



**Teacher Support Materials
2008**

Maths GCE

Paper Reference MFP1

Permission to reproduce all copyrighted material has been applied for. In some cases, efforts to contact copyright holders have been unsuccessful and AQA will be happy to rectify any omissions if notified.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334). Registered address: AQA, Devas Street, Manchester M15 6EX.
Dr Michael Cresswell, Director General

Question 1

1 The equation

$$x^2 + x + 5 = 0$$

has roots α and β .

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. (2 marks)

(b) Find the value of $\alpha^2 + \beta^2$. (2 marks)

(c) Show that $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{9}{5}$. (2 marks)

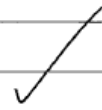
(d) Find a quadratic equation, with integer coefficients, which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. (2 marks)

Student Response

1) a) $x^2 + x + 5 = 0$

$$\alpha + \beta = -1$$

$$\alpha\beta = 5$$



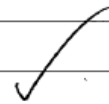
Leave blank

2

b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= (-1)^2 - 2(5)$$

$$= \underline{\underline{-9}}$$



2

	Leave blank
c) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$	
$= \frac{-9}{5}$ ✓	2
d) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{-9}{5}$	
$\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = \frac{\alpha\beta}{\alpha\beta} = \frac{5}{5} = 1$ ✓	1
$= x^2 + \frac{9}{5}x + 1 = 0$ ✓	0
	⑦

Commentary

Most candidates answered this question well. Common errors occurred in part (d), where, as in this example, candidates failed to give integer coefficients. Others failed to write “equals zero” in their equation.

Mark scheme

Q	Solution	Marks	Total	Comments	MFP1
1(a)	$\alpha + \beta = -1, \alpha\beta = 5$	B1B1	2		
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$... = $1 - 10 = -9$	M1 A1F	2	with numbers substituted ft sign error(s) in (a)	
(c)	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$... = $-\frac{9}{5}$	M1 A1	2	AG: A0 if $\alpha + \beta = 1$ used	
(d)	Product of new roots is 1 Eqn is $5x^2 + 9x + 5 = 0$	B1 B1F	2	PI by constant term 1 or 5 ft wrong value for product	
Total			8		

Question 2

2 It is given that $z = x + iy$, where x and y are real numbers.

(a) Find, in terms of x and y , the real and imaginary parts of

$$3iz + 2z^*$$

where z^* is the complex conjugate of z .

(3 marks)

(b) Find the complex number z such that

$$3iz + 2z^* = 7 + 8i$$

(3 marks)

Student response

2		Leave blank
a)	$3iz + 2(x - iy)$	
	$3i(x + iy) + 2(x - iy)$ ✓	M 1
	$3i + 3i^2y + 2x - 2iy$ X	
	$3i - 3y + 2x - 2iy$	M 1
	real: $2x - 3y$ imaginary: $3i - 2iy$ ✓	A 0
b)	$2x - 3y = 7$ ✓	
	$2x - (3x - \frac{5}{2}) = 7$	1
	$2x - (-7.5) = 7$	0
	$2x + 7.5 = 7$	
	$2x = -0.5$	
	$x = \frac{1}{4}$	
	$z = \frac{1}{4} + \frac{-5}{2}i$ X	
		(3)

Commentary

Most candidates attempted to expand and simplify $3i(x + iy) + 2(x - iy)$. Unfortunately this led to many algebraic errors; in this script $3i(x + iy)$ is found to be $3i + 3i^2y$.

Mark Scheme

--

<p>2(a)</p>	<p>Use of $z^* = x - iy$ Use of $i^2 = -1$ $3iz + 2z^* = (2x - 3y) + i(3x - 2y)$</p>	<p>M1 M1 A1</p>	<p>3</p>	<p>Condone inclusion of i in I part</p>
<p>(b)</p>	<p>Equating R and I parts $2x - 3y = 7$, $3x - 2y = 8$ $z = 2 - i$</p>	<p>M1 m1 A1</p>	<p>3</p>	<p>with attempt to solve Allow $x = 2$, $y = -1$</p>
Total			6	

Question 3

- 3 For each of the following improper integrals, find the value of the integral or explain briefly why it does not have a value:

(a) $\int_9^{\infty} \frac{1}{\sqrt{x}} dx$; (3 marks)

(b) $\int_9^{\infty} \frac{1}{x\sqrt{x}} dx$. (4 marks)

Student Response

3a	$\int_9^{\infty} \frac{1}{\sqrt{x}} dx$	
	$\int_9^{\infty} x^{-1/2} dx$	
	$\left[2x^{1/2} + C \right]_9^{\infty}$ ✓	∞ Problem limit
	Let $\infty = k$	
	$\left[2x^{1/2} + C \right]_9^k$	
	$2k^{1/2} - 6$	
	As $k \rightarrow \infty$, $2k^{1/2} - 6 \rightarrow \infty$ ✓	
		3

b)	$\int_9^{\infty} \frac{1}{x\sqrt{x}}$			
	$\int_9^{\infty} x^{-3/2}$			
	$\left[-\frac{2}{3} x^{-1/2} + c \right]_9^{\infty}$	Problem limit to	M	1
			A	0
	Problem limit of ∞			
	let $k = \infty$			
	$-\frac{2}{3\sqrt{k}} + \frac{2}{3\sqrt{9}}$			
	$-\frac{2}{3\sqrt{k}} + \frac{2}{27}$			
	As $k \rightarrow \infty$	$-\frac{2}{3\sqrt{k}} \rightarrow 0$ ✓	E	1
	So $-\frac{2}{3\sqrt{k}} + \frac{2}{27} \rightarrow \frac{2}{27}$	X No FT	A	0

Commentary

Many algebraic errors occurred in the evaluation of $\int \frac{1}{x\sqrt{x}} dx$ in part (b). In this example, the candidate correctly simplified $\frac{1}{x\sqrt{x}}$ to $\frac{1}{x^{3/2}}$, but then after raising the power by one divides by the old power rather than the new power.

Mark Scheme

3(a)	$\int x^{-1/2} dx = 2x^{1/2} (+c)$ $x^{1/2} \rightarrow \infty$ as $x \rightarrow \infty$, so no value	M1A1 E1	3	M1 for correct power in integral
(b)	$\int x^{-3/2} dx = -2x^{-1/2} (+c)$ $x^{-1/2} \rightarrow 0$ as $x \rightarrow \infty$ $\int_9^{\infty} x^{-3/2} dx = -2(0 - \frac{1}{3}) = \frac{2}{3}$	M1A1 E1 A1	4	M1 for correct power in integral PI Allow A1 for correct answer even if not fully explained
Total			7	

Question 4

4 [Figure 1 and Figure 2, printed on the insert, are provided for use in this question.]

The variables x and y are related by an equation of the form

$$y = ax + \frac{b}{x+2}$$

where a and b are constants.

- (a) The variables X and Y are defined by $X = x(x+2)$, $Y = y(x+2)$.

Show that $Y = aX + b$.

(2 marks)

- (b) The following approximate values of x and y have been found:

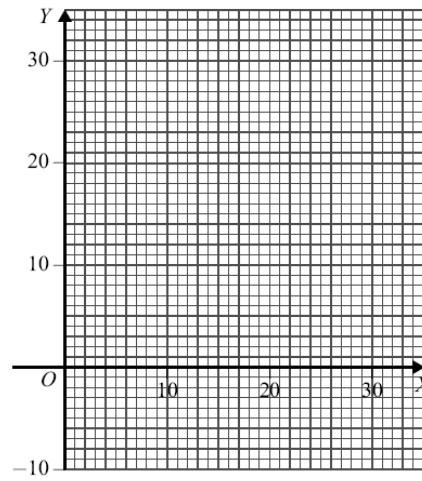
x	1	2	3	4
y	0.40	1.43	2.40	3.35

- (i) Complete the table in **Figure 1**, showing values of X and Y . (2 marks)
- (ii) Draw on **Figure 2** a linear graph relating X and Y . (2 marks)
- (iii) Estimate the values of a and b . (3 marks)

Figure 1 (for use in Question 4)

x	1	2	3	4
y	0.40	1.43	2.40	3.35
X	3			
Y	1.20			

Figure 2 (for use in Question 4)



Student Response

4a.	$y = ax + \frac{b}{x+2}$	→	$y = ax + \frac{b}{x+2}$
	$Y = y(x+2)$		$y - ax = \frac{b}{x+2}$
	$Y = (ax + \frac{b}{x+2})(x+2)$		$(x+2)(y - ax) = b$
	$Y = ax^2 + 2ax + \frac{bx}{x+2} + \frac{2b}{x+2}$		$(x+2)(\frac{b}{x+2}) = b$
	$Y = \cancel{ax}(x+2) + \frac{2b+bx}{x+2}$		$\frac{2bx}{x+2} + \frac{2b}{x+2} = b$ MB
	$Y = ax(x+2) + \frac{2b+bx}{x+2}$		$b = \frac{2b+bx}{x+2}$ X
	$\therefore X = x(x+2) + b = \frac{2b+bx}{x+2}$		
	$\therefore Y = aX + b$ 4		

Commentary

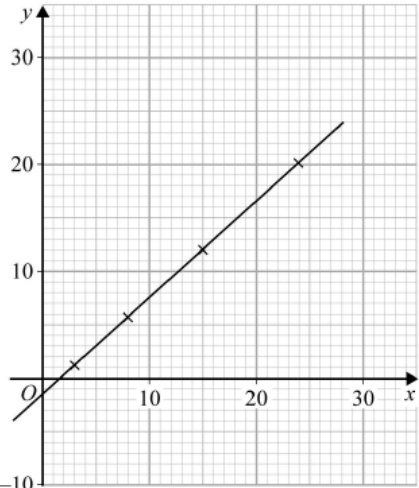
Many candidates appreciated that the first step should be to multiply the equation $y = ax + \frac{b}{x+2}$ by $x+2$.

The equation would then become $y(x+2) = ax(x+2) + b$,
or, $Y = aX + b$

This script shows a typical candidate who struggled with the required algebraic multiplication by $x+2$.

Mark Scheme

4(a)	Multiplication by $x+2$ $Y = aX + b$ convincingly shown	M1 A1	2	applied to all 3 terms AG
(b)(i)	$X = 8, 15, 24$ in table $Y = 5.72, 12, 20.1$ in table	B1 B1	2	Allow correct to 2SF

Q	Solution	Marks	Total	Comments
4(b)(ii)	 <p data-bbox="379 779 608 831">Four points plotted Reasonable line drawn</p>	<p data-bbox="842 779 890 831">B1F B1F</p> <p data-bbox="842 869 890 943">M1 A1 B1F</p>	<p data-bbox="954 808 970 831">2</p> <p data-bbox="954 920 970 943">3</p>	<p data-bbox="1023 779 1278 831">ft incorrect values in table ft incorrect points</p> <p data-bbox="1023 869 1326 1003">or algebraic method for a or b Allow from 0.88 to 0.93 incl Allow from -2 to -1 inclusive; ft incorrect points/line NMS B1 for a, B1 for b</p>
	Total		9	

Question 5

- 5 (a) Find, in radians, the general solution of the equation

$$\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

giving your answer in terms of π .

(5 marks)

- (b) Hence find the smallest positive value of x which satisfies this equation.

(2 marks)

Student Response

5) a) $\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$	$2\pi n + \alpha$	Leave blank
$\alpha = \frac{\pi}{4}$	$\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4} + 2\pi n$	B 1
	$\frac{x}{2} = 2\pi n - \frac{\pi}{12}$	M 1
	$x = 4\pi n - \frac{\pi}{6}$	m 1
		B 0
		A 0
b) <u>0.524</u> X		M 0

Commentary

Many candidates made good progress in this question. Some omitted any term containing $n\pi$ or $2n\pi$, this candidate showed a typical error and wrote the solution of

$$\cos\theta = \frac{1}{\sqrt{2}}$$

as $\theta = \frac{\pi}{4}$ rather than $\pm \frac{\pi}{4}$.

The terms $\frac{\pi}{4} - \frac{\pi}{3}$ gave $-\frac{\pi}{12}$, which was much simpler than the required term $\pm \frac{\pi}{4} -$

$$\frac{\pi}{3}$$

Mark Scheme

<p>5(a)</p> <p>$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ stated or used</p> <p>Appropriate use of \pm</p> <p>Introduction of $2n\pi$</p> <p>Subtraction of $\frac{\pi}{3}$ and multiplication by 2</p> <p>$x = -\frac{2\pi}{3} \pm \frac{\pi}{2} + 4n\pi$</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>m1</p> <p>A1</p>	<p>5</p>	<p>Degrees or decimals penalised in 5th mark only</p> <p>OE</p> <p>OE</p> <p>All terms multiplied by 2</p> <p>OE</p>
<p>5(b)</p> <p>$n = 1$ gives min pos $x = \frac{17\pi}{6}$</p>	<p>M1A1</p>	<p>2</p>	<p>NMS 1/2 provided (a) correct</p>
Total		<p>7</p>	

Question 6

6 The matrices A and B are given by

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

- (a) Calculate the matrix AB . (2 marks)
- (b) Show that A^2 is of the form kI , where k is an integer and I is the 2×2 identity matrix. (2 marks)
- (c) Show that $(AB)^2 \neq A^2B^2$. (3 marks)

Student Response

6a)	$\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \times (BA)$	M	0
	$\begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix} \times$		
b)	$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$		
	$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = 4I \quad k=4 \quad \checkmark$		2
c)	$B^2 = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \checkmark$	B	1
	$(AB)^2 = \begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix} \begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix} = \begin{pmatrix} -16 & 0 \\ 0 & -16 \end{pmatrix} \times \text{FIW}$	B	0
	A^2B^2		
	$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix} \checkmark$	B	1
	$\begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix} \neq \begin{pmatrix} -16 & 0 \\ 0 & -16 \end{pmatrix} \checkmark$		
			$\textcircled{4}$

Commentary

This question was answered well by the majority of candidates. This script shows a common error, finding BA rather than AB (forgetting that matrix multiplication is not commutative). Numerical errors were frequently seen in the multiplication of two vectors.

Mark Scheme

6(a)	$\mathbf{AB} = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct
(b)	$\mathbf{A}^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$... = 4I	B1		
		B1	2	
(c)	$(\mathbf{AB})^2 = -16\mathbf{I}$ $\mathbf{B}^2 = 4\mathbf{I}$ so $\mathbf{A}^2\mathbf{B}^2 = 16\mathbf{I}$ (hence result)	B1 B1 B1		PI Condone absence of conclusion
	Total		7	

Question 7

7 A curve C has equation

$$y = 7 + \frac{1}{x+1}$$

- (a) Define the translation which transforms the curve with equation $y = \frac{1}{x}$ onto the curve C . (2 marks)
- (b) (i) Write down the equations of the two asymptotes of C . (2 marks)
- (ii) Find the coordinates of the points where the curve C intersects the coordinate axes. (3 marks)
- (c) Sketch the curve C and its two asymptotes. (3 marks)

Student Response

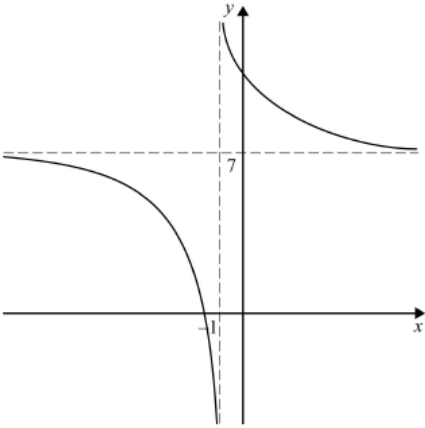
7a)	<p>Add 7 on the y axis. (✓)</p> <p>Minus 1 on the x axis. (✓) Allow 1/2</p>	1
7b)	<p>Horizontal asymptote = $\frac{1}{x} \quad x \rightarrow \infty = 0$ X</p> <p>Vertical asymptote = $x+1=0 \quad \checkmark \quad x=-1 \quad \checkmark = -1$</p>	0
7b)	<p>Vertical asymptote = $x+1=0 \quad \checkmark \quad x=-1 \quad \checkmark = -1$</p>	1
7b)	<p>y int. $7 + \frac{1}{0+1} = 8 \quad \checkmark$</p> <p>x int. $1 \neq 0$: not defined defined defined X</p>	1
7c)		2
		5

Commentary

Candidates were required to use standard mathematical terminology. In part (a), this candidate's description of "add" and "minus" was not adequate. In part (b) (i), most candidates found the vertical asymptote to be $x + 1 = 0$, or $x = -1$. The identification of the horizontal asymptote proved more challenging, as in this script, where

$\frac{1}{x} \pm \infty = 0$, did not identify the equation of a line.

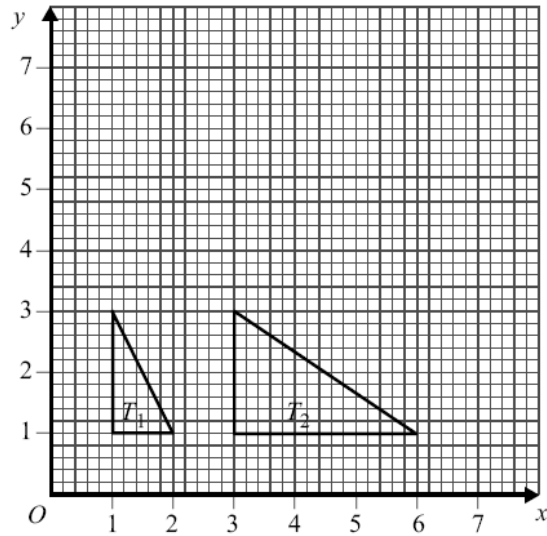
Mark Scheme

Q	Solution	Marks	Total	Comments
7(a)	Curve translated 7 in y direction ... and 1 in negative x direction	B1 B1	2	or answer in vector form
(b)(i)	Asymptotes $x = -1$ and $y = 7$	B1B1	2	
(ii)	Intersections at $(0, 8)$ and $(-\frac{8}{7}, 0)$	B1 M1A1	3	Allow AWRT -1.14 ; NMS $1/2$
(c)	 <p>At least one branch Complete graph All correct including asymptotes</p>	B1 B1 B1	3	of correct shape translation of $y = 1/x$ in roughly correct positions
	Total		10	

Question 8

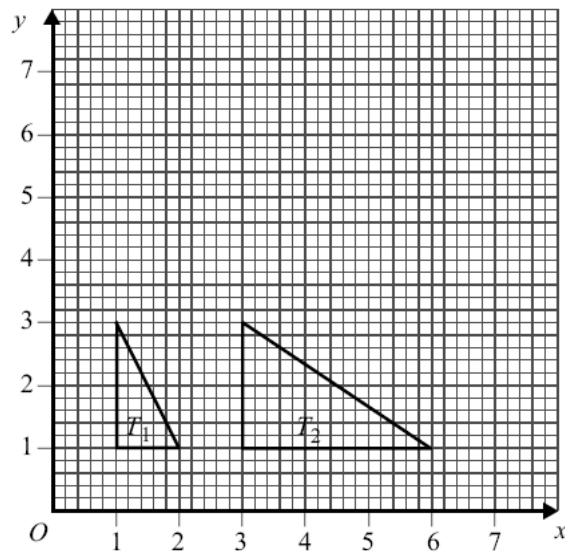
8 [Figure 3, printed on the insert, is provided for use in this question.]

The diagram shows two triangles, T_1 and T_2 .



- (a) Find the matrix of the stretch which maps T_1 to T_2 . (2 marks)
- (b) The triangle T_2 is reflected in the line $y = x$ to give a third triangle, T_3 .
On Figure 3, draw the triangle T_3 . (2 marks)
- (c) Find the matrix of the transformation which maps T_1 to T_3 . (3 marks)

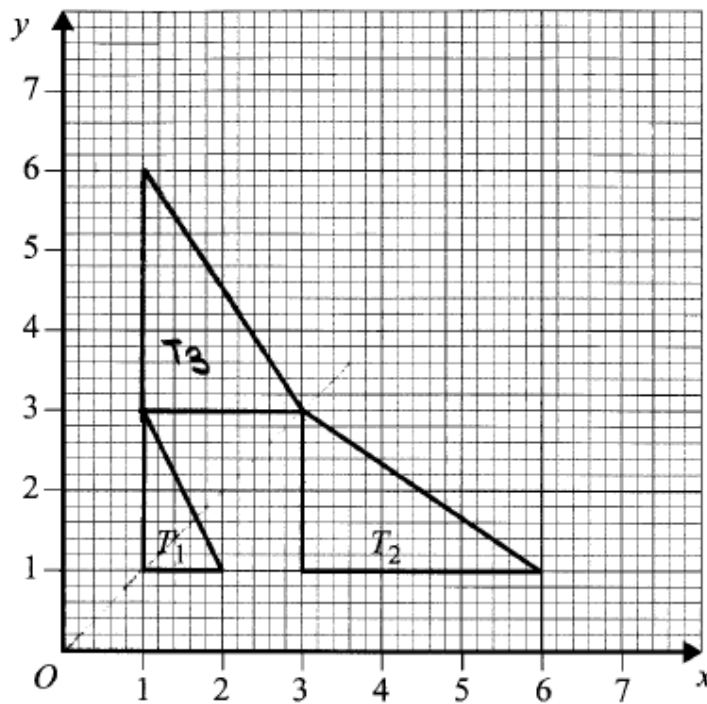
Figure 3 (for use in Question 8)



Student Response

8 a)	$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \rightarrow \begin{pmatrix} 3 \\ 3 \end{pmatrix}$		
	$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 3 \\ 1 \end{pmatrix}$		
	$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 6 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ ✓	2
d	$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 3 \end{pmatrix}$		2
	$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 6 \end{pmatrix}$ ✗		
	$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ ✗	$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ ✗	0
			<u>4</u>

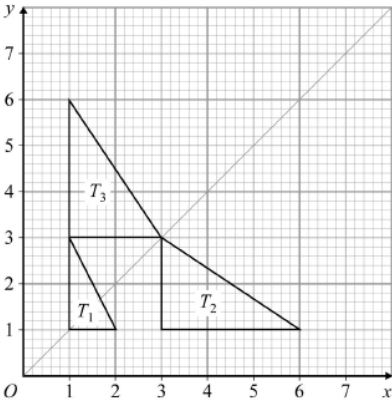
Figure 3 (for use in Question 8)



Commentary

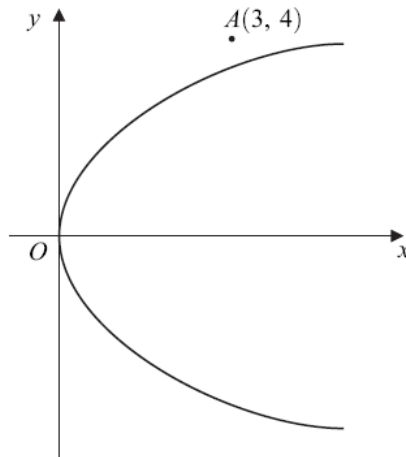
Many candidates found the matrix in part (a) and in part (b) drew the triangle T_3 . As shown in this script, some candidates assumed that T_1 moved “up” into T_3 and did not check the transformation of the points. However, the point (2, 1) on triangle T_1 was transformed into the point (6, 1) in triangle T_2 . This point was reflected into the point (1, 6) in triangle T_3 . Thus the combined transformation did not transform (2, 1) into (3, 3) as this candidate assumed.

Mark Scheme

<p>8(a)</p>	<p>Matrix is $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$</p>	<p>M1A1</p>	<p>2</p>	<p>M1 if zeros in correct positions; allow NMS</p>
<p>(b)</p>	 <p>Third triangle shown correctly</p>	<p>M1A1</p>	<p>2</p>	<p>M1A0 if one point wrong</p>
<p>8(c)</p>	<p>Matrix of reflection is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Multiplication of above matrices Answer is $\begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$</p>	<p>B1 M1 A1F</p>	<p>3</p>	<p>Alt: calculating matrix from the coordinates: M1 A2,1 in correct order ft wrong answer to (a); NMS 1/3</p>
<p>Total</p>			<p>7</p>	

Question 9

9 The diagram shows the parabola $y^2 = 4x$ and the point A with coordinates $(3, 4)$.



- (a) Find an equation of the straight line having gradient m and passing through the point $A(3, 4)$. *(2 marks)*
- (b) Show that, if this straight line intersects the parabola, then the y -coordinates of the points of intersection satisfy the equation

$$my^2 - 4y + (16 - 12m) = 0 \quad (3 \text{ marks})$$

- (c) By considering the discriminant of the equation in part (b), find the equations of the two tangents to the parabola which pass through A .
(No credit will be given for solutions based on differentiation.) *(5 marks)*
- (d) Find the coordinates of the points at which these tangents touch the parabola. *(4 marks)*

Student Response

4	$y^2 = 4x$	$A(3, 4)$	blank	
a)	$y - y_1 = m(x - x_1)$ $y - 4 = m(x - 3)$ $y = mx - 3m + 4$	✓	2	
b)	$y^2 = (mx - 3m + 4)^2 + 9m^2 + 16$ $4x = m^2x^2 + 9m^2 + 16$ $= m^2x^2 - 4x + 9m^2 + 16$	X	0	
c)	$a = m$ $b = -4$ $c = 16 - 12m$	$b^2 - 4ac$ $-4^2 - 4(m)(16 - 12m)$ $16 - 64m + 48m^2 = 0$ ✓ $48m^2 - 64m + 16 = 0$ $12m^2 - 16m + 4 = 0$ $3m^2 - 4m + 1 = 0$ ✓ $(m-1)(3m-1) = 0$ $m = 1 \quad m = \frac{1}{3}$ ✓	✓	3
	\therefore tangents = $y = x + 1$ ✓ $3y = x + 9$ ✓		2	
d)	$\frac{1}{3}y^2 = 4y + (16 - 12(\frac{1}{3})) = 0$ $\frac{1}{3}y^2 - 4y + 12 = 0$ $y = 6$			
	$y^2 - 4y + (16 - 12(1)) = 0$ $y^2 - 4y + 4 = 0$ $y = 2$	✓	M 2	

$y = mx - 3m + 4$		blank
$6 = \frac{1}{3}x - 3(\frac{1}{3}) + 4$		
$6 = \frac{1}{3}x - 5$ X		
$1 = \frac{1}{3}x$		
$3 = x$	(3, 6)	WAAAA A 0
$2 = 1x - 3(1) + 4$		
$2 = x + 1$		
$1 = x$	(1, 2)	✓ A 1
		(10)

Commentary

Part (a) was answered well. Instead of using the simple substitution in part (b), whereby $y = mx - 3m + 4 \Rightarrow 4y = 4mx - 12m + 16$
 $4y = my^2 - 12m + 16$,
some candidates, as shown, assumed that they must eliminate y^2 and hence attempted to square y , but rarely did this correctly. Then in part (c), most candidates, as seen in this script, correctly equated the discriminant to zero, and so found the two values of m .

Mark Scheme

9(a)	Equation is $y - 4 = m(x - 3)$	M1A1	2	OE; M1A0 if one small error
(b)	Elimination of x $4y - 16 = m(y^2 - 12)$ Hence result	M1 A1 A1	3	OE (no fractions) convincingly shown (AG)
(c)	Discriminant equated to zero $(3m - 1)(m - 1) = 0$ Tangents $y = x + 1$, $y = \frac{1}{3}x + 3$	M1 m1A1 A1A1	5	OE; m1 for attempt at solving OE
(d)	$m = 1 \Rightarrow y^2 - 4y + 4 = 0$ so point of contact is (1, 2) $m = \frac{1}{3} \Rightarrow \frac{1}{3}y^2 - 4y + 12 = 0$ so point of contact is (9, 6)	M1 A1 M1 A1	4	OE; $m = 1$ needed for this OE; $m = \frac{1}{3}$ needed for this
	Total		14	
	TOTAL		75	