



Teacher Support Materials 2008

Maths GCE

Paper Reference MFP3

Copyright © 2008 AQA and its licensors. All rights reserved.

Permission to reproduce all copyrighted material has been applied for. In some cases, efforts to contact copyright holders have been unsuccessful and AQA will be happy to rectify any omissions if notified.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334). Registered address: AQA, Devas Street, Manchester M15 6EX.
Dr Michael Cresswell, Director General.

Mark scheme

<p>1</p>	$k_1 = 0.1 \times \ln(2 + 3)$ $= 0.1609(4379\dots) \quad (= *)$ $k_2 = 0.1 \times f(2.1, 3 + * \dots)$ $\dots = 0.1 \times \ln(2.1 + 3.16094\dots)]$ $\dots = 0.1660(31\dots)$ $y(2.1) = y(2) + \frac{1}{2}[k_1 + k_2]$ $= 3 + 0.5 \times 0.3269748\dots$ $= 3.163487\dots = 3.1635 \text{ to 4dp}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p></p> <p>6</p>	<p>PI</p> <p>PI</p> <p>Dep on previous two Ms and numerical values for k's</p> <p>Must be 3.1635</p>
	<p>Total</p>		<p>6</p>	

Question 2

- 2 (a) Find the values of the constants a , b , c and d for which $a + bx + c \sin x + d \cos x$ is a particular integral of the differential equation

$$\frac{dy}{dx} - 3y = 10 \sin x - 3x \quad (4 \text{ marks})$$

- (b) Hence find the general solution of this differential equation. (3 marks)

Student response

Question number		Leave blank
2		
a.	$y = a + bx + c \sin x + d \cos x$	
	$\frac{dy}{dx} = b + c \cos x - d \sin x$ ✓	
	$\frac{d^2y}{dx^2} = -c \sin x - d \cos x$	
	$\frac{dy}{dx} - 3y = 10 \sin x - 3x$	
	$= b + c \cos x - d \sin x - 3a - 3bx - 3c \sin x - 3d \cos x = 10 \sin x - 3x$ ✓ M1	
	$b - 3a = 0$ ✓ $a = \frac{1}{3}$ ✓	
	$-3b = -3$ ✓ $b = 1$ ✓	
	$c - 3d = 0$ ✓ ①	M1 A1
	$-d - 3c = 10$ ✓ ②	
	① × 3 = $3c - 9d = 0$ ③	
	③ + ② = $-10d = 10$ $d = -1$	
	$c = -3$	A1
	$a = \frac{1}{3}$ $b = 1$ $c = -3$ $d = -1$ ✓	4
b.	ⒸF $m - 3 = 0$ $m = 3$ ✓ M1	
	$y = e^{3x} (A + Bx)$ ✓ A0	
	ⒼS $y = e^{3x} (A + Bx) + \frac{1}{3} + x - 3 \sin x - \cos x$ B0	1
		5

Commentary

In the exemplar the candidate gave a correct solution to part (a) by equating coefficients to form and then solve the four equations to find the correct values for the four unknowns a , b , c and d . A significant number of candidates, like the one in the exemplar, wasted time by finding an expression for $\frac{d^2 y}{dx^2}$ which was not required in the solution to find the particular integral of the first order differential equation. In part (b) the exemplar illustrates a common error. The candidate correctly solved the auxiliary equation $m - 3 = 0$ but incorrectly took this to be a repeated root of an auxiliary equation to a second order differential equation and gave the general solution of the first order differential equation with two arbitrary constants instead of the required one.

Mark Scheme

<p>2(a)</p> <p>PI: $y_{PI} = a + bx + c \sin x + d \cos x$ $y'_{PI} = b + c \cos x - d \sin x$ $b + c \cos x - d \sin x - 3a - 3bx - 3c \sin x - 3d \cos x = 10 \sin x - 3x$</p> <p>$b - 3a = 0$; $-3b = -3$; $c - 3d = 0$; $-d - 3c = 10$ $a = \frac{1}{3}$; $b = 1$; $c = -3$; $d = -1$ $y_{PI} = \frac{1}{3} + x - 3 \sin x - \cos x$</p>	<p>M1</p> <p>M1</p> <p>A2,1</p>	<p>4</p>	<p>Substituting into DE</p> <p>Equating coefficients (at least 2 eqns)</p> <p>A1 for any two correct</p>
<p>(b)</p> <p>Aux. eqn. $m - 3 = 0$ $(y_{CF} =) A e^{3x}$ $(y_{GS} =) A e^{3x} + \frac{1}{3} + x - 3 \sin x - \cos x$</p>	<p>M1</p> <p>A1</p> <p>B1F</p>	<p>3</p>	<p>Altn. $\int y^{-1} dy = \int 3 dx$ OE (M1) $A e^{3x}$ OE (c's CF + c's PI) with 1 arbitrary constant</p>
Total		<p>7</p>	

Question 3

- 3 (a) Show that $x^2 = 1 - 2y$ can be written in the form $x^2 + y^2 = (1 - y)^2$. (1 mark)
- (b) A curve has cartesian equation $x^2 = 1 - 2y$.
Find its polar equation in the form $r = f(\theta)$, given that $r > 0$. (5 marks)

Student Response

3. (a) $x^2 = 1 - 2y \Rightarrow x^2 + y^2 = 1 - 2y + y^2 = (1 - y)^2$ ✓	1 5 ⑥
(b) $\because x^2 + y^2 = r^2$ and $r \sin \theta = y$ ✓	
$\therefore x^2 + y^2 = (1 - y)^2 \Rightarrow r^2 = (1 - r \sin \theta)^2$ ✓	
$\Rightarrow r = \pm(1 - r \sin \theta)$ ✓	
If $r = 1 - r \sin \theta$, $(1 + \sin \theta)r = 1 \Rightarrow r = \frac{1}{1 + \sin \theta}$ ✓	
If $r = -(1 - r \sin \theta)$, $r \sin \theta - 1 = r \Rightarrow (r \sin \theta - 1)r = 1 \Rightarrow r = \frac{1}{\sin \theta - 1}$	
Since $-1 \leq \sin \theta \leq 1$, $1 + \sin \theta \geq 0$ while $\sin \theta - 1 \leq 0$ ✓	
$\because r > 0 \therefore r = \frac{1}{1 + \sin \theta}$ ✓	

Commentary

Part (a) was generally answered correctly but it was unusual to see solutions for which the fifth mark was awarded in part (b). In the exemplar the candidate scored this final mark because, within this excellent solution, both square roots (the \pm) had been considered and a full and accurate justification for eliminating the solution $r = \frac{1}{\sin \theta - 1}$ was given by the candidate.

Mark Scheme

3(a)	$x^2 + y^2 = 1 - 2y + y^2 \Rightarrow x^2 + y^2 = (1 - y)^2$	B1	1	AG
(b)	$x^2 + y^2 = r^2$	M1		Or $x = r \cos \theta$
	$y = r \sin \theta$	M1		
	$x^2 = 1 - 2y$ so $x^2 + y^2 = (1 - y)^2$ $\Rightarrow r^2 = (1 - r \sin \theta)^2$	A1		OE eg $r^2 \cos^2 \theta = 1 - 2r \sin \theta$ PI by the next line
	$r = 1 - r \sin \theta$ or $r = -(1 - r \sin \theta)$ $r(1 + \sin \theta) = 1$ or $r(1 - \sin \theta) = -1$	m1		Either
$r > 0$ so $r = \frac{1}{1 + \sin \theta}$	A1	5	CSO	
	Total		6	

Question 4

- 4 (a) A differential equation is given by

$$x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 3x^2$$

Show that the substitution

$$u = \frac{dy}{dx}$$

transforms this differential equation into

$$\frac{du}{dx} - \frac{1}{x}u = 3x \quad (2 \text{ marks})$$

- (b) By using an integrating factor, find the general solution of

$$\frac{du}{dx} - \frac{1}{x}u = 3x$$

giving your answer in the form $u = f(x)$. (6 marks)

- (c) Hence find the general solution of the differential equation

$$x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 3x^2$$

giving your answer in the form $y = g(x)$. (2 marks)

Student Response

4d	$x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 3x^2$	Leave blank
	$u = \frac{dy}{dx}$ ✓	
	$\frac{du}{dx} = \frac{d^2y}{dx^2}$ ✓	
	$x \frac{du}{dx} - u = 3x^2$ ✓	2
	$\frac{du}{dx} - \frac{1}{x}u = 3x$ ✓	
b	$I = e^{\int P(x) dx}$ ✓	
	$\int \frac{1}{x} dx = -\ln x = \ln x^{-1}$ M1 A1	
	$I = e^{\ln x^{-1}} = \frac{1}{x}$ A1	
	$\frac{1}{x}u = \int 3 dx$ ✓ M1	
	$\frac{1}{x}u = 3x$ M0	4
	$u = 3x^2$ A0	
c	$\frac{dy}{dx} = 3x^2$	
	$y = x^3$ M1 A0	1
		⑦

Commentary

A significant number of candidates lost some marks because they forgot to include the constants of integration. The exemplar illustrates this error which resulted in the candidate giving a general solution of the first order differential equation in part (b) with no arbitrary constant and giving a general solution of the second order differential equation in part (c) also with no arbitrary constants. Candidates would have been well advised to check that in their general solution of a differential equation, the number of arbitrary constants was the same as the order of the differential equation.

Mark Scheme

<p>4(a)</p> $u = \frac{dy}{dx} \Rightarrow \frac{du}{dx} = \frac{d^2y}{dx^2}$ $x \frac{du}{dx} - u = 3x^2 \Rightarrow \frac{du}{dx} - \frac{1}{x}u = 3x$	<p>M1</p> <p>A1</p>	<p>2</p>	<p>AG Substitution into LHS of DE and completion</p>
<p>(b)</p> <p>IF is $\exp\left(\int -\frac{1}{x} dx\right)$</p> $= e^{-\ln x}$ $= x^{-1} \text{ or } \frac{1}{x}$ $\frac{d}{dx}[ux^{-1}] = 3$ $\Rightarrow ux^{-1} = 3x + A$ $u = 3x^2 + Ax$	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>m1</p> <p>A1</p>	<p>6</p>	<p>and with integration attempted</p> <p>or multiple of x^{-1}</p> <p>LHS as differential of $u \times \text{IF}$. PI</p> <p>Must have an arbitrary constant (Dep. on previous M1 only)</p>
<p>(c)</p> $\frac{dy}{dx} = 3x^2 + Ax$ $y = x^3 + \frac{Ax^2}{2} + B$	<p>M1</p> <p>A1F</p>	<p>2</p>	<p>Replaces u by $\frac{dy}{dx}$ and attempts to integrate</p> <p>fit on cand's u but solution must have two arbitrary constants</p>
Total		10	

Question 5

5 (a) Find $\int x^3 \ln x \, dx$. (3 marks)

(b) Explain why $\int_0^e x^3 \ln x \, dx$ is an improper integral. (1 mark)

(c) Evaluate $\int_0^e x^3 \ln x \, dx$, showing the limiting process used. (3 marks)

Student Response

5a $\int x^3 \ln x \, dx$ $u = \ln x$ $\frac{du}{dx} = \frac{1}{x}$ $\frac{dv}{dx} = x^3$ $v = \frac{x^4}{4}$ ✓

$\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \int \frac{x^3}{4} \, dx$ $uv - \int v \frac{du}{dx}$

$= \frac{x^4}{4} \ln x - \frac{x^4}{16} + c$ ✓

b ~~Area~~ $x^3 \ln x$ is not defined at point 0 as $\ln 0$ does not exist ✓

c $\int_0^e x^3 \ln x \, dx = \lim_{a \rightarrow 0} \int_a^e x^3 \ln x \, dx$

$= \lim_{a \rightarrow 0} \left[\frac{x^4}{4} \ln x - \frac{x^4}{16} \right]_a^e$

$= \lim_{a \rightarrow 0} \left[\frac{e^4}{4} - \frac{e^4}{16} - \frac{a^4 \ln a}{4} + \frac{a^4}{16} \right]$

as $a \rightarrow 0$ $\frac{-a^4 \ln a}{4} + \frac{a^4}{16} \rightarrow 0$ ✓

$\therefore \int_0^e x^3 \ln x \, dx = \frac{e^4}{4} - \frac{e^4}{16}$ ✓

A \rightarrow 6

3
1
2
6

Commentary

The candidate in the exemplar used integration by parts to find the correct expression for $\int x^3 \ln x \, dx$ and, in part (b), provided a correct explanation for why $\int_0^e x^3 \ln x \, dx$ is an improper integral. In part (c) the candidate showed excellent detail of the limiting process used, in particular the inclusion of $\int_0^e x^3 \ln x \, dx = \left\{ \lim_{a \rightarrow 0} \int_a^e x^3 \ln x \, dx \right\}$ and the statement $\left\{ \text{as } a \rightarrow 0, -\frac{a^4}{4} \ln a \rightarrow 0 \right\}$. The candidate failed to score the final accuracy mark because the expression $\frac{e^4}{4} - \frac{e^4}{16}$ had not been simplified to $\frac{3e^4}{16}$.

Mark Scheme

5(a)	$\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \int \frac{x^4}{4} \left(\frac{1}{x} \right) dx$	M1		... = $kx^4 \ln x \pm \int f(x)$, with $f(x)$ not involving the 'original' $\ln x$
		A1		
 = $\frac{x^4}{4} \ln x - \frac{x^4}{16} + c$	A1	3	Condone absence of '+ c'
(b)	Integrand is not defined at $x = 0$	E1	1	OE
(c)	$\int_0^e x^3 \ln x \, dx = \left\{ \lim_{a \rightarrow 0} \int_a^e x^3 \ln x \, dx \right\}$ $= \frac{3e^4}{16} - \lim_{a \rightarrow 0} \left[\frac{a^4}{4} \ln a - \frac{a^4}{16} \right]$	M1		F(e) - F(a)
		B1		Accept a general form eg $\lim_{x \rightarrow 0} x^k \ln x = 0$
		A1	3	CSO
Total			7	

Question 6

- 6 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 10e^{-2x} - 9 \quad (10 \text{ marks})$$

- (b) Hence express y in terms of x , given that $y = 7$ when $x = 0$ and that $\frac{dy}{dx} \rightarrow 0$ as $x \rightarrow \infty$. (4 marks)

Student Response

Question number		Leave blank
⑥		
a)	$m^2 - 2m - 3$ ✓ $(m-3)(m+1)$ ✓	M1
	$m=3 \quad m=-1$ ✓	A1
	CF: $y = Ae^{3x} + Be^{-x}$ ✓	M1
	PI $y = \lambda e^{-2x} + \frac{1}{3} \beta$ ✓	M1
	$y' = -2\lambda e^{-2x}$ ✓	A1
	$y'' = 4\lambda e^{-2x}$ ✓	A1
	$4\lambda e^{-2x} - 2(-2\lambda e^{-2x}) - 3(\lambda e^{-2x}) - \frac{1}{3}\beta = 10e^{-2x} - 9$ ✓	M1
	$5\lambda e^{-2x} - 3\beta = 10e^{-2x} - 9$ ✓	M1
	$3\beta = 9$ ✓	B1
	$\beta = 3$ ✓	B1
	$5\lambda = 10$ ✓	A1
	$\lambda = 2$ ✓	A1
	\therefore PI = $y = 2e^{-2x} + 3$ ✓	B1
	\therefore general solution = CF + PI ✓	
	$y = Ae^{3x} + Be^{-x} + 2e^{-2x} + 3$ ✓	
b)	$7 = A + B + 2 + 3$ ✓	B1
	$2 = A + B$	
	$y' = 3Ae^{3x} - Be^{-x} - 4e^{-2x}$ ✓	
	$y' = e^{-x}(3Ae^{4x} - B - 4e^{-x})$	
	$y = \frac{1}{5}e^{-x}(3Ae^{4x} - B - 4e^{-x})$	
	$0 = 3Ae - B = 4$ X	
	$(A+B)2A - 2B = 4$	
	$2 + 2A - 2B = 4$	
	$3 \cdot 2 = 2A - 2B \quad 2(A-B) = 3$	
	$\therefore y = e^{3x} - 0.5e^{-x} + 2e^{-2x} + 3$ ✓	A1

10

1
11

Commentary

The exemplar illustrates a typical answer to this mainly unstructured question. The candidate gave a full correct solution to find the general solution of the given second order differential equation in part (a). In part (b) the candidate correctly used the given boundary condition, $y = 7$ when $x = 0$, to get $2 = A + B$ but did not apply the limiting boundary condition $\frac{dy}{dx} \rightarrow 0$ as $x \rightarrow \infty$ correctly. The incorrect equation, $0 = 3A - B - 4$, was obtained by many candidates and effectively came from using the more familiar boundary condition $\frac{dy}{dx} = 0$ when $x = 0$.

Mark Scheme

Q	Solution	Marks	Total	Comments	
(a)	Aux eqn: $m^2 - 2m - 3 = 0$	M1	10	(c's CF+c's PI) with 2 arbitrary constants	
	$m = -1, 3$	A1			PI
	CF ($y_c =$) $Ae^{3x} + Be^{-x}$	M1			
	Try ($y_{PI} =$) $a e^{-2x} (+b)$	M1			
	$\frac{dy}{dx} = -2ae^{-2x}$	A1			
	$\frac{d^2y}{dx^2} = 4ae^{-2x}$	A1			
	Substitute into DE gives $4ae^{-2x} + 4ae^{-2x} - 3ae^{-2x} - 3b = 10e^{-2x} - 9$	M1			
	$\Rightarrow a = 2$ $b = 3$	A1 B1			
	$(y_{GS} =) Ae^{3x} + Be^{-x} + 2e^{-2x} + 3$	B1F			
	(b)	$x = 0, y = 7 \Rightarrow 7 = A + B + 2 + 3$			B1F
$\frac{dy}{dx} = 3Ae^{3x} - Be^{-x} - 4e^{-2x}$					
As $x \rightarrow \infty, e^{-bx} \rightarrow 0, \frac{dy}{dx} \rightarrow 0$ so $A = 0$		B1			
When $A = 0, 5 = 0 + B + 3 \Rightarrow B = 2$		B1F			
$y = 2e^{-x} + 2e^{-2x} + 3$		A1			
Total			14		

Question 7

7 (a) Write down the expansion of $\sin 2x$ in ascending powers of x up to and including the term in x^3 . (1 mark)

(b) (i) Given that $y = \sqrt{3 + e^x}$, find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $x = 0$. (5 marks)

(ii) Using Maclaurin's theorem, show that, for small values of x ,

$$\sqrt{3 + e^x} \approx 2 + \frac{1}{4}x + \frac{7}{64}x^2 \quad (2 \text{ marks})$$

(c) Find

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{3 + e^x} - 2}{\sin 2x} \right] \quad (3 \text{ marks})$$

Student Response

class
blank

⑦

(a) $\sin 2x = 2x - \frac{(2x)^3}{3!}$

$\sin 2x = 2x - \frac{8x^3}{6}$ ✓

B1

$\sin 2x = 2x - \frac{4}{3}x^3$

⑧

(a) $y = (3 + e^x)^{1/2}$

$y' = \frac{1}{2}e^x (3 + e^x)^{-1/2}$ ✓

M1A1

$u = \frac{1}{2}e^x$

$v = (3 + e^x)^{-1/2}$

$u' = \frac{1}{2}e^x$

$v' = -\frac{1}{2}e^x (3 + e^x)^{-3/2}$

$y'' = -\frac{1}{4}e^{2x} (3 + e^x)^{-3/2} + \frac{1}{2}e^x (3 + e^x)^{-1/2}$ ✓ M1A1

$F'(0) = \frac{1}{4}$ ✓

$F''(0) = \frac{7}{32}$ ✓

A1

5

(ii) $F(0) = 2$ ✓

$\therefore \sqrt{3 + e^x} = 2 + \frac{1}{4}x + \frac{7}{32 \times 2}x^2$ ✓

M1

$\Rightarrow = 2 + \frac{1}{4}x + \frac{7}{64}x^2$ ✓

A1

2

(c) $\lim_{x \rightarrow 0} \left[\frac{2 + \frac{1}{4}x + \frac{7}{64}x^2 - 2}{2x - \frac{4}{3}x^3} \right]$ ✓ M1

$\lim_{x \rightarrow 0} \left[\frac{\frac{1}{4}x}{2x} \right] = \frac{1}{8}$ ✓

M0

A0

1

⑨

Commentary

In part (a) the candidate in the exemplar quoted the correct expansion of $\sin 2x$ and, in particular, had replaced $3!$ by 6 . In part (b)(i) the candidate showed good skills in applying the chain rule and product rule for differentiating the function. In (b)(ii) the candidate clearly stated the remaining value, $f(0)=2$, which is required and applied Maclaurin's theorem correctly. In part (c) the candidate had used previously found expansions but did not divide the denominator and numerator by x to get a constant term in each before applying the limit as x tends to zero.

Mark Scheme

7(a)	$\sin 2x \approx 2x - \frac{(2x)^3}{3!} + \dots = 2x - \frac{4}{3}x^3 + \dots$	B1	1	
(b)(i)	$\frac{dy}{dx} = \frac{1}{2}(3+e^x)^{\frac{1}{2}}(e^x)$	M1 A1		Chain rule
	$\frac{d^2y}{dx^2} = \frac{1}{2}e^x(3+e^x)^{\frac{1}{2}} - \frac{1}{4}(3+e^x)^{\frac{3}{2}}(e^{2x})$	M1 A1		Product rule OE OE
	$y'(0) = \frac{1}{4}; y''(0) = \frac{1}{4} - \frac{1}{32} = \frac{7}{32}$	A1	5	CSO
(ii)	$y(0) = 2; y'(0) = \frac{1}{4}; y''(0) = \frac{1}{4} - \frac{1}{32} = \frac{7}{32}$			
	McC. Thm: $y(0) + x y'(0) + \frac{x^2}{2} y''(0)$ $\sqrt{3+e^x} \approx 2 + \frac{1}{4}x + \frac{7}{64}x^2$	M1 A1	2	CSO; AG
(c)	$\left[\frac{\sqrt{3+e^x} - 2}{\sin 2x} \right] = \left[\frac{2 + \frac{1}{4}x + \frac{7}{64}x^2 - 2}{2x - \frac{4}{3}x^3} \right]$	M1		
	$= \left[\frac{\frac{1}{4} + \frac{7}{64}x + \dots}{2 - \frac{4}{3}x^2 + \dots} \right]$	m1		Dividing numerator and denominator by x to get constant term in each
	$\lim_{x \rightarrow 0} \left[\frac{\sqrt{3+e^x} - 2}{\sin 2x} \right] = \frac{1}{2} = \frac{1}{8}$	A1F	3	Ft on cand's answer to (a) provided of the form $ax+bx^3$
Total			11	

Question 8

8 The polar equation of a curve C is

$$r = 5 + 2 \cos \theta, \quad -\pi \leq \theta \leq \pi$$

- (a) Verify that the points A and B , with **polar coordinates** $(7, 0)$ and $(3, \pi)$ respectively, lie on the curve C . (2 marks)
- (b) Sketch the curve C . (2 marks)
- (c) Find the area of the region bounded by the curve C . (6 marks)
- (d) The point P is the point on the curve C for which $\theta = \alpha$, where $0 < \alpha \leq \frac{\pi}{2}$. The point Q lies on the curve such that POQ is a straight line, where the point O is the pole. Find, in terms of α , the area of triangle OQB . (4 marks)

Student Response

8	a) when $\theta = 0$.	
	$r = 5 + 2 \times \cos 0 = 5 + 2 \times 1 = 7 \therefore A$ lies on the curve	
	when $\theta = \pi$.	
	$r = 5 + 2 \times \cos \pi = 5 + 2 \times (-1) = 3 \therefore B$ lies on the curve	
	b)	

2

B | B₀

c) $A = \frac{1}{2} \int_{-\pi}^{\pi} (5 + 2\cos\theta)^2 d\theta$
 $= \frac{1}{2} \int_{-\pi}^{\pi} (25 + 20\cos\theta + 4\cos^2\theta) d\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\cos 2\theta = 2\cos^2\theta - 1$
 $\frac{\cos 2\theta + 1}{2} = \cos^2\theta$
 $\therefore 4\cos^2\theta = 2\cos 2\theta + 2$

$\therefore = \frac{1}{2} \int_{-\pi}^{\pi} (25 + 20\cos\theta + 2\cos 2\theta + 2) d\theta$
 $= \frac{1}{2} [25\theta + 20\sin\theta + \sin 2\theta + 2\theta]_{-\pi}^{\pi}$
 $= \frac{1}{2} [(27\pi + 20 \times 0 + \sin 2\pi) - (-27\pi)]$
 $= \frac{1}{2} (54\pi) = 27\pi$

d) $OB = 3$ $OQ = 5 + 2\cos(-\pi + \alpha)$ $(\angle POA = \angle BOQ)$ M1
 area of $\triangle BOQ = \frac{1}{2} OB \cdot OQ \times \sin \angle BOQ$ B1 M1
 $= \frac{3}{2} \times [5 + 2\cos(\alpha - \pi)] \times \sin \alpha$

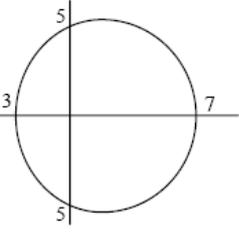
6
3
12

Commentary

In the exemplar the candidate produced full correct solutions to parts (a) and (c). Although the candidate's sketch in part (b) should not have had a 'dent' on the left hand side, this 'error' was condoned, but full marks were not scored because there was no indication of vertical scaling. A '5' at the top of the vertical dotted line would have been sufficient. Only a minority of candidates scored all the four marks in part (d). The candidate in the exemplar produced a very good attempt and found the correct expression for OQ by finding r when $\square = -\square + \square$.

The correct formula for the area of the triangle was then used but the final step, to reach an expression in \square only (not in \square and \square), was not carried out. The identity $\cos(A-B) = \cos A \cos B + \sin A \sin B$, or equivalent, should have been used to write $\cos(\square - \square)$ as $-\cos \square$.

Mark Scheme

<p>8(a)</p>	<p>$\theta = 0, r = 5 + 2\cos 0 = 7$ {A lies on C}</p> <p>$\theta = \pi, r = 5 + 2\cos \pi = 3$ {B lies on C}</p>	<p>B1</p> <p>B1</p>	<p></p> <p>2</p>	
<p>(b)</p>		<p>B1</p> <p>B1</p>	<p></p> <p>2</p>	<p>Closed single loop curve, with (indication of) symmetry</p> <p>Critical values, 3,5,7 indicated</p>
<p>(c)</p>	<p>Area = $\frac{1}{2} \int (5 + 2\cos \theta)^2 d\theta$</p> <p>$= \frac{1}{2} \int_{-\pi}^{\pi} (25 + 20\cos \theta + 4\cos^2 \theta) d\theta$</p> <p>$= \frac{1}{2} \int_{-\pi}^{\pi} (25 + 20\cos \theta + 2(\cos 2\theta + 1)) d\theta$</p> <p>$= \frac{1}{2} [27\theta + 20\sin \theta + \sin 2\theta]_{-\pi}^{\pi}$</p> <p>$= 27\pi$</p>	<p>M1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1F</p> <p>A1</p>	<p></p> <p></p> <p></p> <p>6</p>	<p>Use of $\frac{1}{2} \int r^2 d\theta$</p> <p>OE for correct expansion of $(5 + 2\cos \theta)^2$</p> <p>For correct limits</p> <p>Attempt to write $\cos^2 \theta$ in terms of $\cos 2\theta$</p> <p>Correct integration ft wrong non-zero coefficients in $a + b\cos \theta + c\cos 2\theta$</p> <p>CSO</p>
<p>(d)</p>	<p>Triangle OBQ with $OB = 3$ and angle $BOQ = \alpha$</p> <p>$OQ = 5 + 2\cos(-\pi + \alpha)$</p> <p>Area of triangle $OQB = \frac{1}{2} OB \times OQ \sin \alpha$</p> <p>$= \frac{3}{2} (5 - 2\cos \alpha) \sin \alpha$</p>	<p>B1</p> <p>M1</p> <p>m1</p> <p>A1</p>	<p></p> <p></p> <p></p> <p>4</p>	<p>PI</p> <p>OE</p> <p>Dep. on correct method to find OQ</p> <p>CSO</p>
<p>Total</p>			<p>14</p>	