Teacher Support Materials

Maths GCE

Paper Reference MM04

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Question 1

1 Two forces, $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$, act at the points $P(1, -1, 6)$ and $Q(0, 3, -2)$ respectively. These two forces together with a force $\mathbf{F}$, acting through the point Q, form a couple.

(a) (i) Find the force $\mathbf{F}$. *(3 marks)*

(ii) Show that the magnitude of $\mathbf{F}$ is $3\sqrt{10}$. *(2 marks)*

(b) Find the moment of the couple. *(5 marks)*

Student Response

Leave blank

① $\underline{i} + 2\underline{j} + 3\underline{k}$ $\quad$ $4\underline{i} - 3\underline{j} + 5\underline{k}$

$P(1, -1, 6)$ $\quad$ $Q(0, 3, -2)$

a) i) $\underline{i} + 2\underline{j} + 3\underline{k} + 4\underline{i} - 3\underline{j} + 5\underline{k} + a\underline{i} + b\underline{j} + c\underline{k} = 0$

$(5 + a)\underline{i} + (b - 1)\underline{j} + (8 + c)\underline{k} = 0$

Force $F = -5\underline{i} + \underline{j} - 8\underline{k}$

3

Leave blank

ii) $-5\underline{i} + \underline{j} - 8\underline{k}$

$\text{magnitude} = \sqrt{(5)^2 + (1)^2 + (8)^2}$

$= \sqrt{25 + 1 + 64}$

$= \sqrt{90} = 3\sqrt{10}$ (QED)

2

b)
$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -1 & 6 \\ 1 & 2 & 3 \end{vmatrix} = (-3-12)\underline{i} - (3-6)\underline{j} + (2+1)\underline{k}$$
$$= -15\underline{i} + 3\underline{j} + 3\underline{k}$$

M1

M0.

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 3 & -2 \\ 4 & -3 & 5 \end{vmatrix} = (15-6)\underline{i} - (0+8)\underline{j} + (0-12)\underline{k}$$
$$= 9\underline{i} - 8\underline{j} ~~-15\underline{k}~~ -12\underline{k}$$

Determinant missing

A1

A0

A0.

$\therefore -15\underline{i} + 3\underline{j} + 3\underline{k} + 9\underline{i} - 8\underline{j}$ ~~$-15\underline{k}$~~ $-12\underline{k}$

~~$-6\underline{i} - 5\underline{j} - 12\underline{k}$~~

$\therefore -6\underline{i} - 5\underline{j} - 9\underline{k}$

(7)

Commentary

Topic – Couples in 3D. A very good response to this question. Most candidates knew that the sum of forces must be zero and used it to find F, clearly showing their reasoning. The most common error was to add the given two forces and give this as the answer. Showing that the magnitude of F was $3\sqrt{10}$ proved easy, although $\sqrt{90}$ had to be seen to secure both marks. Almost all candidates correctly used r x F although a common error was not to use all the necessary determinants. The most efficient solutions found $\overrightarrow{QP}$ and then used just one determinant.

This candidate was chosen to illustrate a common error. Part a) is fully correct with good working shown. In part b) the candidate uses two determinants – missing one completely. This eventually loses three marks, largely because it is an incomplete method. The first M1 is awarded for any attempt at one appropriate determinant. All appropriate determinants must be used to be awarded the second M1.

The first A1 is for any three correct entries from any determinant. The second A1 requires all entries to be correct. The final A1 is for the correct answer – no follow through allowed. The rigid mark scheme enforced here ensures better differentiation. Candidate must be encouraged to check their method is complete.

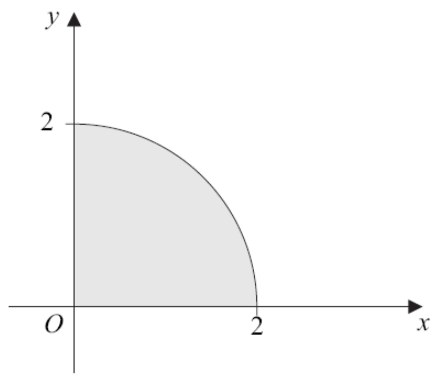
Mark scheme

MM04

Q	Solution	Mark	Total	Comments
1(a)(i)	$\begin{pmatrix}1\\2\\3\end{pmatrix}+\begin{pmatrix}4\\-3\\5\end{pmatrix}+\mathbf{F}=\begin{pmatrix}0\\0\\0\end{pmatrix}$	M1		sum of forces $=\mathbf{0}$ must be seen for M1
	$\Rightarrow \mathbf{F}=\begin{pmatrix}-5\\1\\-8\end{pmatrix}$	B1 A1	3	$\pm(5\mathbf{i}-\mathbf{j}+8\mathbf{k})$ seen correct sign
(ii)	$\lvert\mathbf{F}\rvert=\sqrt{5^2+1^2+8^2}=\sqrt{90}=3\sqrt{10}$	M1 A1	2	$\sqrt{\text{their } \mathbf{F} \text{ components}}$ AG
(b)	Moment $=\mathbf{r}\times\mathbf{F}$ $=\begin{vmatrix}\mathbf{i}&1&1\\\mathbf{j}&-1&2\\\mathbf{k}&6&3\end{vmatrix}+\begin{vmatrix}\mathbf{i}&0&4\\\mathbf{j}&3&-3\\\mathbf{k}&-2&5\end{vmatrix}+\begin{vmatrix}\mathbf{i}&0&-5\\\mathbf{j}&3&1\\\mathbf{k}&-2&-8\end{vmatrix}$	M1 M1		attempt at one $\mathbf{r}\times\mathbf{F}$ (all attempted)
	$=\begin{pmatrix}-15\\3\\3\end{pmatrix}+\begin{pmatrix}9\\-8\\-12\end{pmatrix}+\begin{pmatrix}-22\\10\\15\end{pmatrix}$	A1√ A1√		any three components correct all components correct
	$=\begin{pmatrix}-28\\5\\6\end{pmatrix}$	A1√	5	sum of vectors; √ their $\mathbf{F}$ from part (a)
	1st Alternative for (b):			
	$\overrightarrow{QP}=\begin{pmatrix}1\\-4\\8\end{pmatrix}$	(M1) (A1)		intention to use $\mathbf{r}\times\mathbf{F}$ about Q $\overrightarrow{QP}$ obtained correctly
	Moments about Q $QP\times\mathbf{F}_1=\begin{vmatrix}\mathbf{i}&1&1\\\mathbf{j}&-4&2\\\mathbf{k}&8&3\end{vmatrix}=\begin{pmatrix}-28\\5\\6\end{pmatrix}$	(M1) (A1) (A1)	(5)	determinant attempted one component correct all correct
	2nd Alternative for (b):			
	$\overrightarrow{PQ}=\begin{pmatrix}-1\\4\\-8\end{pmatrix}$	(M1) (A1)		intention to use $\mathbf{r}\times\mathbf{F}$ about P $\overrightarrow{PQ}$ obtained correctly
	$\begin{vmatrix}\mathbf{i}&-1&-5\\\mathbf{j}&4&1\\\mathbf{k}&-8&-8\end{vmatrix}=\begin{pmatrix}-24\\32\\19\end{pmatrix}$	(M1)		one determinant correct
	$\begin{vmatrix}\mathbf{i}&-1&4\\\mathbf{j}&4&-3\\\mathbf{k}&-8&5\end{vmatrix}=\begin{pmatrix}-4\\-27\\-13\end{pmatrix}$	(A1)		both correct
	$\begin{pmatrix}-24\\32\\19\end{pmatrix}+\begin{pmatrix}-4\\-27\\-13\end{pmatrix}=\begin{pmatrix}-28\\5\\6\end{pmatrix}$	(A1)	(5)	all correct
	Total		**10**	

Question 2

2 The region bounded by the positive x-axis, the positive y-axis and the curve with equation $y = \sqrt{4 - x^2}$ is shown in the diagram.



The region is rotated through 2π radians around the x-axis to form a uniform solid.

(a) Use integration to show that the volume of the solid is $\frac{16\pi}{3}$. *(3 marks)*

(b) Use integration to find the distance of the centre of mass of the solid from the y-axis. *(4 marks)*

(c) The solid is suspended from a point on the edge of its circular face and hangs in equilibrium.

Find the angle between the circular face and the vertical. *(3 marks)*

Student response (next page)

Leave blank

2a) ~~$V = \pi\int_a^b xy^2\,dx$~~

$V = \pi\int_0^b y^2\,dx$

$V = \pi\int_0^2 4 - x^2\,dx = \pi\left[4x - \frac{x^3}{3}\right]_0^2$

$\pi\left(8 - \frac{8}{3}\right) = \frac{16\pi}{3}$ [Q.E.D.] 3

b) $\bar{x} = \dfrac{\int_a^b xy^2\,dx}{\int_a^b y^2\,dx}$ M1

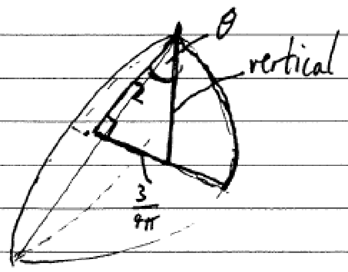
π not cancelled.

$\bar{x} = \dfrac{\int_0^b 4x - x^3\,dx}{\frac{16\pi}{3}} = \dfrac{\left[2x^2 - \frac{x^4}{4}\right]_0^2}{\frac{16\pi}{3}}$ A1

M1

$= \dfrac{(8-4)}{\frac{16\pi}{3}} = \dfrac{4\times 3}{16\pi} = \dfrac{3}{4\pi}$ A0.

c)



$\tan\theta = \dfrac{\frac{3}{4\pi}}{2}$ M1

Structure correct. A1

$\tan\theta = \dfrac{3}{8\pi}$

$\theta = 6.81°$ (3 s.f.) A1

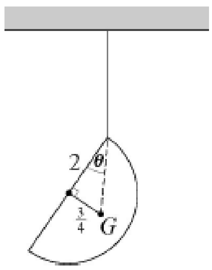
Commentary

Topic – Centres of mass by integration. A very good response to this question with candidates showing sound knowledge of formulae and good integration skills. Part a) was universally correct. A common error in part b) was to lose a π symbol resulting in the wrong answer. Candidates showed good knowledge of suspension problems and even if errors were made in part b) full recovery marks were available.

The chosen candidate displays good integration skills. However in part b) in the quoted formula for $\bar{X}$, which is correct, π has been cancelled. The candidate still uses the answer from part a) which of course has not lost π. This means that the candidate loses only the last accuracy mark. In part c) the candidate then uses part b) to find the angle. The structure of the candidate's answer is correct and no further penalty is incurred.

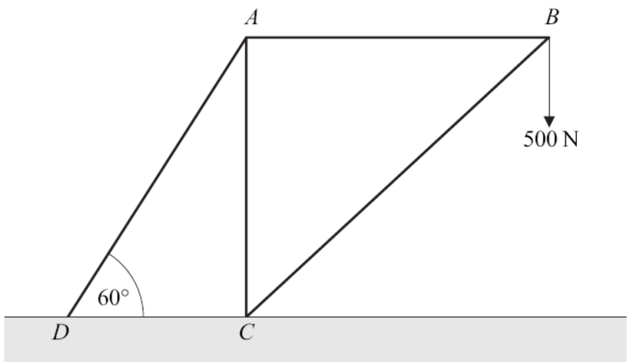
Mark Scheme

MM04 (cont)

Q	Solution	Mark	Total	Comments
2(a)	$\text{volume} = \pi\int y^2\,dx$			
	$= \pi\int_0^2 (4 - x^2)\,dx$	M1		evidence of attempt at $\int y^2 dx$
	$= \pi\left[4x - \frac{x^3}{3}\right]_0^2$	A1		integrating
	$= \pi\left[8 - \frac{8}{3} - 0\right]$			
	$= \frac{16\pi}{3}$	A1	3	AG
(b)	$\frac{16\pi}{3}\bar{x} = \pi\int_0^2 x(4 - x^2)\,dx$			
	$= \pi\int_0^2 (4x - x^3)\,dx$	M1		attempt at $\int xy^2 dx$
	$= \pi\left[2x^2 - \frac{x^4}{4}\right]_0^2$	A1		integrating correctly
	$= \pi[8 - 4 - 0]$	m1		equation to find $\bar{x}$ (dependent on first M1)
	$= 4\pi$			
	$\Rightarrow \bar{x} = \frac{3}{4}$	A1	4	
(c)				
	$\tan\theta = \frac{\frac{3}{4}}{2}$	M1		$\tan\theta$ seen
	$= \frac{3}{8}$	A1√		structure correct $\frac{\bar{x}}{2}$
	$\Rightarrow \theta = 20.6°$	A1√	3	accept AWFW $20° - 21°$; √ their $\bar{x}$
	Total		10	

Question 3

3 Dominic designs a crane. He models the crane by four light smoothly-jointed rods AB, AC, AD and BC. The crane is smoothly jointed to the horizontal ground at C and D. The framework is at rest in a vertical plane, with AB horizontal and AC vertical. The framework is such that $AC = AB$ and the angle CDA is 60°. A force of magnitude 500 N acts vertically downwards on the framework at B, as shown in the diagram.



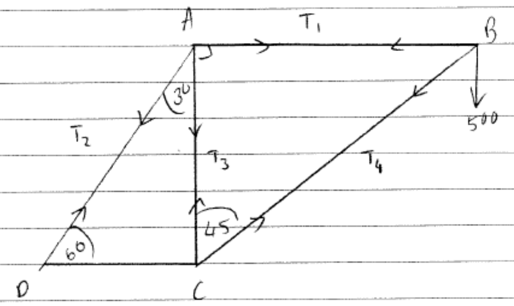
(a) (i) Find the magnitudes of the forces in the rods BC, AB and AD. *(9 marks)*

(ii) State which of the rods BC, AB and AD could be replaced by ropes, giving a reason for your answer. *(3 marks)*

(b) Find the magnitude of the force acting on the framework at D. *(1 mark)*

Student Response

3a)i)



Resolving vertically at B:

$500 + T_4 \sin 45 = 0$

(BC) $\Rightarrow$ $\underline{T_4 = -500\sqrt{2}\,N}$ (In compression)

Resolving horizontally at B:

$T_1 - 500\sqrt{2}\cos 45 = 0$

(AB) $\Rightarrow$ $\underline{T_1 = 500N}$ (In tension).

Leave blank

Resolving horizatally at A:

$500 = T_2 \sin 30 = \frac{T_2}{2}$ ✓ ✓

(AD) $\Rightarrow$ $T_2 = 1000N$ (In tension) ✓ 9

ii) T_4 (tension in BC) ~~t~~ was negative, so BC is in compression.

T_1 and T_2 (tension in AB and AD) were positive, so they are both in tension. Hence, AB and AD could be replaced by ropes, as ropes can be in tension but not compression. ✓ 3

b) ~~Resolving vertically for the whole system, force at D = 500N~~

AD = 1000N. ✓

$\Rightarrow$ Force at D = 1000N 1

(All joints in equilibrium.

(13)

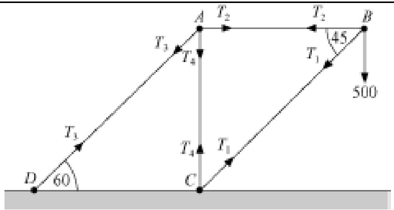
Commentary

Topic – Frameworks. An excellent response to this question. The most successful solutions included clear labelling of tensions or compressions in the framework, using TAB. The best solutions clearly indicated what they were doing thus making it easier to award marks even if minor errors occurred. A common error in part a) was to evaluate 500/sin 45 as 354. Explanations in a)ii) were good. No follow through marks were available here as the question could be answered with reference to the diagram. Part b) proved very discriminating with many candidates “resolving the whole system” to get the answer as 500 N – failing to understand the implication of a reaction force at point C. It was expected that candidates would use Newton’s third law at point D to get the correct answer of 1000N.

The chosen candidate scored full marks on every question. This question was chosen because of the clarity of the diagram and explanation. With the diagram the tensions are clearly marked and labelled. Linking text explains what the candidate is doing at each step. Negative signs are interpreted as they occur (compressions). The explanation in part a)ii) is good because it clearly states what the candidate found, identifies the ones to be replaced by ropes and gives a reason. In part b) the candidate realises that forces balance and quickly obtains the right answer.

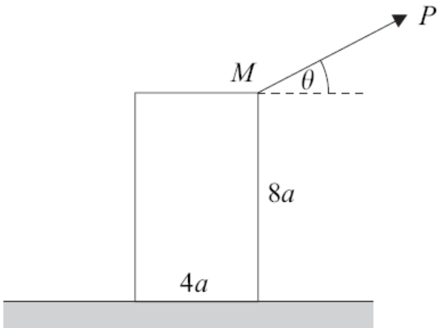
Mark Scheme

MM04 (cont)

Q	Solution	Mark	Total	Comments
3(a)(i)				
	Resolve vertically at B:			
	$T_1 \sin 45° + 500 = 0$	M1A1		forces can be marked as tensions and/or compressions; signs **must** be consistent
	$\Rightarrow T_1 = \dfrac{-500}{\sin 45°} = -500\sqrt{2}$ or -707 N [magnitude = 707 N]			NB if moments are used, reaction forces at C, D must be identified for first M1
	Resolve horizontally at B:			
	$T_2 + T_1 \cos 45° = 0$	M1A1		
	$\Rightarrow T_2 = -T_1 \cos 45° = 500\text{ N}$	A1√		√ their T_1
	Resolve horizontally at A:			
	$T_2 = T_3 \sin 30°$	M1A1		
	$\Rightarrow T_3 = \dfrac{T_2}{\sin 30°} = 1000\text{ N}$	A1√	9	√ their T_2
(ii)	AD and AB are in tension and could be replaced by ropes. BC is in thrust and cannot be replaced by ropes.	B1 B1 E1	3	identification of AD/AB identification of BC (can be implied) reference to tension/thrust
(b)	magnitude $= T_3 = 1000\text{ N}$	B1√	1	
	Total		**13**	

Question 4

4 A uniform block in the shape of a cuboid has weight W, a square base of side $4a$, a height of $8a$ and stands on a rough horizontal surface. The coefficient of friction between the block and the surface is μ. A rope is attached to the point M, the midpoint of a top edge of the block.



The rope is pulled with a force P, which acts at an angle of θ above the horizontal and is perpendicular to the top edge.

(a) Find P, in terms of W and θ, if the block is on the point of toppling. *(4 marks)*

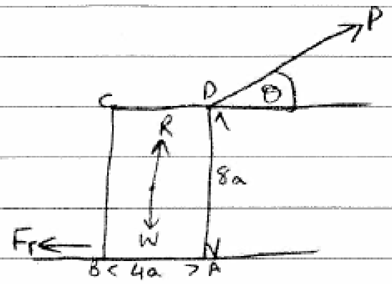
(b) Show that $P = \dfrac{\mu W}{\cos\theta + \mu \sin\theta}$ if the block is on the point of sliding. *(7 marks)*

(c) Given that $\tan\theta = 1$, find an inequality that μ must satisfy if the block slides before it topples. *(5 marks)*

Student Response

(4) a) On point of toppling, ~~weight (W) acts~~ ~~through~~ the reaction force (R) acts through A:

Taking moments about A: ✓ (11)



$W \times 2a - P\cos\theta \times 8a = 0$

$2Wa = 8aP\cos\theta$ ✓✓

$P\cos\theta = \frac{1}{4}W$

$P = \frac{W}{4\cos\theta}$ ✓ 4

b) On point of sliding, $F_r = \mu R$

R↑ for whole system

$P\sin\theta + R = W$ ✓✓

R → for whole system

$P\cos\theta = F_r$ ✓✓ (using $F_r = \mu R$)

$\mu R = P\cos\theta$

$R = \frac{P\cos\theta}{\mu}$ ✓

$\Rightarrow P\sin\theta + \frac{P\cos\theta}{\mu} = W$ ✓

$\mu P\sin\theta + P\cos\theta = \mu W$ 7

$P = \frac{\mu W}{\mu\sin\theta + \cos\theta}$ ✓ Q.E.D.

c) if the block slides before it topples:

$\frac{\mu W}{\mu\sin\theta + \cos\theta} < \frac{W}{4\cos\theta}$ ✓

$\mu 4\cos\theta < \mu\sin\theta + \cos\theta$ ✓

$4\mu < \mu\tan\theta + 1$ ✓ (if $\tan\theta = 1$)

$4\mu - \mu < 1$ ✓✓ 5

$\mu < \frac{1}{3}$ (16)

Commentary

Topic – Toppling and sliding. An excellent response to this question. Part a) was almost universally correct however when slips did occur it was because of the use of W (4a). Part b) was rarely seen incorrect, however inequality signs sometimes lost a mark. Part c) was very good with sound knowledge of cancelling, and trigonometrical identities. Once again a mark was sometimes lost because of an equality or greater than sign.

The chosen candidate has written a model solution. In every part there is linking text which explains what the candidate is doing. This makes it easy to award marks even if a minor slip is made. Moreover in each part the candidate has observed the key point of understanding right at the start of the question – this impresses examiners as they expect good work to follow – straight to the point.. Algebraic work is clearly legible and it is easy to follow each step. The candidate deservedly is awarded full marks.

Mark Scheme

MM04 (cont)

Q	Solution	Mark	Total	Comments
4(a)	On point of toppling, take moments about bottom right corner	M1		attempt at moments
	$W(2a) = P\cos\theta(8a)$	A1,A1		A1 each side
	$P = \dfrac{W}{4\cos\theta}$	A1	4	
(b)	On point of sliding			
	vertically, $N + P\sin\theta = W$	M1A1		
	horizontally, $F = P\cos\theta$	M1A1		
	friction $F = \mu N$			
	$\Rightarrow P\cos\theta = \mu(W - P\sin\theta)$	M1A1		substitute; use of $F = \mu N$
	$P\cos\theta = \mu W - \mu P\sin\theta$			
	$P(\cos\theta + \mu\sin\theta) = \mu W$			
	$P = \dfrac{\mu W}{\cos\theta + \mu\sin\theta}$	A1	7	AG
(c)	Slides before topples $\Rightarrow$			
	$\dfrac{\mu W}{\cos\theta + \mu\sin\theta} < \dfrac{W}{4\cos\theta}$	M1		inequality formed
	$4\mu\cos\theta < \cos\theta + \mu\sin\theta$	A1		elimination of fractions / cancel W
	$4\mu < 1 + \mu\tan\theta$	A1		$\div$ by $\cos\theta$ and use of $\tan\theta = 1$
	$\tan\theta = 1 \Rightarrow 3\mu < 1$	M1		collect μ terms
	$\mu < \dfrac{1}{3}$	A1	5	
	Total		**16**	

Question 5

5 A thin uniform rectangular plate, of mass m and with sides of length $4a$ and $6a$, is hanging freely from a horizontal axis along a side of length $4a$. Initially the plate is stationary and hangs in a vertical plane.

(a) Use integration to show that the moment of inertia of the plate about the horizontal axis is $12ma^2$. *(5 marks)*

(b) The plate is struck at its centre of mass by a small lump of clay of mass $\frac{1}{2}m$ moving with speed u horizontally at right angles to the plate.

Assume that the clay sticks to the plate throughout the subsequent motion. Find, in terms of u and a, the angular speed of the plate immediately after impact. *(7 marks)*

Student Response

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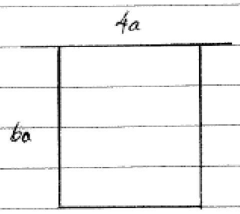
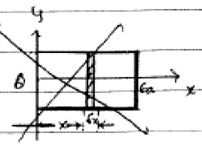
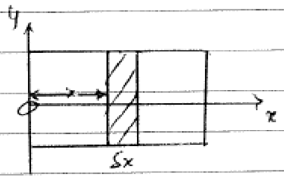


(a)



(a)



Density : $\rho = \frac{m}{4a \times 6a} = \frac{m}{24a^2}$. ✓

$\delta I = x^2 \, \delta m$

$= x^2 (\rho \, \delta A)$

$= x^2 \times \frac{m}{24a^2} \times (4a \, \delta x)$

$= \frac{m}{6a} x^2 \, \delta x$ ✓

$\Rightarrow \quad I = \frac{m}{6a} \int_0^{6a} x^2 \, dx$

$= \frac{m}{6a} \left(\frac{x^3}{3} \right) \Big|_0^{6a}$ ✓

$= \frac{m}{6a} \times \frac{(6a)^3}{3}$ ✓

$= 12 ma^2$. ✓

5

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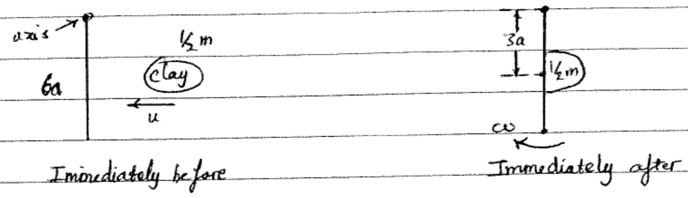
(b) Seeing from a side :



~~Conservation of angular momentum :~~

~~$(\frac{1}{2}m)u(3a) =$~~

Immediately after impact :

Moment of inertia of the whole thing will be :

$I_1 = 12ma^2 + (\frac{1}{2}m)(3a)^2$

$= \frac{33}{2}ma^2$ ✓

Applying the conservation of angular momentum :

$(\frac{1}{2}m)u(3a) = (I_1)\omega$ ✓

$\Rightarrow \frac{3}{2}mua = \frac{33}{2}ma^2\omega$ ✓✓ ✓

$\Rightarrow \omega = \frac{u}{11a}$ ✓✓

7

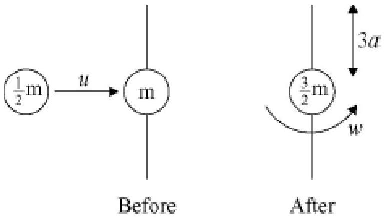
Commentary

Topic – MI and angular momentum. *This was the question that proved to be most challenging. In part a) a few candidates quoted results from the formulae book ignoring the request " by integration". It was pleasing to see methods using strips or particles being used correctly – explanation was good too with references to mass of strip, MI of rectangle etc. Part b) was probably the most difficult on the paper. A number of candidates tried to use energy methods to no avail. When candidates did attempt to use conservation of angular momentum* they often *made errors with the before scenario. For example using* $\frac{1}{2}mu$ *(no distance) or* $\frac{1}{2}mu6a$ *(wrong distance) or* $\frac{3}{2}mu3a$ *(wrong mass). Iw was correctly identified and used by nearly all candidates, but the momentum of the particle was again incorrect.*

The chosen candidate has clearly labelled diagrams that help them think out the solution. In part a) the MI is built up carefully using the appropriate strip. In part b) the solution is completed efficiently largely due to the thinking work being done in relation to the diagram. All candidates should be encouraged to do this.

Mark Scheme

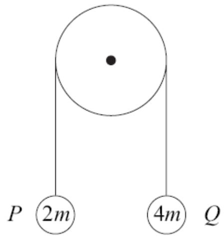
MM04 (cont)

Q	Solution	Mark	Total	Comments
5	**Alternative for (a):**			
	$\rho = \dfrac{m}{24a^2}$	(B1)		
	Mass of strip = $4a\delta x\rho$			
	MI of rectangle = $\sum (4a\delta x\rho)x^2$	(M1)		use of mx^2
	$= \int_0^{6a} 4a\dfrac{m}{24a^2}x^2 \, dx$	(m1)		integration attempt
	$= \left[\dfrac{mx^3}{18a}\right]_0^{6a} = 12ma^2$	(A1, A1)	(5)	AG
(b)				
	angular momentum before			
	$= \dfrac{1}{2}mu(3a) = \dfrac{3mua}{2}$	M1A1		'ka' required for M1
	angular momentum after			
	$= Iw + \dfrac{1}{2}m(3a)^2 w$	M1 A1		either term correct both correct
	$= 12ma^2 w + \dfrac{9ma^2}{2}w$	B1		use of $I = 12ma^2$ anywhere
	$= \dfrac{33ma^2 w}{2}$			
	use C of momentum to set			
	$\dfrac{3mua}{2} = \dfrac{33ma^2 w}{2}$	M1		equation – C of m ('their' expression)
	$\Rightarrow w = \dfrac{u}{11a}$	A1	7	
	Total		**12**	

Question 6

6 A uniform circular disc, of radius a, can rotate freely in a vertical plane about a fixed horizontal axis through its centre perpendicular to its plane face. The moment of inertia of the disc about this axis is $10ma^2$.

A light inextensible string passes over the rough rim of the disc, and two particles P and Q, of masses $2m$ and $4m$ respectively, are attached to its ends.



Initially the system is held at rest with the particles hanging freely in equilibrium. The system is then released. In the subsequent motion, no slipping occurs between the string and the disc. When the disc has turned through an angle θ, the particle P has **not** reached the disc.

(a) (i) Show that the gain in kinetic energy of the system is $8ma^2\dot{\theta}^2$. *(4 marks)*

(ii) Hence show that $a\dot{\theta}^2 = \frac{1}{4}g\theta$. *(3 marks)*

(b) Find the force exerted by the string:

(i) on the particle P;

(ii) on the particle Q. *(7 marks)*

Student Response (next page)

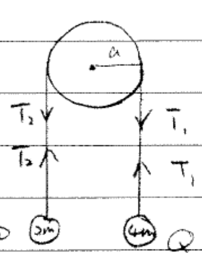
(b) (a) (i) [crossed out] velocity $= a\dot{\theta}$ (3)

$KE_Q = \frac{1}{2} \times 2m \times (a\dot{\theta})^2 = ma^2\dot{\theta}^2$ ✓

$KE_P = \frac{1}{2} \times 4m \times (a\dot{\theta})^2 = 2ma^2\dot{\theta}^2$

$KE_{disc} = \frac{1}{2} \times 10ma^2 \dot{\theta}^2 = 5ma^2\dot{\theta}^2$ ✓

$KE_{total} = ma^2\dot{\theta}^2 + 2ma^2\dot{\theta}^2 + 5ma^2\dot{\theta}^2 = 8ma^2\dot{\theta}^2$ 4



(ii) $PE_Q = -4mga\theta$, $PE_P = 2mga\theta$

$PE_{total} = -2mga\theta$

$PE_{total} + KE_{total} = 0$

$\Rightarrow 8ma^2\dot{\theta}^2 - 2mga\theta = 0 \Rightarrow 8ma^2\dot{\theta}^2 = 2mga\theta$

$\Rightarrow a\dot{\theta}^2 = \frac{1}{4}g\theta$ ✓ 3

(b) (i) Assume acceleration is f

$T_2 - 2mg = 2mf$ ✓ ①

$4mg - T_1 = 4mf$ ✓ ②

$T_1 a - T_2 a = 10ma^2\ddot{\theta}$ ✓

$f = a\ddot{\theta} \Rightarrow T_1 a - T_2 a = 10maf \Rightarrow (T_1 - T_2) = 10mf$ ③

① + ② $= T_2 - 2mg + 4mg - T_1 = 2mf + 4mf$

Leave blank

$\Rightarrow T_1 - T_2 = 2mg - 6mf$ ④

substitude ④ into ③

$2mg - 6mf = 10mf \Rightarrow 2mg = 16mf$

$\Rightarrow f = \frac{1}{8}g$ ✓

$T_2 - 2mg = 2mf \Rightarrow T_2 = 2mg + 2m \times \frac{1}{8}g = \frac{9}{4}mg = 22.1m$

$4mg - T_1 = 4mf \Rightarrow T_1 = 4mg - 4m \times \frac{1}{8}g = \frac{7}{2}mg = 34.3m$

∴ The force exerted on P is 22.1m (N) ✓

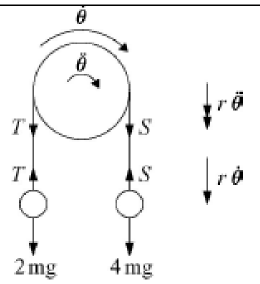
The force exerted on Q is 34.3m (N) 7

Commentary

Topic – Rotational dynamics (connected particles). A challenging question although the structure helped. The majority of candidates successfully identified the KE of particles and the the disc to get the printed answer. Conservation of energy was often correctly used in a)ii). Part b) proved more discriminating. The method which involved differentiating the result in a)ii) was least popular. Many types of errors were made – the worst of which was failing to use acceleration ie velocity was used. Some candidates failed to appreciate how couple = inertia x angular acceleration could be used to form a third equation. Again however the mark scheme rewarded key understanding enabling all candidates to score marks somewhere.

This candidate was chosen for a number of reasons. In part a)i) each KE term is clearly identified and correctly calculated to verify the result stated. Similarly in a)ii) clear but brief explanation is given. Candidates should be encouraged to do this. In part b) the candidate avoids using the standard angular acceleration symbol but eventually evaluates his f correctly before obtaining the required answers. Thus their solution is a variation of the ones given in the mark scheme but fully valid and correct.

MM04 (cont)

Q	Solution	Mark	Total	Comments
6(a)				
(i)	KE =			
	$\frac{1}{2}(4m)(a\dot{\theta})^2+\frac{1}{2}(2m)(a\dot{\theta})^2+\frac{1}{2}(10ma^2)\dot{\theta}^2$	B1 B1		$a\dot{\theta}$ used disc KE
	$=2ma^2\dot{\theta}^2+ma^2\dot{\theta}^2+5ma^2\dot{\theta}^2$	M1		particles KE
	$=8ma^2\dot{\theta}^2$	A1	4	AG
(ii)	PE lost $=4mga\theta-2mga\theta$			
	$=2mga\theta$	B1		PE seen - any term
	C of E $\Rightarrow 8ma^2\dot{\theta}^2=2mga\theta$	M1		C of E
	$a\dot{\theta}^2=\frac{g\theta}{4}$	A1	3	AG
(b)	differentiating $2a\dot{\theta}\ddot{\theta}=\frac{g\dot{\theta}}{4}$	M1		
	$\Rightarrow a\ddot{\theta}=\frac{g}{8}$	A1		
	For P,			
	$T-2mg=2ma\ddot{\theta}\Rightarrow T=2mg+\frac{mg}{4}=\frac{9mg}{4}$	M1		equation for P
		A1		for $\frac{9mg}{4}$
	For Q,			
	$4mg-S=4ma\ddot{\theta}\Rightarrow S=4mg-\frac{mg}{2}=\frac{7mg}{2}$	M1A1		equation for Q
		A1	7	for $\frac{7mg}{2}$
	Alternative for (b):			
	Use $C=I\ddot{\theta}$ for disc			
	$Sa-Ta=10ma^2\ddot{\theta}$	(M1)		M1 for LHS attempt
	$\Rightarrow S-T=10ma\ddot{\theta}$	(A1)		RHS correct
	For P, $T-2mg=2ma\ddot{\theta}$	(M1)		
	For Q, $4mg-S=4ma\ddot{\theta}$	(M1)		
	Solving	(M1)		
	For T	(A1)		
	For S	(A1)	(7)	
	Total		**14**	