A-level

Mathematics

MM03

Mark scheme

6360

June 2015

Version 1.0: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

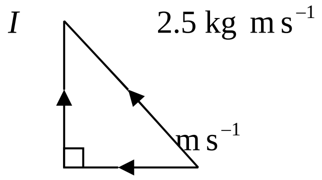
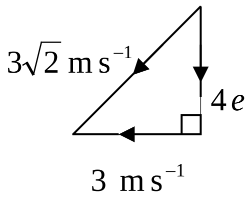
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

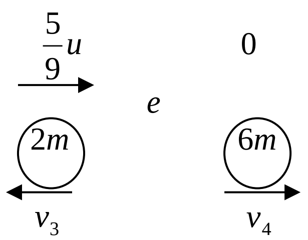
Question	Solution	Marks	Total	Comments
1	$[F] = MLT^{-2}$	B1		B1: Correct dimensions of F
	$MLT^{-2} = (LT^{-1})^{\alpha} (L^2)^{\beta} (ML^{-3})^{\gamma}$	M1		M1: Substituting the dimensions of the quantities into the given equation to obtain RHS correctly.
	$= M^{\gamma} L^{\alpha+2\beta-3\gamma} T^{-\alpha}$	m1		m1: Collecting indices on RHS. Could be implied by later work.
	$\left.\begin{array}{l}\gamma = 1 \\ \alpha + 2\beta - 3\gamma = 1 \\ -\alpha = -2\end{array}\right\}$	A1 m1	6	A1: $\gamma = 1$ m1: Two correct equations for α and β.
	$\alpha = 2$, $\beta = 1$	A1		A1: Correct values for α and β. Condone use of units instead of dimensions.
	Total		**6**	

Question	Solution	Marks	Total	Comments
2 (a)	$x = u\cos\alpha t$	M1		M1: Correct expression for horizontal displacement.
	$t = \dfrac{x}{u\cos\alpha}$	A1		A1: Correct expression for *t*.
	$y = u\sin\alpha t - \frac{1}{2}gt^2$	M1		M1: Correct expression for vertical displacement. Allow sign errors.
	$y = u\sin\alpha \times \dfrac{x}{u\cos\alpha} - \dfrac{1}{2}(9.8)\left(\dfrac{x}{u\cos\alpha}\right)^2$	m1		m1: Elimination of *t* from equation for vertical displacement.
	$y = x\tan\alpha - \dfrac{4.9x^2}{u^2\cos^2\alpha}$ **AG**	A1	5	A1: Correct result from correct working. Penalise use of $g = 9.81$.
(b)(i)	$-s = s\tan 55^\circ - \dfrac{4.9s^2}{21^2\cos^2 55^\circ}$	M1		M1: Substituting $\pm s$ for *x* and *y*.
	$s = \dfrac{(1+\tan 55^\circ)21^2\cos^2 55^\circ}{4.9}$	m1		m1: Making *s* the subject of their equation.
	$s = 71.9$	A1	3	A1: AWRT 71.9 Condone use of $g = 9.81$ which gives 71.8.
(ii)	$\dot{x} = 21\cos 55^\circ = 12.045$	B1		B1: Correct expression or value for horizontal component of velocity.
	$\dot{y} = 21\sin 55^\circ - 9.8 \times \dfrac{71.895}{21\cos 55^\circ}$ or $\dot{y}^2 = (21\sin 55^\circ)^2 - 2(9.8)(-71.895)$	M1		M1: Correct expression or value for vertical component of velocity, with their answer to (b)(i).
	$\dot{y} = -41.292$	A1		A1: Correct expression or value.
	$\tan^{-1}\dfrac{-41.292}{21\cos 55^\circ}$	m1		m1: Use of tan with their velocity components.
	$= -74^\circ$ or 74°	A1	5	A1: Correct angle to nearest degree. CAO.
	Total		**13**	

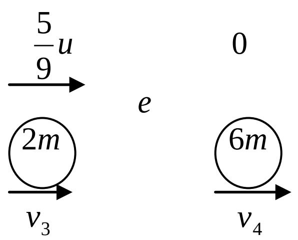
(b)(ii)	Alternative: $y = x\tan\alpha - \dfrac{4.9x^2}{u^2\cos^2\alpha}$ $\dfrac{dy}{dx} = \tan\alpha - \dfrac{2(4.9)x}{u^2\cos^2\alpha}$ $= \tan 55^\circ - \dfrac{2(4.9)(71.895)}{21^2 \times \cos^2 55^\circ}$ $= -3.428$ The angle $= \tan^{-1}(-3.428)$ $= -74^\circ$ or 74°	 B1 M1 A1 m1 A1	 5	 B1: Correct derivative. M1: Substituting values. A1: Correct value of the derivative m1: Use of tan to find the angle. A1: Correct angle to nearest degree. CAO.

Question	Solution	Marks	Total	Comments
3 (a)		B1		B1: Momentum – Impulse triangle with right angle. Can be implied by a correct equation.
	$2.5^2 = 1.5^2 + I^2$	M1	3	M1: Use of Pythagoras to obtain a correct equation. OE for example $5^2 = 3^2 + \left(\frac{I}{0.5}\right)^2$
	$I = 2$ N s	A1		A1: Correct impulse.
(b)	After the impact:	B1		B1: Sight of perpendicular component as $4e$. Could be implied by a correct equation.
		B1	4	B1: Correct velocity diagram, PI by a correct equation
	$\left(3\sqrt{2}\right)^2 = (4e)^2 + 3^2$	M1		M1: Use of Pythagoras to obtain a correct equation.
	$e = \frac{3}{4}$ or 0.75	A1		A1: Correct coefficient of restitution.
	Total		**7**	

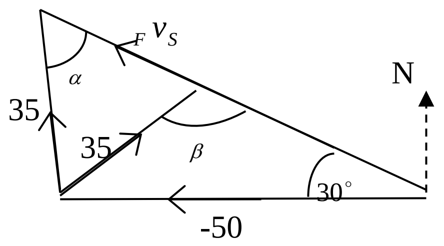
(a)	Alternative: $I\mathbf{j} = 0.5(5\cos\alpha\mathbf{i} + 5\sin\alpha\mathbf{j}) - 0.5(3\mathbf{i})$ $2.5\cos\alpha - 1.5 = 0$ $\cos\alpha = 0.6$ $\sin\alpha = 0.8$ $I = 0.5(5 \times 0.8)$ $I = 2$	B1 M1 A1	3	B1: Correct vector equation. M1: Correct value for $\sin\alpha$. A1: Correct impulse.
(b)	Alternative: $3 = 3\sqrt{2}\sin\beta$ $\cos\beta = \frac{1}{\sqrt{2}}$ $e = \frac{3\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)}{\frac{2}{0.5}}$ $e = \frac{3}{4}$ or 0.75	B1 B1 M1 A1	4	B1: Correct equation for motion parallel t B1: Value for $\cos\beta$ or $\beta = 45°$. M1: Correct expression for e or correct ec A1: Correct impulse.

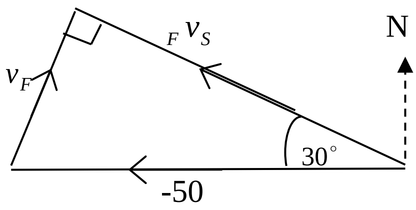
Question	Solution	Marks	Total	Comments
4 (a) (i)	$mu = mv_1 + 2mv_2$ OE	M1 A1		M1: Equation with three momentum terms. A1: Correct equation.
	$u = v_1 + 2v_2$			
(ii)	$\frac{2}{3}u = v_2 - v_1$ OE	M1 A1		M1: Newton's Law of Restitution. (Allow sign errors.) A1: Correct equation.
	$3v_2 = \frac{5}{3}u$			
(b)	$v_2 = \frac{5}{9}u$ **AG**	A1	6	A1: Correct speed of *B*, from correct working.
	$v_1 = u - \frac{10}{9}u$			
	$v_1 = -\frac{1}{9}u$			
	The speed of A is $\frac{1}{9}u$	A1		A1: Correct speed of *A*. Do not accept negative speed
				
	$2m\left(\frac{5}{9}u\right) = -2mv_3 + 6mv_4$ OE	M1 A1		M1: Equation with three momentum terms. A1: Correct equation
	$\frac{10}{9}u = -2v_3 + 6v_4$			
	$e\left(\frac{5}{9}u\right) = v_3 + v_4$ OE	M1 A1		M1: Newton's Law of Restitution. (Allow sign errors.) A1: Correct equation
(c)	$\frac{10}{9}u = -2v_3 + 6\left(\frac{5}{9}ue - v_3\right)$	m1 A1F	8	
	$8v_3 = \frac{10}{3}ue - \frac{10}{9}u$			m1: Solving equations to find the speed of *B* after the second collision. A1F: Correct speed of *B* after the second collision. FT their equations
	$v_3 = \frac{5}{12}ue - \frac{5}{36}u$ OE			
			2	

Q	Solution	Marks	Total	Comments
	second collision $\Rightarrow$ $\frac{5}{12}ue - \frac{5}{36}u > \frac{1}{9}u$ $\frac{5}{12}ue > \frac{9}{36}u$ $e > \frac{3}{5}$ or 0.6 Equal radii $\Rightarrow$ Velocities are parallel to the line of centr	M1 A1F B1 B1		M1: For the inequality $v_3 > v_1$ A1F: Correct value of *k*. FT their $v_3 > v_1$. The value of *k* must be less than 1 and greater than 0 to score A1F B1: Comment about equal radii or same size. B1: Comment about the line of centres.
	Total		**16**	

	(b) Alternative: 			
	$2m\left(\frac{5}{9}u\right) = 2mv_3 + 6mv_4$ $\frac{10}{9}u = 2v_3 + 6v_4$	M1A1		M1: Equation with three momentum terms. A1: Correct equation.
	$e\left(\frac{5}{9}u\right) = v_4 - v_3$ $\frac{10}{9}u = 2v_3 + 6\left(\frac{5}{9}ue + v_3\right)$ $8v_3 = \frac{10}{9}u - \frac{10}{3}ue$	M1A1		M1: Newton's Law of Restitution. (Allow sign errors.) A1: Correct equation.
	$v_3 = \frac{5}{36}u - \frac{5}{12}ue$ OE	m1A1F		m1: Solving equations to find the velocity of *B* after the second collision. A1F: Correct velocity of *B* after the second collision. FT their equations.
	second collision $\Rightarrow$ $\frac{5}{36}u - \frac{5}{12}ue < -\frac{1}{9}u$	M1		M1: For the inequality $v_3 < v_1$
	$\frac{5}{12}ue > \frac{9}{36}u$ $e > \frac{3}{5}$ or 0.6	A1F		A1F: Correct value of *k*. The value of *k* must be less than 1 and greater than 0 to score A1F

Question	Solution	Marks	Total	Comments
5	$\cos\alpha = \frac{3}{5}$ or 0.6 and $\cos\beta = \frac{5}{13}$ or 0.3846...	B1		B1: Correct values for $\cos\alpha$ and $\cos\beta$.
	$2(4\cos\alpha) + 1(2.6\cos\beta) = 2v_A + 1v_B$ $2(2.4) + 1(1) = 2v_A + 1v_B$	M1A1		M1: Four term momentum equation along the line of centres. A1: Correct equation. May be in terms of α and β.
	$\frac{4}{7}(4\cos\alpha - 2.6\cos\beta) = v_B - v_A$ $\frac{4}{7}(2.4 - 1) = v_B - v_A$ $\begin{cases} 5.8 = 2v_A + v_B \\ 0.8 = v_B - v_A \end{cases}$	M1 A1		M1: Newton's Law of Restitution. (Allow sign errors.) A1: Correct equation.
	$v_A = \frac{5}{3}$ ms^{-1}	A1	11	A1: Correct velocity of *A*. AWRT 1.67
	$v_B = \frac{37}{15}$ ms^{-1}	A1		A1: Correct velocity of *B*. AWRT 2.47
	$V_A = \sqrt{\left(\frac{5}{3}\right)^2 + (4\sin\alpha)^2}$	m1		m1: Finding speed of *A* with their v_A. May be in terms of α and β.
	$V_A = \sqrt{\left(\frac{5}{3}\right)^2 + (3.2)^2} = 3.61$ ms^{-1}	A1		A1: Correct speed. AWRT 3.61
	$V_B = \sqrt{\left(\frac{37}{15}\right)^2 + (2.6\sin\beta)^2}$	m1		m1: Finding speed of *B* with their v_B. May be in terms of α and β.
	$V_B = \sqrt{\left(\frac{37}{15}\right)^2 + (2.4)^2} = 3.44$ ms^{-1}	A1		A1: Correct speed. AWRT 3.44
	Total		**11**	

Q	Solution	Marks	Total	Comments
6 (a)(i)	 	B1 B1		B1: For one velocity triangle, could be implied by later working. B1: For the other velocity triangle drawn together or separately, could be implied by the correct 2nd angle
(ii)	$\dfrac{\sin\alpha}{50}=\dfrac{\sin 30^\circ}{35}$ or $\dfrac{\sin\beta}{50}=\dfrac{\sin 30^\circ}{35}$	M1	5	M1: Correct use of sine rule to find α or β.
	$\left.\begin{array}{l}\alpha = 45.58^\circ\\ \beta = 134.42^\circ\end{array}\right\}$	A1		A1: **Either** angle correct.
	Bearings: $\left.\begin{array}{l}346^\circ\\ 074^\circ\end{array}\right\}$	A1		A1: **Two** correct bearings. Accept 74°.
	Angle for shorter time: 45.58°	B1	5	B1: Selecting the smaller of their two angles from part (a).
(b)	$\dfrac{_Fv_S}{\sin 104.42^\circ}=\dfrac{35}{\sin 30^\circ}$	M1		M1: Using the sine rule to find the speed of the frigate relative to the ship, with their angle.
	$_Fv_S = 67.79 \quad \text{km h}^{-1}$	A1		A1: Correct speed.
	$t=\dfrac{8}{67.79}$	m1		m1: Using distance over speed.
	$= 0.118$ h or 7.08 min	A1F		A1F: Correct time. FT their speed. Full marks can be scored by using both angles **and** choosing the shorter time. If both times calculated and none selected do not award final A1 mark.

	 $v_F = 50\sin 30^\circ$ OE $v_F = 25$ kmh^{-1}	B1 M1 A1	3	B1: Correct right angled velocity triangle. Could be implied by later working. M1: Use of trigonometry to find speed. A1: Correct speed. CAO.
	Total		**13**	

(a)(ii)	Alternative: Angle for shorter time : 45.58° $t(50\cos 30^\circ + 35\cos 45.58^\circ) = 8$ $\left(t = \dfrac{8}{50\cos 30^\circ + 35\cos 45.58^\circ}\right)$ $t = 0.118$ h or 7.08 min	B1 M1A1 m1 A1F	 5	B1: Selecting the smaller of their two angles from part (a). M1: For $50\cos 30^\circ \pm 35\cos 46^\circ$ A1: Correct expression. m1: Using distance over speed. A1F: Correct time. FT their angle. Full marks can be scored by using both angles **and** choosing the shorter time. If both times calculated and none selected do not award final A1 mark.
	Alternative: Angle for shorter time : 45.58° $\dfrac{d}{\sin 30^\circ} = \dfrac{8}{\sin 104.42^\circ}$ $d = 4.130$ km $\left(t = \dfrac{4.130}{35}\right)$ $t = 0.118$ h or 7.08 min	B1 M1 A1 m1 A1F	 5	B1: Selecting the smaller of their two angles from part (a). M1: Using the sine rule to find the distance travelled by the frigate with their angle. A1: Correct distance m1: Using distance over speed. A1: Correct time. FT their angle. Full marks can be scored b using both angles **and** choosing the shorter time. If both times calculated and none selected do not award final A1 mark.

Question	Solution	Marks	Total	Comments
7 (a)	$y = u\sin(\alpha - \vartheta)t - \frac{1}{2}g\cos\vartheta t^2$	M1		M1: Expression for perpendicular height of particle above the plane. Accept wrong angles for M1 but **not** sin and cos in wrong places.
	$0 = u\sin(\alpha - \vartheta)t - \frac{1}{2}g\cos\vartheta t^2$	A1	4	A1: Correct expression with $y = 0$.
	$t = \dfrac{2u\sin(\alpha - \vartheta)}{g\cos\vartheta}$	m1 A1		m1: Solving for non-zero t. A1: Correct t.
(b)	$u\sin\alpha - gt = 0$ $t = \dfrac{u\sin\alpha}{g}$	M1		M1: Velocity equation to find time to A.
	$\dfrac{u\sin\alpha}{g} = \dfrac{2u\sin(\alpha - \vartheta)}{g\cos\vartheta}$	A1		A1: Correct time.
	$\sin\alpha\cos\vartheta = 2\sin(\alpha - \vartheta)$ $\sin\alpha\cos\vartheta = 2\sin\alpha\cos\vartheta - 2\cos\alpha\sin\vartheta$	m1		m1: Forming an equation using their time from part (a) and this time.
	$\left.\begin{array}{l}\sin\alpha\cos\vartheta = 2\cos\alpha\sin\vartheta \\ \dfrac{\sin\alpha}{\cos\alpha} = 2\dfrac{\sin\vartheta}{\cos\vartheta}\end{array}\right\}$	M1	5	M1: Use of identity to eliminate compound expressions. It is not enough to only expand $\sin(\alpha - \theta)$ in the expression in part (a) without anything else.
	$\tan\alpha = 2\tan\vartheta$	A1		A1: Seeing required expression derived with $k = 2$.
	Total		**9**	
	TOTAL		**75**	

(b)	Alternative: Taking x and y axes parallel and perpendicular to the plane respectively and using $\tan\theta = \frac{-\dot{y}}{\dot{x}}$ or equivalent,			
	$\left(u\cos(\alpha-\theta) - g\frac{2u\sin(\alpha-\theta)}{g\cos\theta}\sin\theta\right)\tan\theta = -u\sin(\alpha-\theta) + \frac{g2u\sin(\alpha-\theta)}{g\cos\theta}\cos\theta$	M1 A1		M1: Correct terms, allow sign errors. A1: All correct
	$\cos(\alpha-\theta)\tan\theta = \sin(\alpha-\theta)(2\tan^2\theta+1)$			
	$(\cos\alpha\cos\theta + \sin\alpha\sin\theta)\tan\theta = (\sin\alpha\cos\theta - \sin\theta\cos\alpha)(2\tan^2\theta+1)$	M1		M1: Use of identities to eliminate compound expressions.
	$\tan\alpha\tan^2\theta + \tan\alpha - 2\tan^3\theta - 2\tan\theta = 0$			
	$\tan\alpha(1+\tan^2\theta) = 2\tan\theta(1+\tan^2\theta)$	m1		m1: Rearranging to the required form.
	$\tan\alpha = 2\tan\theta$	A1	5	A1: Seeing required expression derived with $k = 2$.