Teacher Support Materials

Maths GCE

Paper Reference MPC2

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Question 1

1 (a) Simplify:

(i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}}$; *(1 mark)*

(ii) $x^{\frac{3}{2}} \div x$; *(1 mark)*

(iii) $\left(x^{\frac{3}{2}}\right)^2$. *(1 mark)*

(b) (i) Find $\int 3x^{\frac{1}{2}}\,dx$. *(3 marks)*

(ii) Hence find the value of $\int_1^9 3x^{\frac{1}{2}}\,dx$. *(2 marks)*

Student Response

Leave blank

b) i) $\int 3x^{\frac{1}{2}}\,dx$

$3\int x^{\frac{1}{2}}\,dx$

$3\left[\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}\right] = \dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ ∧ M1 A1 A0 2

ii) $\left[\dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^9$ $54 - 2 = 52$ M1 A1 2

(7)

Commentry

Part (a) was generally answered very well. The only common error was in (iii) where the wrong answer '$x^{\frac{9}{4}}$' was given by a minority of candidates. In part (b)(i) most candidates illustrated the correct method for integration although some others seemingly differentiated '$3x^{\frac{1}{2}}$' rather than integrated it. By far the most common loss of a mark is indicated by the exemplar. A significant number of candidates did not evaluate '$3 \div \frac{3}{2}$' (or evaluated it incorrectly) and omitted the constant of integration. In part (b)(ii) the majority of candidates knew how to use the given limits but a significant minority could not evaluate '$2\left(9^{\frac{3}{2}}\right) - 2\left(1^{\frac{3}{2}}\right)$' correctly. It was not unusual to see '729 − 1'. It is worth pointing out that in the exemplar the candidate did not show the relevant substitution, namely, '$\frac{3\times 9^{\frac{3}{2}}}{\frac{3}{2}} - \frac{3\times 1^{\frac{3}{2}}}{\frac{3}{2}}$', and would have lost both the method mark and the accuracy mark if, for example, the '54' had been replaced by '729'.

Mark scheme

Q	Solution	Marks	Total	Comments
1(a)(i)	x^2	B1	1	
(ii)	$x^{\frac{1}{2}} = \sqrt{x}$	B1	1	Accept either form
(iii)	x^3	B1	1	
(b)(i)	$\int 3x^{\frac{1}{2}}\,dx = \frac{3}{\frac{3}{2}}x^{\frac{3}{2}}$ $\{+c\}$	M1 A1		Index raised by 1 Simplification not yet required
	$= 2x^{\frac{3}{2}} + c$	A1	3	Need simplification **and** the + c OE
(ii)	$\int_1^9 3x^{\frac{1}{2}}\,dx = (2\times 9^{\frac{3}{2}}) - (2\times 1^{\frac{3}{2}})$	M1		F(9) – F(1), where F(x) is candidate's answer to (b)(i) [or clear recovery]
	= 52	A1ft	2	Ft on (b)(i) answer of form $kx^{1.5}$ i.e. $26k$
	Total		8	

Question 2

2 The nth term of a geometric sequence is u_n, where

$$u_n = 3 \times 4^n$$

(a) Find the value of u_1 and show that $u_2 = 48$. *(2 marks)*

(b) Write down the common ratio of the geometric sequence. *(1 mark)*

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is $4^k - 4$, where k is an integer. *(3 marks)*

(ii) Hence find the value of $\sum_{n=2}^{12} u_n$. *(1 mark)*

Student response

Leave blank

2 a) $u_1 = 3 \times 4^1 = 12$ ✓

$u_2 = 3 \times 4^2 = 3 \times 16 = 48$ ✓ 2

b) Common ratio $= 4$ ✓ 1

c) i/ $S_{12} = \dfrac{12 \times (1 - 4^{12})}{(1-4)} = \dfrac{12 \times (4^{12} - 1)}{(4-1)}$ ✓

$= \dfrac{12(4^{12} - 1)}{3}$ ✓

$= 4 \times (4^{12} - 1)$ ✓

$= 4^{13} - 4$ 3

$k = 13$.

ii/ $\sum_{n=2}^{12} u_n = \sum_{n=1}^{13} u_n - u_1$ ✓

$= 4^{13} - 4 - 12$ ✓

$= 4^{13} - 16$

$= 67108864 - 16$

$= 67108848$. ✓ 1

(7)

Commentry

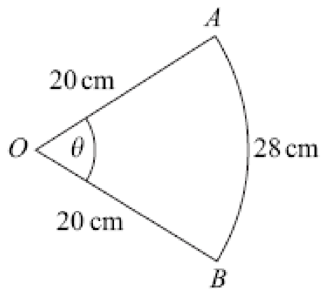
Most candidates scored full marks in part (a). As indicated in the exemplar, candidates generally showed sufficient detail to convince examiners that the printed value of 48 for u_2 had been obtained legitimately. The correct value, 4, for the common ratio was usually given. The most popular wrong answer was '36' (= 48 − 12).
In part (c)(i) many candidates applied the correct formula for the sum to 12 terms of the geometric sequence but could not find the correct value for *k*. It was not uncommon to see the error '$-4(1-4^{12}) = -4+16^{12}$' resulting in '*k* = 24'. Some other candidates made sign errors and just altered their final line rather than going back through the solution to find and correct the original error. In the exemplar the candidate overcomes the common sign error by writing $\frac{1-4^{12}}{1-4}$ as $\frac{4^{12}-1}{4-1}$ before eliminating the fraction. This candidate goes on to find the correct value for *k* by using $4\times 4^{12} = 4^{13}$.
In part (c)(ii) most candidates did not seem to know how to use the sigma sign. The exemplar shows an excellent solution to this part of the question. The first line shows a thorough understanding of the sigma notation and sets up the use of earlier parts of the question. The candidate goes on to score the mark by evaluating '4^{13} − 16' correctly.

Mark Scheme

2(a)	$u_1 = 12$	B1		
	$u_2 = 3\times 4^2 = 48$	B1	2	CSO AG (be convinced)
(b)	$r = 4$	B1	1	
(c)(i)	$\{S_{12} =\}\frac{a(1-r^{12})}{1-r}$	M1		OE Using a correct formula with $n = 12$
	$= \frac{12(1-4^{12})}{1-4}$	A1ft		Ft on answer for u_1 in (a) and r in (b)
	$= \frac{12(1-4^{12})}{-3} = -4(1-4^{12}) = 4^{13}-4$	A1	3	CAO Accept $k = 13$ for 4^{13} term
(ii)	$\sum_{n=2}^{12} u_n = (4^{13}-4) - u_1$ $= 67108848$	B1	1	
	Total		7	

Question 3

3 The diagram shows a sector OAB of a circle with centre O and radius 20 cm. The angle between the radii OA and OB is θ radians.

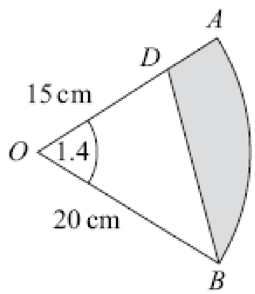


The length of the arc AB is 28 cm.

(a) Show that $\theta = 1.4$. *(2 marks)*

(b) Find the area of the sector OAB. *(2 marks)*

(c) The point D lies on OA. The region bounded by the line BD, the line DA and the arc AB is shaded.



The length of OD is 15 cm.

(i) Find the area of the shaded region, giving your answer to three significant figures. *(3 marks)*

(ii) Use the cosine rule to calculate the length of BD, giving your answer to three significant figures. *(3 marks)*

Student Response

3a) $r\theta = 28$ ✓

$20\theta = 28$

$\theta = \frac{28}{20} = 1.4$ ✓ 2

b) $\frac{1}{2}r^2\theta$ ✓ $\frac{1}{2} \times (20\times20)\times1.4 = 280\text{cm}^2$ ✓ 2

Δ area ✓

c) $\frac{1}{2}ab\sin C = \frac{1}{2}(20\times15)\sin 1.4 = 147.817$

≈ 148 (3sf) cm^2 3

$280 - 148 = 132\text{cm}^2$ ✓ area of shaded region

$c^2 = a^2 + b^2 - 2ab\sin C$ M0

$= 20^2 + 15^2 - 2\times20\times15\sin 1.4 = 33.730$ 0

$\sqrt{33.730}$

$= 5.8078$

≈ 5.81 (3sf) cm (7)

Commentary

Most candidates scored high marks on this question. The common errors, other than using wrong formulae, included using the wrong mode on the calculator or failing to write final answers to the degree of accuracy asked for in the question. In part (a) candidates need to show sufficient detail in solutions to reach a printed answer. In the exemplar the candidate does this by first quoting the general expression *r*□ for the arc length. Again, in part (b), the candidate correctly quotes the general expression $\frac{1}{2}r^2\theta$ for the area of the sector before substituting the relevant values to obtain the correct answer. The examiner is then able to give the method mark even if the candidate had made a numerical slip in the subsequent calculation. In part (c)(i), most candidates were able to obtain the correct value for the area of the shaded region but some stopped after finding the area of the triangle. In the exemplar the candidate obtained the correct 3 sf answer for the required area although it would perhaps be wiser in similar questions to do the rounding at the end rather than at the area of the triangle stage. Considering that the cosine rule is in the AQA formulae booklet it was surprising to find a significant minority of candidates misquoting it. In the exemplar the candidate has quoted the cosine rule as having a sine rather than a cosine! This error appeared more often this year than in previous sittings of the MPC2 examination. By considering the diagram it should have been clear from the size of angle *DOB* (=1.4 rads ≈ 80º) that the length of *BD* would be at least greater than 15 cm.

Mark Scheme

3(a)	Arc = $r\theta$	M1		For $r\theta$ or 20θ or PI by 20×1.4
	$28 = 20\theta \Rightarrow \theta = 1.4$	A1	2	AG
(b)	Area of sector = $\frac{1}{2}r^2\theta$	M1		$\frac{1}{2}r^2\theta$ OE seen
	$= \frac{1}{2}20^2(1.4) = 280$ (cm^2.)	A1	2	Condone absent cm^2.
(c)(i)	Area triangle $= \frac{1}{2}\times15\times20\times\sin1.4$ (= 147.8….)	M1		Use of $\frac{1}{2}ab\sin C$ OE
	Shaded area = Area of sector – area of triangle	M1		
	= 280 – 147.8 = 132 (cm^2.) (3sf)	A1ft	3	Ft on [ans (b) – 147.8…] to 3sf provided [….] > 0
(ii)	$\{BD^2 =\}15^2 + 20^2 - 2\times15\times20\cos1.4$	M1		RHS of cosine rule used
	$= 225 + 400 - 101.98...$	m1		Correct order of evaluation
	$\Rightarrow BD = \sqrt{523.019...} = 22.86..$ = 22.9 (cm) to 3 sf	A1	3	Condone absent cm
	Total		**10**	

Question 4

4 An arithmetic series has first term a and common difference d.

The sum of the first 29 terms is 1102.

(a) Show that $a + 14d = 38$. *(3 marks)*

(b) The sum of the second term and the seventh term is 13.

Find the value of a and the value of d. *(4 marks)*

Student Response

4 a)

$S_n = \frac{1}{2}n[2a + (n-1)d]$

$S_{29} = \frac{1}{2} \times 29[2a + (28)d]$

$1102 = 14.5 \times (2a + 28d)$

$1102 = 14.5 \times 2(a + 14d)$

$1102 = 14.5 \times 2(a + 14d)$

$76 = 2(a + 14d)$

$38 = a + 14d$

$a + 14d = 38$

3

b) $u_2 = a + (2-1)d$ $a + d = u_2$ B1

$u_7 = a + (7-1)d$ $a + 6d = u_7$

$U_2 + U_7 = 13$ $13 = a + d + a + 6d$

$13 = 2a + 7d$ M1

$a + d = U_2$ $a + 6d = U_7$

$a + 6d = U_7$ $- a + d = U_2$

$5d = U_7 - U_2$

1 M0

2

5

Commentary

Many candidates substituted n = 29 to obtain a correct expression in terms of a and d for the sum of the first 29 terms of the arithmetic series. These candidates usually equated their expression to 1102 but some then jumped too quickly to the printed result. In the exemplar the candidate shows good technique by firstly writing down the general formula for S_n before substituting n = 29. The candidate then writes down sufficient steps in a convincing solution to reach the printed answer. In part (b) a significant number of candidates failed to interpret the given information correctly by again using the formula for S_n to obtain expressions for both S_2 and S_7 and equated each to 13. More successful candidates, like the one in the exemplar, were able to obtain correct expressions for u_2 and u_7 and equate their sum to 13 but failed to make any further progress. They did not recognise that the printed result in part (a) was also relevant and that the values of a and d could be found by solving the pair of simultaneous linear equations.

Mark Scheme

4(a)	$\{S_{29} =\}\frac{29}{2}[2a+28d]$	M1		Formula for S_n with n = 29 substituted and with a and d
	$29\ (a + 14d)\ = 1102$	m1		Equation formed then some manipulation
	$a + 14d = \frac{1102}{29} \Rightarrow a + 14d = 38$	A1	3	CSO AG
(b)	$u_2 = a + d \quad u_7 = a + 6d$	B1		Either expression correct
	$u_2 + u_7 = 13 \Rightarrow 2a + 7d = 13$	M1		Forming equation using u_2 & u_7 both in form $a + kd$
	e.g. $21d = 63;\ \ 3a = -12$	m1		Solving $a + 14d = 38$ with candidate's '$2a + 7d = 13$' to at least stage of elimination of either a or d
	$a = -4 \quad d = 3$	A1	4	Both correct
	Total		7	

Question 5

5 A curve is defined for $x > 0$ by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point P lies on the curve where $x = 2$.

(a) Find the y-coordinate of P. *(1 mark)*

(b) Expand $\left(1 + \frac{2}{x}\right)^2$. *(2 marks)*

(c) Find $\frac{dy}{dx}$. *(3 marks)*

(d) Hence show that the gradient of the curve at P is -2. *(2 marks)*

(e) Find the equation of the normal to the curve at P, giving your answer in the form $x + by + c = 0$, where b and c are integers. *(4 marks)*

Student Response

5 a) $y = \left(1 + \frac{2}{2}\right)^2$

$= (2)^2 = 4$ = y-coord P ✓ 1

b) $\left(1 + \frac{2}{x}\right)^2$

$1^2 + \binom{2}{1} 1 \times \frac{2}{x} + \binom{2}{2}\left(\frac{2}{x}\right)^2$

$= 1 + 2 \times \frac{2}{x} + \left(\frac{2}{x}\right)^2 = 1 + \frac{4}{x} + \frac{4}{x^2}$ ✓ 2

c) $y = 1 + 4x^{-1} + 4x^{-2}$ ✓ 3

$\frac{dy}{dx} = -4x^{-2} + (-8x^{-3})$

$= -4x^{-2} - 8x^{-3}$

d) $\frac{dy}{dx} = -4x^{-2} - 8x^{-3}$ ✓ 2

$= -4(2)^{-2} - 8(2)^{-3}$

$= -1 - 1 = -2$ ✓

e) $m_1 m_2 = -1$ if lines are perpendicular

$\therefore$ gradient of normal line $= \frac{1}{2}$ ✓ M1

as $-2 \times \frac{1}{2} = -1$ P (2,4)

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{1}{2}(x - 2)$ ✓ M1 A1

$y - 4 = \frac{1}{2}x - 1$

$y - \frac{1}{2}x - 4 + 1 = 0$ $-\frac{1}{2}x + y - 3 = 0$

~~$+\frac{1}{2}x + y + 3$~~ $= 0$

or $\frac{1}{2}x + y + 3 = 0$ A0 3

1 (11)

Commentary

Most candidates found the correct value for the *y*-coordinate of *P*. The most common wrong answer was 3, presumably from the calculation '$1+\frac{2^2}{2}$'. In the exemplar the candidate illustrates a good use of brackets which overcomes this common error. In part (b) some candidates only gave two terms. In the exemplar, the candidate uses the binomial expansion, again with good use of brackets, although most candidates expanded $\left(1+\frac{2}{x}\right)\left(1+\frac{2}{x}\right)$ directly. The differentiation of *x* raised to a negative power was generally well understood in part (c) and in part (d) most candidates realised that the value of $y'(2)$ was required although others attempted to find $y'(-2)$. In part (e) a significant number of candidates obtained a valid equation for the normal but could not rearrange it correctly into the required form. Common errors included writing $y=\frac{1}{2}x+3$ as $2y = x + 3$ or leaving the final answer with a non-unitary coefficient of *x* as illustrated by the exemplar.

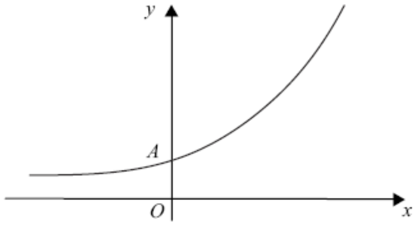
Mark Scheme

Q	Solution	Marks	Total	Comments
5(a)	$y_P = 4$	B1	1	
(b)	$y=1+\frac{2}{x}+\frac{2}{x}+\frac{4}{x^2}$ $y=1+4x^{-1}+4x^{-2}$	B2,1,0	2	(B1 if only one error in the expansion) For B2 the last line of the candidate's solution must be correct
(c)	$\frac{dy}{dx}=-4x^{-2}-8x^{-3}$	M1		Index reduced by 1 after differentiating x to a negative power
		A1ft		At least 1 term in x correct ft on expn
		A1	3	CSO Full correct solution. ACF
(d)	When $x=2$, $\frac{dy}{dx}=-4\times2^{-2}-8\times2^{-3}$	M1		Attempt to find $y'(2)$.
	Gradient $= -1\ -1 = -2$	A1	2	AG (be convinced-no errors seen)
(e)	$-2\times m' = -1$	M1		$m_1\times m_2=-1$ OE stated or used. PI
	$y-4=m(x-2)$	M1		C's y_P from part (**a**) if not recovered; m must be numerical.
	$y-4=\frac{1}{2}(x-2)$	A1ft		Ft on candidate's y_P from part (**a**) if not recovered.
	$x-2y+6=0$	A1	4	CAO Must be this or $0 = x-2y+6$
	Total		**12**	

MPC2

Question 6

6 The diagram shows a sketch of the curve with equation $y = 3(2^x + 1)$.



The curve $y = 3(2^x + 1)$ intersects the y-axis at the point A.

(a) Find the y-coordinate of the point A. *(2 marks)*

(b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 3(2^x + 1)\,dx$. *(4 marks)*

(c) The line $y = 21$ intersects the curve $y = 3(2^x + 1)$ at the point P.

(i) Show that the x-coordinate of P satisfies the equation

$$2^x = 6$$ *(1 mark)*

(ii) Use logarithms to find the x-coordinate of P, giving your answer to three significant figures. *(3 marks)*

Student Response (below)

Leave blank

6. $y = 3(2^x + 1)$

a) $x = 0$

$y = 3(2^0 + 1)$

$y = 3(1+1)$

$y = 6$

$(0, 6)$ ✓ 2

b) $\int_0^6 3(2^x + 1)\,dx$

$\int_a^b y\,dx \approx \frac{1}{2}h\left((y_0 + y_n) + 2(y_1 + y_{n-1})\right)$

x	0	1	2	3	4	5	6
y	6	9	15	27	51	99	195

B1

$\int_6^6 3(2^x + 1)\,dx \approx \frac{1}{2}\,2\left((6 + 195) + 2(9 + 15 + 27 + 51 + 99)\right)$ M0

$= 1\left((201) + 402\right)$ 1

$= 603$ ✗

c) i) $y = 21$

$21 = 3(2^x + 1)$

$7 = 2^x + 1$ ✓

$6 = 2^x$

$2^x = 6$ ✓ 1

Commentary

Part (a) was generally answered well but some candidates failed to gain any credit due to poor examination technique. In the exemplar the candidate shows good examination technique by writing '$y = 3(2^0 + 1)$' which clearly shows that x = 0 has been substituted into the correct equation so even if the evaluation had been incorrect the method mark would still have been scored. The majority of candidates showed a good understanding of the trapezium rule but others could not even gain the method mark. The two main errors involved either the use of seven ordinates rather than the required four or the failure to cover the full range of the limits by use of x=0, 1, 2, 3 only. The exemplar illustrates the first of these errors. The candidate indicates h on the sketch and score the B1 mark for using/stating h = 2. The sketch is useful as it would seem to help the candidate to choose the relevant values of x; indeed the candidate has highlighted, using vertical bars, the values 0, 2, 4 and 6 for x but then in the trapezium rule itself the candidate has used seven ordinates instead of the four required and so the method mark, along with the two associated accuracy marks, have been lost. In part (c)(i) the common error was to write 3×2^x as 6^x. In the exemplar the candidate overcomes this error by immediately dividing both sides by 3 to get '$7=2^x+1$' which is then only one step away from the printed result. The majority of candidates were able to able to answer part (c)(ii) correctly but a significant minority lost the accuracy mark by incorrect rounding. In the exemplar the candidate presents an acceptable solution but perhaps it would have been 'safer' to explicitly show the base 10 and also to show the value 2.5849…before rounding to three significant figures. A significant minority gave the wrong answer 2.59.

Mark Scheme

6(a)	$y_A = 3\left(2^0+1\right)$	M1		Substituting $x = 0$ PI
	$= 6$	A1	2	
(b)	$h = 2$	B1		PI
	Integral = $h/2$ {……}			
	{….} = f(0) + 2[f(2) + f(4)] + f(6)	M1		OE summing of areas of the three traps.
	{} = $6+2\left[3\times5+3\times17\right]+3\times65$ = 6 + 2[15 + 51] + 195	A1		Condone 1 numerical slip {ft on (a) for f(0) if not recovered} [Sum of 3 traps. = 21 + 66 + 246]
	Integral = 333	A1	4	CAO
(c)(i)	$21=3\left(2^x+1\right)\Rightarrow 2^x=6$	B1	1	AG (be convinced)
(ii)	$\log_{10}2^x=\log_{10}6$	M1		Take ln or $\log_{10}$ of both sides of $a^x = b$ or other relevant base if clear. The equation $a^x = b$ used must be correct.
	$x\log_{10}2=\log_{10}6$	m1		Use of $\log 2^x = x\log 2$ OE
	$x=\dfrac{\lg 6}{\lg 2}=2.5849\ldots=2.58$ to 3sf	A1	3	Both method marks must have been awarded.
	Total		**10**	

Question 7

7 (a) Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) Write down the **two** solutions of the equation $\tan x = \tan 61°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(c) (i) Given that $\sin\theta + \cos\theta = 0$, show that $\tan\theta = -1$. *(1 mark)*

(ii) Hence solve the equation $\sin(x - 20°) + \cos(x - 20°) = 0$ in the interval $0° \leqslant x \leqslant 360°$. *(4 marks)*

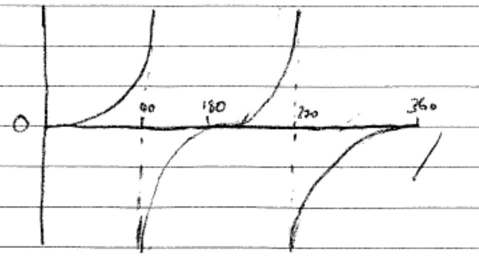
(d) Describe the single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x - 20°)$. *(2 marks)*

(e) The curve $y = \tan x$ is stretched in the x-direction with scale factor $\frac{1}{4}$ to give the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(1 mark)*

Student Response

Leave blank

7



✓ 3

b) $x = 61, 241$ ✓ 2

c) $\sin\theta = -\cos\theta$ ✓ 0

$\tan\theta =$

ii) $x = 155, 335$ ← answer need evidence of 'hence' 0

$\sin(x - 20°) = -\cos(x - 20°)$

d) translation ✓ $\begin{bmatrix} 0 \\ 20 \end{bmatrix}$ ✗ B1 B0 1

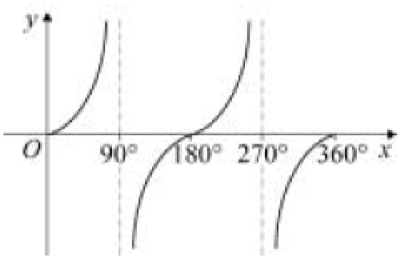
e) $y = 4\tan x$ ✗ ✗ 0

6

Commentary

Sketching the graph of $y = \tan x$ was generally not done well. In the exemplar the candidate produces a sketch which is much better than the usual ones seen. The asymptotes have been drawn in to help position the extremities of the branches and all relevant values in the given interval are shown to indicate clearly where the main features of the graph occur. Part (b) was not always answered correctly as many candidates gave the incorrect solutions tan61 and tan241. In the exemplar the candidate correctly writes down the two values of x as required.
Candidates' solutions to show that $\tan\theta = -1$ in part (c)(i) were often unconvincing. In the exemplar the candidate has performed a correct rearrangement but then does not show the necessary division of both sides by $\cos\theta$ to reach the printed answer. Part (c)(i) was not answered well with many average candidates failing to use the previous part or starting their solution by writing the incorrect statement '$\sin(x-20)+\cos(x-20)=\sin x-\sin 20+\cos x-\cos 20$'. In the exemplar the candidate basically just writes down the two solutions without the necessary evidence that the previous part has been used and so gained no marks. Many candidates gained at least 1 mark for their description of the geometrical transformation in part (d). In the exemplar the candidate gains a mark for using the correct word 'translation' but gives the wrong vector (a vertical rather than a horizontal translation) so does not pick up the second mark. The final part of the question was answered well by the above average candidates but some left their answer as f(4x) which was not awarded the mark. In the exemplar the candidate gives the other common wrong answer which indicates a vertical rather than a horizontal stretch.

Mark Scheme

7(a)		M1 A1 A1	 3	Correct shape of branch from O {to 90°} or correct shapes of branches from 90°-360° Complete graph for $0° \le x \le 360°$ (Asymptotes not explicitly required but graphs should show 'tendency') Correct scaling on x-axis $0° \le x \le 360°$
(b)	61° ; 241°	B1 B1	 2	For 61° For 241° and no 'extras' in the interval $0° \le x \le 360°$
(c)(i)	$\sin\theta = -\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = -1$ $\Rightarrow \tan\theta = -1.$	B1	1	AG; be convinced that the identity $\frac{\sin\theta}{\cos\theta} = \tan\theta$ is known and validly used
(ii)	$\Rightarrow \tan(x-20°) = -1$ $x - 20° = \tan^{-1}(-1)$ $x - 20° = 135°, 315° \ldots$ $x = 155°$; 335°	M1 m1 A1 A1ft	 4	 Ft on (180 + "155") and no 'extras' in the given interval.
(d)	Translation $\begin{bmatrix} 20 \\ 0 \end{bmatrix}$	B1 B1	 2	'Translation'/'translate(d)' Accept equivalent in words provided linked to 'translation/move/shift' (Note: B0B1 is possible)
(e)	f(x) = tan 4x	B1	1	For tan 4x
	Total		**13**	

Question 8

8 (a) It is given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

Find the value of n. *(3 marks)*

(b) Given that $\log_a x = 3$ and $\log_a y - 3\log_a 2 = 4$:

(i) express x in terms of a; *(1 mark)*

(ii) express xy in terms of a. *(4 marks)*

Student Response

Leave blank

8)

b) $\log_a x = 3$

(i) $\therefore a^3 = x$ ✓ B1 1

(ii) $\log_a y - 3\log_a 2 = 4$

$\log_a y - \log_a 2^3 = 4$ ✓ M1

$\log_4 [8y] = 4$ ✗ M0

$\therefore 8y = a^4$ (M0)

$\frac{1}{8}a^4 = y$

$xy = \left(\frac{a^4}{8}\right) \times \left(a^3\right)$ 1

$xy = \frac{a^7}{8}$ ✗

→ more.

4

Commentary

The question produced a wide range of marks. Candidates who started part (a) by writing
'$\log_a n = \log_a 3 + \log_a 2n - \log_a 1$' rarely scored more than 1 mark in the whole question whilst those who had a thorough understanding of logarithms sometimes slipped up in solving a resulting linear equation. In the exemplar the candidate in part (a) shows a thorough understanding of the laws of logarithms to reach the linear equation '$n = 6n - 3$' but then fails to solve this equation correctly. Like many others who attempted part (b)(i) the candidate in the exemplar gives the correct expression, a^3, for x. The candidate in part (b)(ii) displays knowledge of the '3rd law of logarithms' by writing '$3\log_a 2 = \log_a 2^3$' but then makes a costly mistake by writing the difference of two logarithms as the logarithm of the product. This method error has cost the candidate 3 marks in the final question on the paper.

Mark Scheme

8(a)	$\log_a n = \log_a 3(2n-1)$	M1		OE Log law used PI by next line
	$\Rightarrow n = 3(2n-1)$	m1		OE, but must **not** have any logs.
	$\Rightarrow 3 = 5n \Rightarrow n = \frac{3}{5}$	A1	3	
(b)(i)	$\log_a x = 3 \Rightarrow x = a^3$	B1	1	
(ii)	$\log_a y - \log_a 2^3 = 4$	M1		$3\log 2 = \log 2^3$ seen or used any time in **(ii)**
	$\log_a \frac{y}{2^3} = 4$ $\begin{cases} xy = a^7 \times a^{(3\log_a 2)} \\ \text{or} \\ y = a^4 \times a^{(3\log_a 2)} \end{cases}$	M1		Correct method leading to an equation involving y (or xy) and a log but **not** involving + or −
	$\frac{y}{2^3} = a^4$ $\begin{cases} xy = a^7 \times 2^3 \\ \text{or} \\ y = a^4 \times 2^3 \end{cases}$	ml		Correct method to eliminate **ALL** logs e.g. using $\log_a N = k \Rightarrow N = a^k$ or using $a^{\log_a c} = c$
	$by = a^3 \times 8a^4$ or $8a^7$	A1	4	
	Total		**8**	