Teacher Support Materials
2008

Maths GCE

Paper Reference MM05

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Dr Michael Cresswell, Director General.

Question 1

1 A simple pendulum of length 2 metres is set in motion.

(a) (i) Show that the period of the motion is 2.84 seconds, correct to three significant figures. *(2 marks)*

(ii) Show that the frequency of the motion is 0.352 cycles per second, correct to three significant figures. *(1 mark)*

(b) The length of the pendulum is adjusted so that the period of its motion is 2.5 seconds. Find the adjusted length of the pendulum. *(2 marks)*

Student Response

Leave blank

1)a) i) $w = \sqrt{\frac{g}{l}}$

$w = \sqrt{\frac{9.8}{2}}$ ~~tan~~

$p = \frac{2\pi}{w} = \frac{2\pi}{\sqrt{\frac{9.8}{2}}} = 2.43845 = 2.84secs$ (3.s.f). 2

ii) $f = \frac{1}{p} = \frac{1}{2.83845} = 0.35204955 = 0.352$ (3.s.f). 1

ⓑ $p = \frac{2\pi}{w}$

$2.5 = \frac{2\pi}{\sqrt{\frac{g}{l}}}$

$\sqrt{\frac{g}{l}} = \frac{2\pi}{2.5}$

$\frac{g}{l} = \frac{4\pi^2}{6.25}$

$l = \frac{6.25g}{4\pi^2} = 1.55148 = 1.55m$ (3.s.f). 2

The pendulum has been adjusted by 0.45m

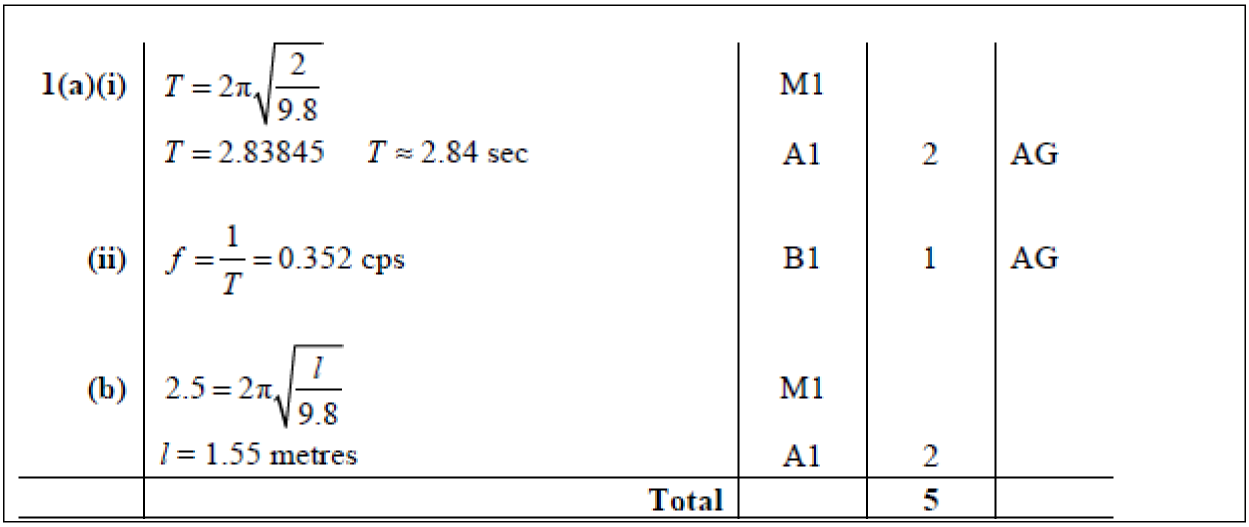
⑤

Commentary

This question was answered well, with highly accurate and clearly set out responses. The majority of candidates scored full marks

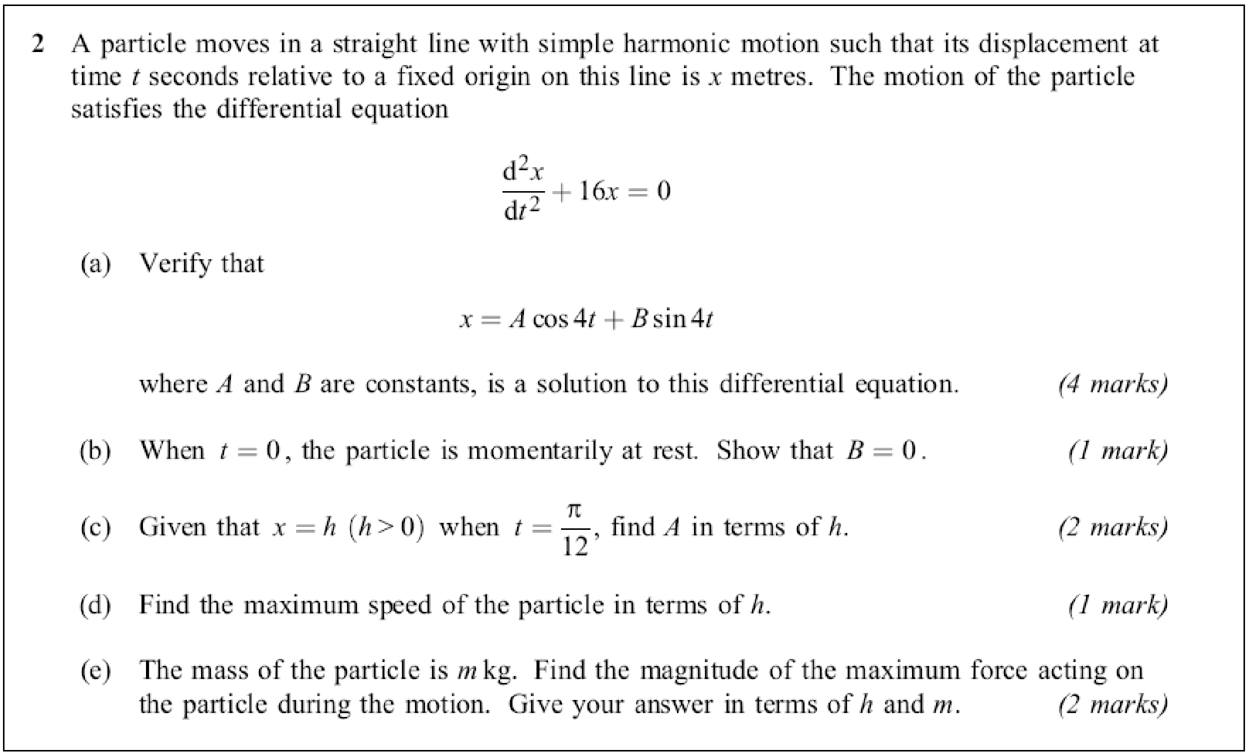
An exemplary ‘good practice’ response, showing full, clear and accurate working.

Mark scheme



1(a)(i)	$T = 2\pi\sqrt{\frac{2}{9.8}}$	M1		
	$T = 2.83845 \quad T \approx 2.84$ sec	A1	2	AG
(ii)	$f = \frac{1}{T} = 0.352$ cps	B1	1	AG
(b)	$2.5 = 2\pi\sqrt{\frac{l}{9.8}}$	M1		
	$l = 1.55$ metres	A1	2	
	Total		**5**	

Question 2



2 A particle moves in a straight line with simple harmonic motion such that its displacement at time t seconds relative to a fixed origin on this line is x metres. The motion of the particle satisfies the differential equation

$$\frac{d^2x}{dt^2} + 16x = 0$$

(a) Verify that

$$x = A\cos 4t + B\sin 4t$$

where A and B are constants, is a solution to this differential equation. *(4 marks)*

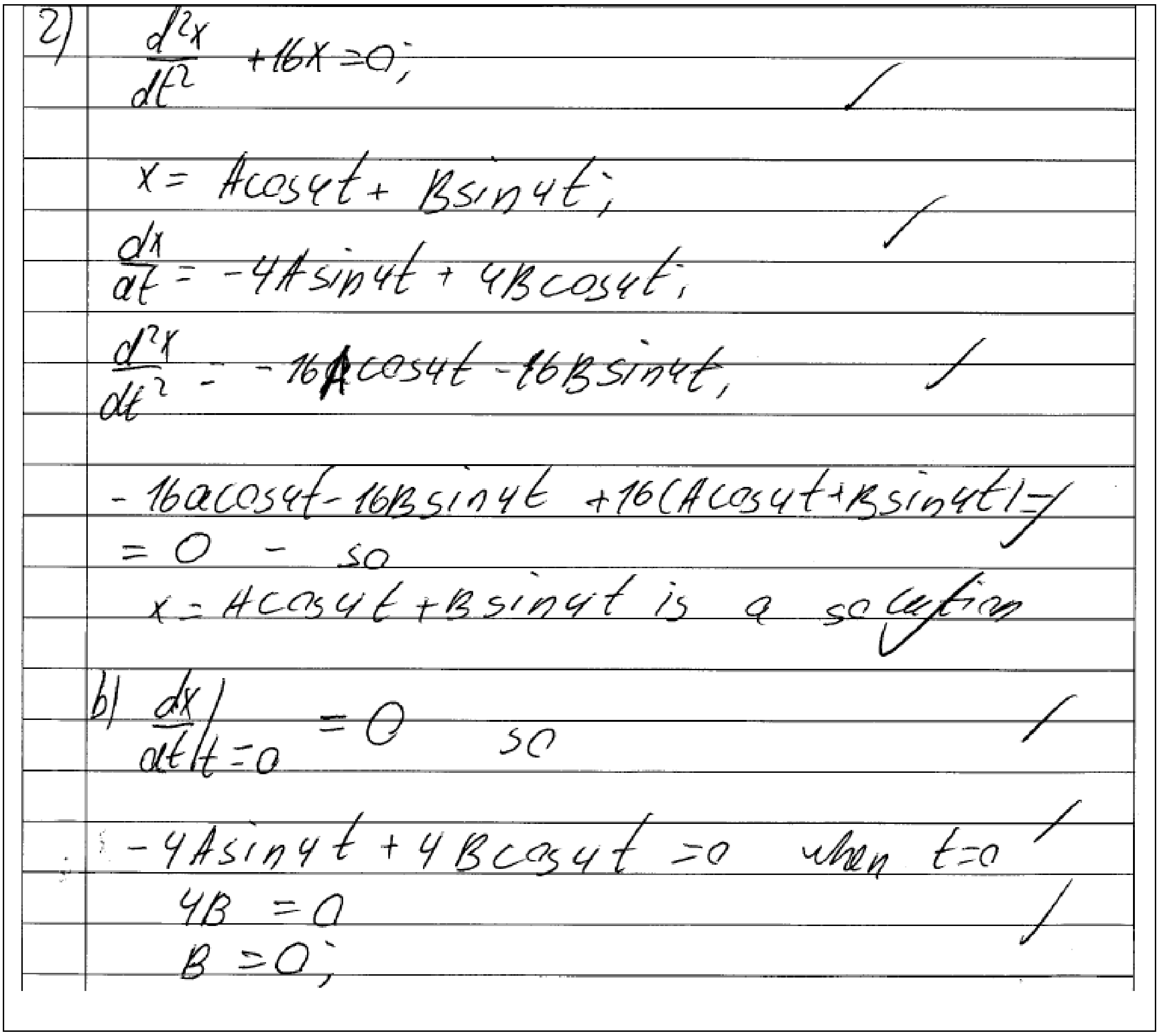
(b) When $t = 0$, the particle is momentarily at rest. Show that $B = 0$. *(1 mark)*

(c) Given that $x = h$ $(h > 0)$ when $t = \frac{\pi}{12}$, find A in terms of h. *(2 marks)*

(d) Find the maximum speed of the particle in terms of h. *(1 mark)*

(e) The mass of the particle is m kg. Find the magnitude of the maximum force acting on the particle during the motion. Give your answer in terms of h and m. *(2 marks)*

Student response



2) $\frac{d^2x}{dt^2} + 16x = 0;$ ✓

$x = A\cos 4t + B\sin 4t;$ ✓

$\frac{dx}{dt} = -4A\sin 4t + 4B\cos 4t;$

$\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t,$ ✓

$-16a\cos 4t - 16B\sin 4t + 16(A\cos 4t + B\sin 4t) =$ ✓

$= 0$ - so

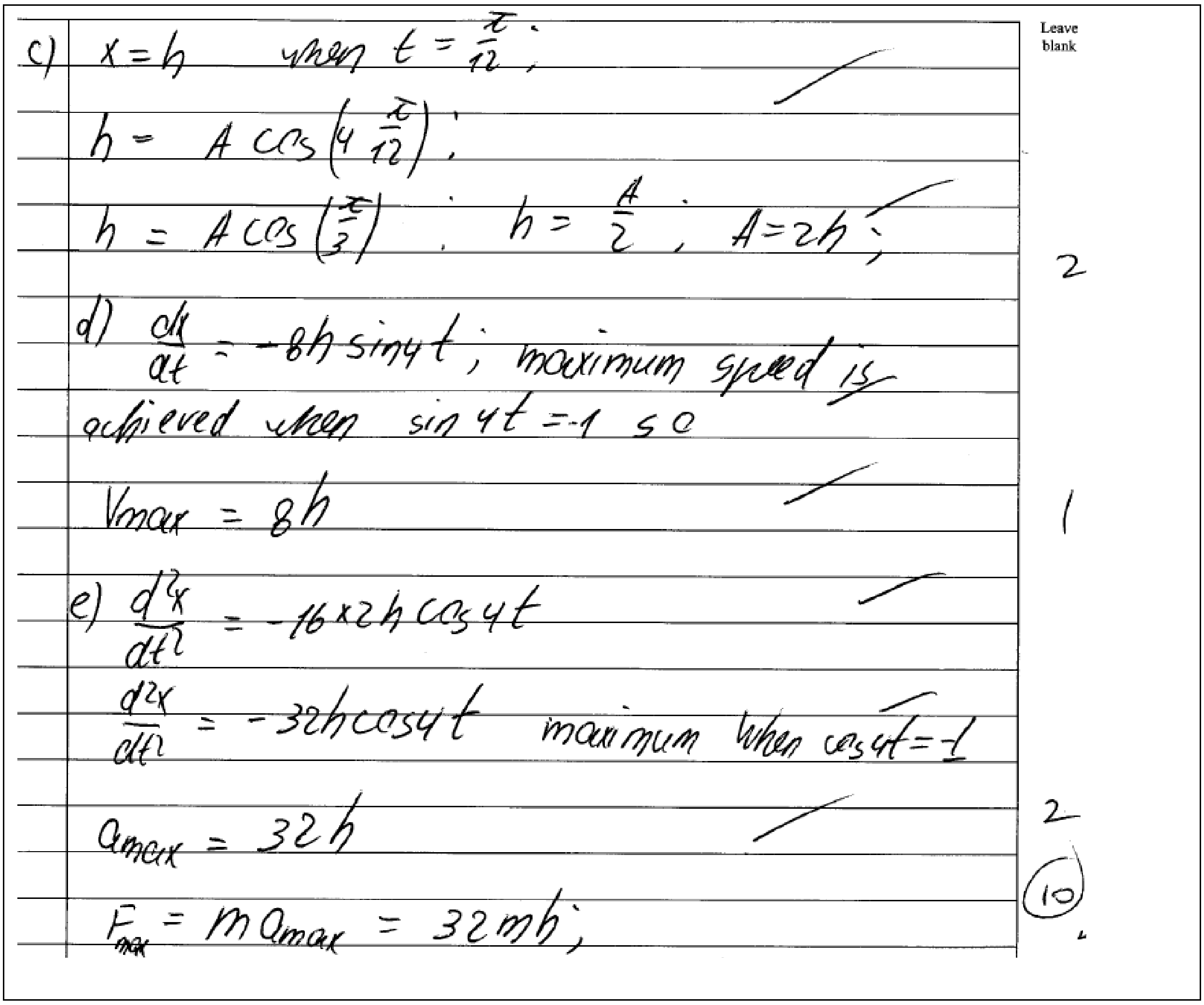
$x = A\cos 4t + B\sin 4t$ is a solution ✓

b) $\left.\frac{dx}{dt}\right|_{t=0} = 0$ so ✓

$-4A\sin 4t + 4B\cos 4t = 0$ when $t=0$ ✓

$4B = 0$ ✓

$B = 0;$



c) $x = h$ when $t = \frac{\pi}{12}$;

$h = A\cos\left(4\frac{\pi}{12}\right)$;

$h = A\cos\left(\frac{\pi}{3}\right)$; $h = \frac{A}{2}$, $A = 2h$;

2

d) $\frac{dx}{dt} = -8h\sin 4t$; maximum speed is achieved when $\sin 4t = -1$ so

$V_{max} = 8h$

1

e) $\frac{d^2x}{dt^2} = -16 \times 2h\cos 4t$

$\frac{d^2x}{dt^2} = -32h\cos 4t$ maximum when $\cos 4t = -1$

$a_{max} = 32h$

2

⑩

$F_{max} = m a_{max} = 32mh$;

Commentary

Once again the question was answered well, showing good knowledge and understanding of this topic. The appropriate techniques were usually applied efficiently to produce correct solutions.

A very sound response, with an especially well explained and clearly worked out solution.

Mark Scheme

2(a)	$x = A\cos 4t + B\sin 4t$ $\dot{x} = -4A\sin 4t + 4B\cos 4t$ $\ddot{x} = -16A\cos 4t - 16B\sin 4t$ Substitute into $\ddot{x} + 16x = 0$ Satisfactory conclusion	 B1 B1 M1 A1	 4	**Alt:** $m^2 + 16 = 0$ B1 $m = \pm 4i$ B1 $x = A\cos 4t + B\sin 4t$ M1 A1
(b)	$t = 0,\ \dot{x} = 0:\ \ 0 = 0 + 4B \ \rightarrow\ B = 0$	B1	1	AG
(c)	$x = A\cos 4t$ $t = \frac{\pi}{12},\ x = h:\quad h = A\cos\frac{\pi}{3}$ $A = 2h$	 M1 A1	 2	
(d)	$\dot{x} = -8h\sin 4t$ Max speed $= 8h$ m s^{-1}	 B1	 1	
(e)	$F = m(-32h\cos 4t)$ $\lvert F_{\max}\rvert = 32hm$ N	M1 A1	 2	
	Total		**10**	

Question 3

3 A particle P moves in a plane so that, at time t, its polar coordinates (r, θ) with respect to a fixed origin, O, are given by

$$r = t^2 \qquad \theta = \frac{9}{\pi^2}\sin\frac{\pi t}{6}$$

(a) Find the radial and transverse components of the velocity of P when $t = 3$. *(4 marks)*

(b) Find the radial and transverse components of the acceleration of P when $t = 3$. *(5 marks)*

(c) Determine the angle between the acceleration of P and OP when $t = 3$. *(2 marks)*

Student Response

3.

a) $\dot{r} = 2t$ $\qquad \dot{\theta} = \frac{9\pi}{6\pi^2} \cos \pi t/6$

$= \frac{3}{2\pi} \cos \pi t/6$ ✓

radial component og velocity $= \dot{r}$

$= 2t$ ✓

when $t = 3$,

radial component og velocity $= 6$ ✓

~~tra~~ transvese component og velocity $= r\dot{\theta}$ M1

$= t^2 \times \frac{3}{2\pi} \cos \pi t/6$ A1

$= \frac{27t}{\pi} \cos \pi t/6$

M1

when $t = 3$

transvese component og velocity $= 3 \times \frac{3}{\pi} \cos 3\pi/6$ A0

$= \frac{9}{\pi} \cos \pi/2$

$= 0$

b) $\ddot{r} = 2$ $\qquad \ddot{\theta} = -\frac{3\pi}{12\pi} \sin \pi t/6$

$= -\frac{1}{4} \sin \pi t/6$

when $t = 3$:

radial component of acceleration $= \ddot{r} - r\dot{\theta}^2$

$= 2 - \frac{9}{4} \times \left(\frac{3}{2\pi} \cos \frac{3\pi}{6}\right)^2$

$= 2 - 0$

$= 2$

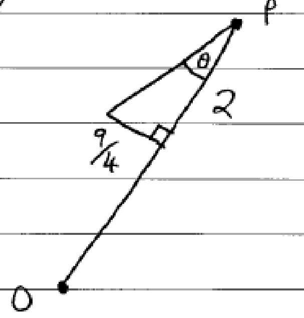
transverse component of acceleration $= r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 3^2\left(-\frac{1}{4} \sin \frac{\pi}{2}\right) + 2(6)\left(\frac{3}{2\pi} \cos \frac{\pi}{2}\right)$

$= -\frac{9}{4} \times 1 + 0$

$= -\frac{9}{4}$

c)



$\tan \theta = \frac{\left(\frac{9}{4}\right)}{2}$

$\theta = \tan^{-1}\left(\frac{9}{8}\right)$

$= 0.844$ rad.

$= 0.84$ rad. fw

2

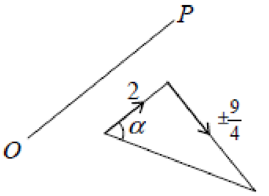
10

Commentary

This question was also popular, mostly with careful and accurate use of the relevant equations governing the motion. There were a few errors in differentiation and algebraic work, and occasionally inappropriate choice of components in part (c).

This student's solution shows the most frequent errors in working this question, in both the differentiation and manipulation of necessary algebra.

Mark Scheme

3(a)	$r=t^2$ $\quad\theta=\frac{9}{\pi^2}\sin\left(\frac{\pi t}{6}\right)$			
	$\dot{r}=2t$ $\quad\dot{\theta}=\frac{3}{2\pi}\cos\left(\frac{\pi t}{6}\right)$	M1 A1		differentiation $\dot{\theta}$
	$t=3,\quad \dot{r}=6,\ r=9,\ \dot{\theta}=0$			
	Components: $\quad\dot{r}\quad r\dot{\theta}$	M1		subs attempted
	$\dot{r}=6\quad r\dot{\theta}=0$	A1	4	
(b)	$\ddot{r}=2$ $\quad\ddot{\theta}=-\frac{1}{4}\sin\left(\frac{\pi t}{6}\right)$	M1 A1F		differentiation ft slip in $\dot{\theta}$
	$t=3,\ \ddot{r}=2$ $\quad\ddot{\theta}=-\frac{1}{4}$	A1F		ft slip in $\dot{\theta}$
	Components: $\quad\ddot{r}-r\dot{\theta}^2\quad r\ddot{\theta}+2\dot{r}\dot{\theta}$	M1		subs attempted
	$=2\quad =-\frac{9}{4}$	A1F	5	ft slip in $\dot{\theta}$
(c)				
	$\tan\alpha=\pm\frac{9}{8}$	M1		
	$\alpha=\pm0.844$ rads or ±2.30 rads	A1	2	any one of these; allow degrees (48.4°)
	Total		**11**	

Question 4

4 A rocket is launched from the ground so that it travels vertically upwards. The rocket ejects burnt fuel vertically downwards at a speed of $1400\,\mathrm{m\,s^{-1}}$ relative to the rocket at a constant rate of $10\,\mathrm{kg\,s^{-1}}$.

The initial mass of the rocket and its fuel is 1000 kg.

The velocity of the rocket at time t seconds after it is launched is $v\,\mathrm{m\,s^{-1}}$.

It may be assumed that the only external force acting on the rocket is gravity. The acceleration due to gravity should be taken as constant.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -9.8 + \frac{1400}{100 - t}$$

(8 marks)

(b) Given that $v = 0$ when $t = 0$, show that

$$v = -9.8t + 1400 \ln\left(\frac{100}{100 - t}\right)$$

(3 marks)

(c) When $t = 80$, the fuel in the rocket has all been burnt. Find the total time taken for the rocket to reach its maximum height. *(4 marks)*

Student Response

blank

4. a) $M_0 = 1000$ ✓ ↗ $\frac{dm}{dt} = -10$ ✓

$M = 1000 - 10t$ ✓

B1

B1

$(m+\delta m)(v+\delta v) - mv = F\delta t$

$mv + m\delta v + v\delta m + \delta v\delta m - mv = F\delta t$

M1

A1

A0

~~m~~ $\therefore F = m\frac{dv}{dt} + v\frac{dm}{dt}$

M0

$-(1000-10t)g = (1000-10t)\frac{dv}{dt}$ ~~m~~ $+ 1400\frac{dm}{dt}$

$(1000-10t)\frac{dv}{dt} =$ ~~m~~$14000 - (1000-10t)g$

$\frac{dv}{dt} = \frac{1400\cancel{0}}{100\cancel{0}-1\cancel{0}t} - 9.8$

$\frac{dv}{dt} = \frac{1400}{100-t} - 9.8$ as required

A0

b) ~~$\int\frac{dv}{dt}$~~ ~~m~~ $\int dv = \int\frac{1400\,dt}{100-t} - \int 9.8\,dt$ ✓

M1

$v = 1400\ln\left(\frac{100}{100-t}\right) - 9.8t + c$

A0

when $t=0, v=0,$ $0 = 0 - 0 + c$

$\therefore c = 0$

$\therefore v = 1400\ln\left(\frac{100}{100-t}\right) - 9.8t$

as required

Lea
bla

c) $t = 80, \quad v = 1400 \ln\left(\frac{100}{20}\right) - 784$ ✓

$v = 1469.213077$ m/s

let $u = 1469.21\ldots, \quad v = 0, \quad a = -9.8$ ✓

$v = u + at$

$0 = 1469.213\ldots - 9.8t$

$9.8t = 1469.213\ldots$ ✓

$t = 149.919\ldots.$

∴ total time to reach max height

$T = 80 + 149.919$ ✓

$T = 230$ secs (3sf.)

4

(9)

Commentary

This question was less well done, and in particular part (a) proved very challenging, with fully correct responses in the minority; the most serious error being the omission of the gravity term. In part (b), the printed result was sometimes obtained erroneously, without the inclusion of a constant of integration or any equivalent technique. Part (c) was done well by many, but some failed to appreciate the two separate stages of the motion.

The solution to part (a) shows a lack of understanding of the forces affecting the motion, with the omission of a vital term. The integration in part (b) is incorrect and working appears to be directed be the printed result. Part (c) shows a good understanding of the two stages of the motion.

Mark Scheme

4(a)	$(m+\delta m)(v+\delta v)-mv-\delta m(v-V)=-mg\delta t$	M1A2		
	$m\delta v+\delta mV=-mg\delta t$			
	$\Rightarrow \frac{m\mathrm{d}v}{\mathrm{d}t}+\frac{V\mathrm{d}m}{\mathrm{d}t}=-mg$	M1		
	$m=(1000-10t) \qquad \frac{\mathrm{d}m}{\mathrm{d}t}=-10$	B1 B1		
	$(1000-10t)\frac{\mathrm{d}v}{\mathrm{d}t}+1400(-10)=-(1000-10t)9.8$	m1		
	$\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{420+9.8t}{100-t}$	A1	8	
	$\left(=\frac{1400-980+9.8t}{100-t}=-9.8+\frac{1400}{100-t}\right)$			
(b)	$\int_0^v \mathrm{d}v=\int_0^t\left\{-9.8+\frac{1400}{100-t}\right\}\mathrm{d}t$	M1		separate variables ⇒ integration
	$v=\left[-9.8t-1400\ln(100-t)\right]_0^t$	A1		
	$v=-9.8t+1400\ln\left(\frac{100}{100-t}\right)$	A1	3	AG
(c)	$t=80, \quad v=1469$	B1		
	$0=1469-9.8t$	M1		
	$t=150$	A1		
	Total time = 230 sec	A1	4	
	Total		**15**	

Question 5

5 A particle, of mass 2 kg, is suspended from a fixed point O by a light spring of natural length 0.5 metres and modulus of elasticity 49 N.

(a) Initially, the particle hangs at rest in equilibrium below O. Find the extension of the spring in this position. *(2 marks)*

(b) A force, F newtons, is then applied to the particle in a vertically downwards direction. The displacement of the particle below its equilibrium position at time t seconds later is x metres. Given that $F = 12\cos nt$, where n is a positive constant, show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+49x=6\cos nt$$

(5 marks)

(c) In the case where $n=5$, find an expression for x at time t. *(10 marks)*

(d) State the value of n for which resonance occurs. *(1 mark)*

Student Response

blank

(

5)a) $F = \frac{\lambda x}{l}$

$20 = \frac{49}{0.5} x$

$x = 0.2m$

2

b) Using $F = ma$

$\ddot{x} = 12\cos nt - \frac{49}{0.5}(x)$

M0

$2\ddot{x} = 12\cos nt - 98x$

$\ddot{x} + \frac{(98x)}{2} = \frac{12\cos nt}{2}$

B0.

$\frac{d^2x}{dt^2} + 49x = 6\cos nt$

A0

c) $\frac{d^2x}{dt^2} + 49x = 6\cos 5t$ (let $x = Ae^{mt}$)

$u^2 + 49 = 0$

M1

$u^2 = -49$

A1

$u = \pm i7$

$\therefore x = A\cos 7t + B\sin 7t$

M1

let $y = C\cos 5t + D\sin 5t$

A1

$dy/dt = -5C\sin 5t + 5D\cos 5t$

A1

$d^2y/d^2t = -25C\cos 5t - 25D\sin 5t$

Substituting

$-25C\cos 5t - 25D\sin 5t + 49(C\cos 5t + D\sin 5t) = 6\cos 5t$

$\Rightarrow -25C + 49C = 6$

$C = 1/4$

A1

blan

$\Rightarrow -25D + 49D = 0$ ✓

$D = 0$ ✓

$\therefore PI = \frac{1}{4}\cos 5t$ ✓

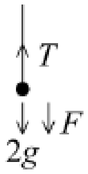
$x = (A\cos 7t + B\sin 7t) + \frac{1}{4}\cos 5t$ ✓ M

d) $x(0)$

Commentary

Part (a) was completed successfully by most candidates. Part (b) was often successful, with a common error being the omission of the weight and an incorrect expression for the tension in the general position; often these two errors cancelled each other out in subsequent working, leading to an apparently correct solution. The main error in part (c) was to stop work having found the full general solution and not to evaluate the remaining constants. Part (d) was not known well.

In this solution, the candidate does not include the weight in the equation of motion in part (b), and the expression for the tension in the general position is also incorrect. The solution in part (c) stops when the candidate has found the general solution of the differential equation, so the full solution including values of constants is not found. The request in part (d) is not known.

5(a)	 $2g=\frac{49\times e}{0.5}$	M1		
	$e=0.2$ metres	A1	2	
(b)	 $2\ddot{x}=2g+F-T$	M1A1		
	$2\ddot{x}=2g+12\cos nt-\frac{49(x+0.2)}{0.5}$	B1F		Tension ft (a)
	$2\ddot{x}=2g+12\cos nt-98x-19.6$	A1F		ft (a)
	$\ddot{x}+49x=6\cos nt$	A1	5	AG
(c)	$n=5$ PI, $x=A\cos 5t+B\sin 5t$	M1		Accept cos term only
	$\dot{x}=-5A\sin 5t+5B\cos 5t$	A1		
	$\ddot{x}=-25A\cos 5t-25B\sin 5t$	A1		
	Subs: $-25A\cos 5t-25B\sin 5t+$ $49A\cos 5t+49B\sin 5t=6\cos 5t$			
	$B=0,\quad A=\frac{1}{4}$	A1		
	A. eqn: $m^2+49=0 \quad m=\pm 7\mathrm{i}$	M1		
	C.F.: $x=C\cos 7t+D\sin 7t$	A1		
	Gen sol: $x=C\cos 7t+D\sin 7t+\frac{1}{4}\cos 5t$	M1		
	$t=0,\ x=0:\quad 0=C+\frac{1}{4}\quad C=-\frac{1}{4}$	A1		
	$\dot{x}=-7C\sin 7t+7D\cos 7t-\frac{5}{4}\sin 5t$	m1		
	$t=0,\ \dot{x}=0:\quad D=0$			
	$x=\frac{1}{4}(\cos 5t-\cos 7t)$	A1	10	
(d)	Resonance occurs when $n=7$	B1	1	
	Total		**18**	

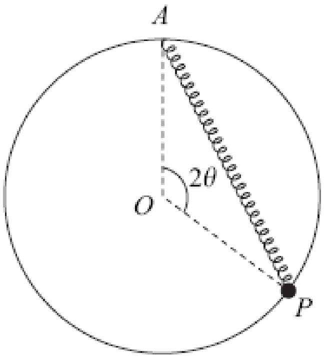
Question 6

6 A smooth circular wire, of radius a and centre O, is fixed in a vertical plane.

A small smooth bead, P, of mass m, can move freely on the wire.

The bead is attached to one end of a light spring, which has modulus of elasticity $4mg$ and natural length a. The other end of the spring is attached to A, the highest point on the wire.

The angle subtended by the spring at O is 2θ, as shown in the diagram, where $0 < \theta \leqslant \frac{\pi}{2}$.



(a) (i) Show that the elastic potential energy stored in the spring in this position is given by

$$2mga(2\sin\theta - 1)^2 \qquad \textit{(3 marks)}$$

(ii) The gravitational potential energy is taken to be zero at the level of the lowest point on the wire. Show that the total potential energy, V, is given by

$$V = 2mga(3\sin^2\theta - 4\sin\theta + 2) \qquad \textit{(5 marks)}$$

(b) Find the two values of θ for which the bead is in equilibrium, giving your answers to two decimal places. *(4 marks)*

(c) Determine, for each of these values, whether the bead is in stable or unstable equilibrium. *(4 marks)*

Student Response

6 (a)(i) length of AP = $2a\sin\theta$ ✓

$e = 2a\sin\theta - a$

$\therefore E.P.E = \frac{\lambda e^2}{2l} = \frac{4mg(2a\sin\theta - a)^2}{2 \times a}$ ✓

$= \frac{\cancel{4}^{2}mg a^{\cancel{2}}(2\sin\theta - 1)^2}{\cancel{2a}}$ ✓

$= 2mga(2\sin\theta - 1)^2$ ✓ 3

(ii) $h = 2a - 2a\sin\theta \cdot \sin\theta$ ✓

$h = 2a - 2a\sin^2\theta$ ✓

$G.P.E = mgh = mg(2a - 2a\sin^2\theta) = 2mga - 2mga\sin^2\theta$ ✓

$\therefore V = G.P.E + E.P.E$ ✓

$= 2mga - 2mga\sin^2\theta + 2mga(2\sin\theta - 1)^2$ ✓

$= 2mga[(1 - \sin^2\theta) + (2\sin\theta - 1)^2]$ ✓

$= 2mga[1 - \sin^2\theta + 4\sin^2\theta - 4\sin\theta + 1]$ ✓

$= 2mga(3\sin^2\theta - 4\sin\theta + 2)$ ✓ 5

✓

(b) $\frac{dV}{d\theta} = 2mga(3\sin\theta\cos\theta + 3\sin\theta\cos\theta - 4\cos\theta) = 0$ M1

$\therefore 6\sin\theta\cos\theta - 4\cos\theta = 0$ ✓ A1

$\cos\theta(6\sin\theta - 4) = 0 \quad \therefore \cos\theta = 0$ or $\sin\theta = \frac{2}{3}$ ✓ M1

$\therefore \theta = 90°$ or $\theta = 41.81°$ A0

(c) $\frac{d^2V}{d\theta^2} = 2mga(-6\sin\theta\sin\theta + 6\cos\theta\cos\theta + 4\sin\theta)$ ✓ M1

$= 2mga(-6\sin^2\theta + 6\cos^2\theta + 4\sin\theta)$ ✓ A1

when $\theta = 90°$ $\frac{d^2V}{d\theta^2} = -6 + 4 = -2 < 0$ unstable A0

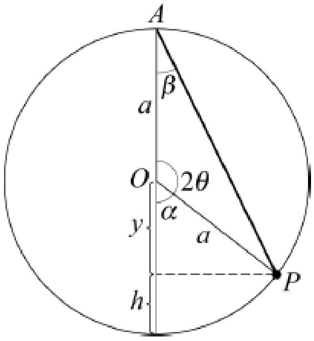
when $\theta = 41.8°$ $\frac{d^2V}{d\theta^2} = -6 \times 0.44 + 6 \times 0.56 + 4 \times 0.67 = 3.4 > 0$ A0

Commentary

This question was well done, with, in part (a) in particular, efficient use of a range of trignometrical techniques to obtain the required expression. Part (b) produced further sound work, the only disappointment being the use of degrees in solutions. Part (c) was also done well although there was a tendency for expressions for the second derivative to lose the *mga* term and become solely functions of the angle *Θ.*

Part (a) is answered well, with very clear and concise working. Part (b) shows all the correct techniques until the values of *Θ* are given in degrees. Working in part (c) is good but becomed careless when the candidate loses the *mga* multiple in the expression to determine the nature of the equilibrium in the two positions under consideration.

Mark Scheme

6				
(a)(i)	Extension $2a\sin\theta - a$	B1		
	$\text{EPE} = \frac{4mg}{2a}(2a\sin\theta - a)^2$	M1		
	$= 2mga(2\sin\theta - 1)^2$	A1	3	AG
(ii)	$y = a\cos\alpha = a\cos(\pi - 2\theta) = -a\cos 2\theta$	B1		**Alt :** $AP\cos\beta = 2a\sin\theta\ \cos\beta = 2a\sin^2\theta$ B1
	$h = a - y = a + a\cos 2\theta$	B1		$h = 2a - 2a\sin^2\theta$ B1
	$V = mga(1 + \cos 2\theta) + 2mga(2\sin\theta - 1)^2$	M1		$V\ = mga\left(2a - 2a\sin^2\theta\right) + \text{EPE}$ M1
	$= mga\left(1 + 1 - 2\sin^2\theta + 8\sin^2\theta - 8\sin\theta + 2\right)$	A1		Simplify A1
	$V = 2mga\left(3\sin^2\theta - 4\sin\theta + 2\right)$	A1	5	AG A1
(b)	$\frac{\mathrm{d}v}{\mathrm{d}\theta} = 2mga(6\sin\theta\cos\theta - 4\cos\theta)$	M1A1		
	$\frac{\mathrm{d}v}{\mathrm{d}\theta} = 0$ if $\cos\theta = 0$ or $6\sin\theta - 4 = 0$	m1		
	ie $\theta = 1.57$ or $\theta = 0.73$	A1	4	
(c)	$\frac{\mathrm{d}v}{\mathrm{d}\theta} = 2mga(3\sin 2\theta - 4\cos\theta)$			
	$\frac{\mathrm{d}^2v}{\mathrm{d}\theta^2} = 2mga(6\cos 2\theta + 4\sin\theta)$	M1A1F		OE PE slip
	$\theta = 1.57,\ \frac{\mathrm{d}^2v}{\mathrm{d}\theta^2} = -4mga$ equil unstable	A1		
	$\theta = 0.73,\ \frac{\mathrm{d}^2v}{\mathrm{d}\theta^2} = 6.7mga$ equil stable	A1	4	
	Total		**16**	
	TOTAL		**75**	