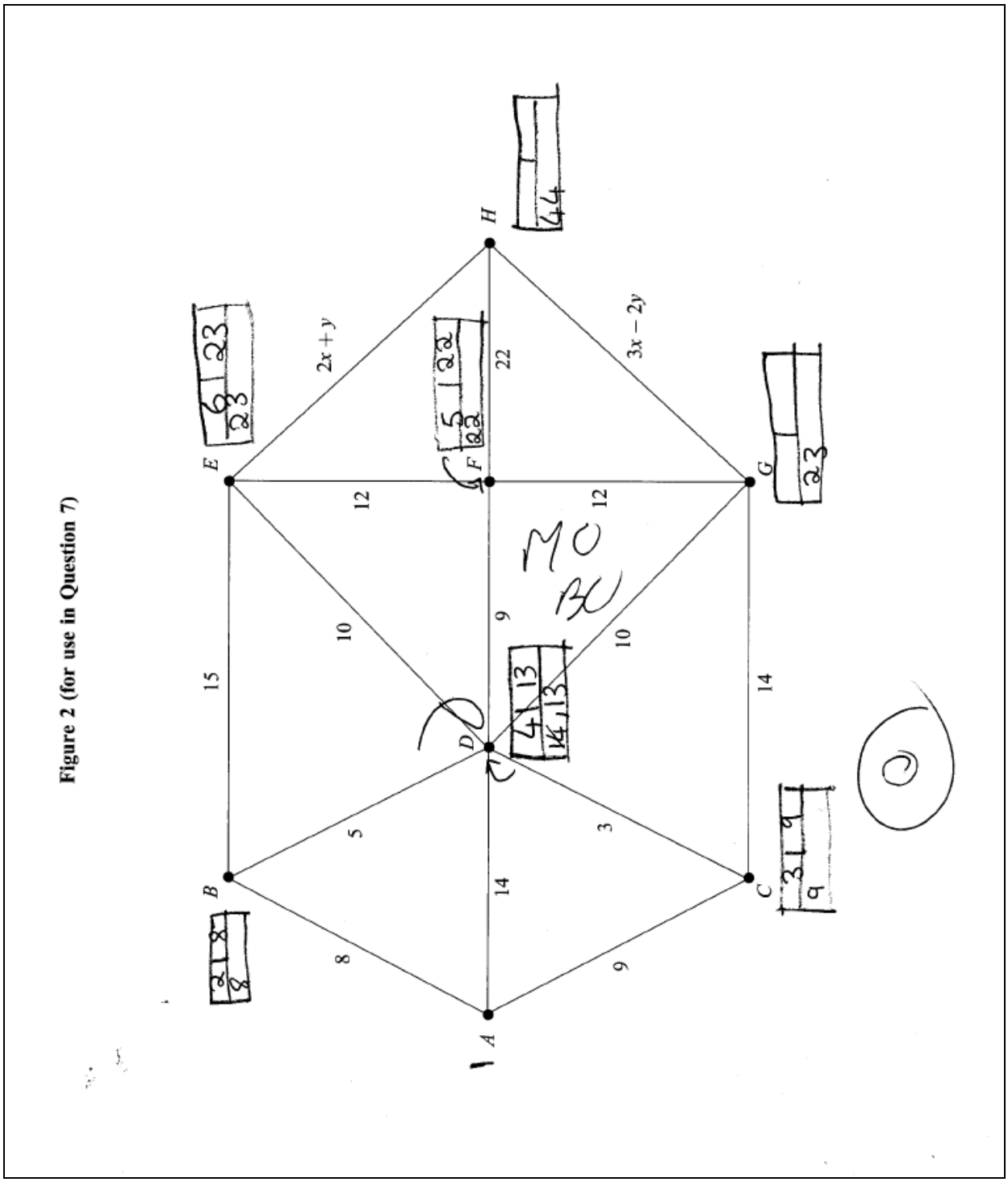


Given that there are three routes from A to H with the same minimum weight, use Dijkstra's algorithm on **Figure 2** to find:

(a) this minimum weight; *(6 marks)*

(b) the values of x and y. *(3 marks)*

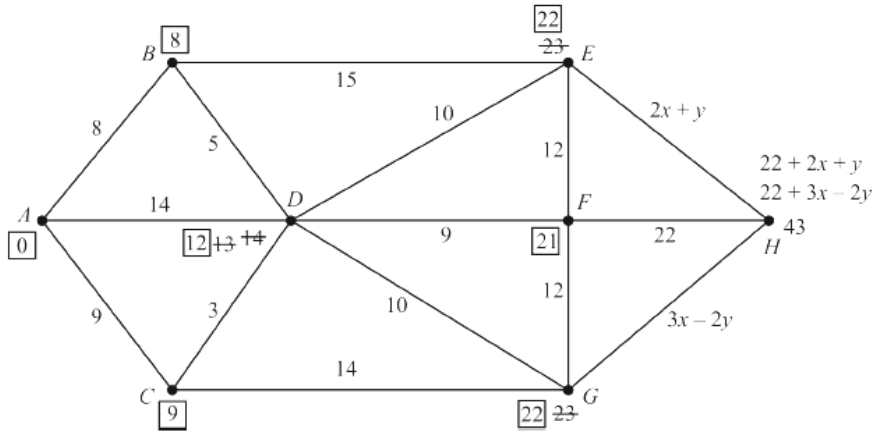
Student Response



Commentary

Dijkstra's algorithm is a fundamental topic in Decision 1. Candidates cannot expect to be rewarded if they choose to answer a question by inspection or by complete enumeration.
This solution shows a candidate writing down values at vertices with **no** working.
The only marks that are available for candidates in this case are the final mark for 43 at H (and a mark for the route, if required)

Mark Scheme

7(a)				
		M1		SCA: cancelling at 2 (or more) vertices
		A1		Correct at D
		M1		2 values at E
		M1		2 values at G
		A1		All correct (condone 0 missing at A and missing expressions in x and y at H)
	(Min =) 43	B1	6	Accept 43 at H
(b)	$2x + y = p$ $3x - 2y = q$	M1		Obtaining a pair of equations in this form *or* $(22) + 2x + y = (43)$ and $(22) + 3x - 2y = (43)$ $2x + y = 21$ and $3x - 2y = 21$
	$x = 9$	A1		CAO
	$y = 3$	A1	3	CAO NMS: both correct M1A2 one/none correct M0A0
	Total		**9**	