Teacher Support Materials

Maths GCE

Paper Reference MPC3

Question 1(b)(c)

1 (a) Differentiate $\ln x$ with respect to x. *(1 mark)*

(b) Given that $y = (x+1)\ln x$, find $\dfrac{dy}{dx}$. *(2 marks)*

(c) Find an equation of the normal to the curve $y = (x+1)\ln x$ at the point where $x = 1$. *(4 marks)*

Student Response

(b) $y = (x+1)\ln x$. $u = (x+1)$ $v = \ln x$.
$u' = 1$ $v' = \frac{1}{x}$.

$uv' + vu' \rightarrow (x+1)(\frac{1}{x}) + (\ln x)(1)$ M1 A1

$= \frac{1}{x}(x+1) + \ln x$ ✓

$= \frac{x+1}{x} + \ln x = 1 + \ln x$ X ISW

2

(c) $y - y_1 = m(x - x_1)$.

~~$x = 1$ $y = \ln x$~~
~~$y = \ln(1)$~~
~~$y = 0$ $(1,0)$.~~

$x = 1$ $y = (x+1)\ln x$
$y = (1+1)\ln(1)$
$y = 2 \times 0$
$y = 0$.
$(1,0)$.

$m = 1 + \ln x$ THEIRS

$= \frac{1}{-1+\ln x}$.

$\rightarrow y - 0 = \frac{1}{-1+\ln x}(x-1)$.

$y = \frac{1}{-1+\ln x}(x-1)$ 0

No sub^n $x=1$

$y(-1+\ln x) = 1(x-1)$. M0

$-y + y\ln x = x - 1$. M0

A0

A0

2

Commentary

For this paper candidates were not penalised if they obtained a correct answer but then went on to spoil their answer but this may not be the case in the future.
In this example the script arrives at the correct answer but then through dodgy algebra changes the answer. However they would be penalised on accuracy marks on the later parts of the question.
This script also highlights a second common mistake. Having written the gradient, incorrectly, as (1+lnx) they have not substituted the value for x and as such could not gain the marks for the gradient of the normal.

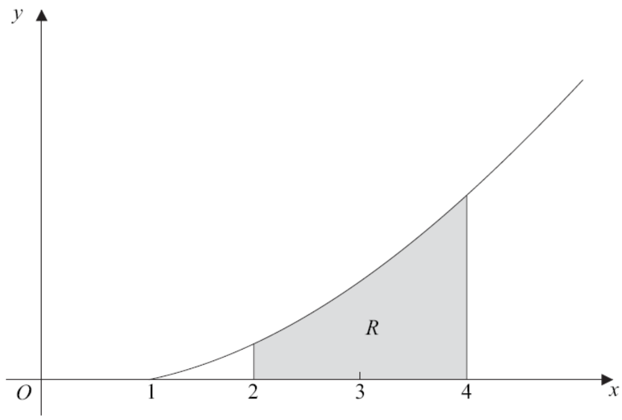
Mark scheme

Q	Solution	Marks	Total	Comments
1(a)	$y = \ln x$			penalise + c once on 1(a) or 2(a)
	$\frac{dy}{dx} = \frac{1}{x}$	B1	1	
(b)	$y = (x+1)\ln x$			
	$\frac{dy}{dx} = (x+1) \times \frac{1}{x} + \ln x$	M1 A1	2	product rule
(c)	$y = (x+1)\ln x$			
	$\frac{dy}{dx} = \frac{1}{x} + 1 + \ln x$			
	$x = 1$: $\frac{dy}{dx} = 1 + 1 = 2$	M1		substitute $x = 1$ into their $\frac{dy}{dx}$
	Grad normal $= -\frac{1}{2}$	M1 A1		use of $m_1 m_2 = -1$ CSO
	$y = -\frac{1}{2}(x-1)$	A1	4	OE
	Total		7	

Question 2(b)

2 (a) Differentiate $(x-1)^4$ with respect to x. *(1 mark)*

(b) The diagram shows the curve with equation $y = 2\sqrt{(x-1)^3}$ for $x \geqslant 1$.



The shaded region R is bounded by the curve $y = 2\sqrt{(x-1)^3}$, the lines $x = 2$ and $x = 4$, and the x-axis.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x-axis. *(4 marks)*

(c) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sqrt{x^3}$ onto the graph of $y = 2\sqrt{(x-1)^3}$. *(4 marks)*

Student response

(b) $y = 2\sqrt{(x-1)^3}$ (for $x \geqslant 1$)

$= \int_a^b \pi y^2 \, dx$ M1

$= \int_2^4 \pi \left(2\sqrt{(x-1)^3}\right)^2 dx$.

$= \pi \int_2^4 \left(2\sqrt{x-1)^3}\right)^2 dx$.

$\pi \int_2^4 \cancel{\left(2\sqrt{(x-1)^3}\right)}\cancel{\left(2\sqrt{(x-1)^3}\right)} dx$.

$\pi \int_2^4 \left(2((x-1)^3)^{1/2}\right)^2 dx$.

$a^3 \times a^4 = a^7$

$\pi \int_2^4 \left(2(x-1)^{3\frac{1}{2}}\right)^2$

$= \pi \int_2^4 4(x-1)^{5\frac{1}{2}} dx$.

$= \pi \left[\frac{4(x-1)^{6\frac{1}{2}}}{6\frac{1}{2}}\right]_2^4$ M0

$= \left(\frac{4(4-1)^{6.5}}{6.5}\right) - \left(\frac{4(2-1)^{6.5}}{6.5}\right)$

$= \pi[(777.0246) - (0.6153)]$

$= \pi(776.409)$ M0

$= 2439.161487$ A0

$= 2439.16$ (2.d.p)

$= 2440$ (3.s.f).

Commentary

Many candidates had forgotten from C2 how to handle indices.

They had learnt the new work on volume of rotations but fell by the wayside due to the indices.

The question was simple as they had to square a square root but many scripts dealt with the powers as this script obtaining powers to the ½ or similar. This meant that further correct progress on the question was difficult.

Centres should ensure that candidates revise working with indices as this is a pre-requisite for volumes of rotation.

Mark Scheme

2(a)	$4(x-1)^3$ or in expanded form	B1	1	allow $-4(1-x)^3$
(b)	$V = 4\,(\pi)\int_2^4 (x-1)^3\,dx$	M1		$(\pi)\int y^2\,dx$
	$= \cancel{4}\pi\left[\dfrac{(x-1)^4}{\cancel{4}}\right]_2^4$	M1 m1		$k(x-1)^4\ (\pi)$ or in expanded form correct substitution of limits into $k(x-1)^4$
	$= \pi(81-1) = 80\pi$	A1	4	CAO
(c)	Translate	E1		
	$\begin{pmatrix}1\\0\end{pmatrix}$	B1		OE
	Stretch (I) SF 2 (II) // y axis (III)	M1 A1	4	for I and (II or III) for I and II and III
	Total		9	

Question 3(a)

3 (a) Solve the equation $\operatorname{cosec} x = 2$, giving all values of x in the interval $0° < x < 360°$.
(2 marks)

Student Response

Leave blank

3.a. $\operatorname{cosec} x = 2$ $\quad \operatorname{cosec} x = \frac{1}{\sin x}$

$\frac{1}{\sin x} = 2$

$\sin^{-1} \frac{1}{2} = x$

$\therefore$

$x = 30°$ or $\frac{\pi}{6}$

2nd = 180 − 1st = 180° − 30° = 150°.

3rd. 1st + 360 = 390° Outside range

Solutions → $x = 30°$ and $x = 150°$

2

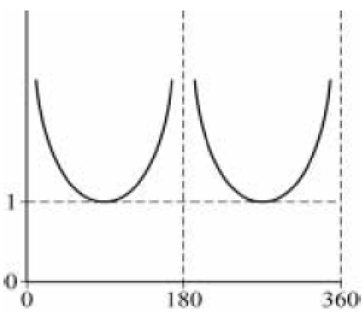
1

Commentary

Candidates will not be penalised on trigonometry questions if they give extra answers that are OUTSIDE of the given range.

However for extra answers within the range will be penalised.

Mark Scheme

3(a)	$\operatorname{cosec} x = 2$			
	$\Rightarrow \sin x = \frac{1}{2}$	M1		30° scores M1 implied
	$x = 30, 150$	A1	2	and no extras in range
(b)(i)	1	B1	1	
(ii)		M1		all positive, 2 U shapes
		A1	2	minima consistent > 0, **not** intersecting with each other or y-axis
(c)	$x = 30, 150, 210, 330$	B1F		3 correct values from their (a), which must be $\theta, 180 - \theta$
		B1	2	all correct and no extras in range
	Total		7	

Question 4b(ii)

(b) The curve $y = 3^x$ intersects the line $y = x + 3$ at the point where $x = \alpha$.

(i) Show that α lies between 0.5 and 1.5. *(2 marks)*

(ii) Show that the equation $3^x = x + 3$ can be rearranged into the form

$$x = \frac{\ln(x+3)}{\ln 3}$$ *(2 marks)*

Student Response

Leave blank

b) $y = 3^x$ intersects $y = x + 3$, $x = \alpha$

~~$x = 0.5, y = 3^{0.5} = 1.73$~~
~~$x = 1.5, y = 3^{1.5} = 5.196$~~

$x = 0.5, y = \sqrt[0.5]{0.5+3} - 3$
$= 9.25$ ✓

$x = 1.5$, $y = \sqrt[1.5]{1.5+3} - 3$ ✓
$= -0.2743$

$3^x = x + 3$
$3 = \sqrt[x]{x+3}$
$0 = \sqrt[x]{x+3} - 3$ ✓

// Change of sign

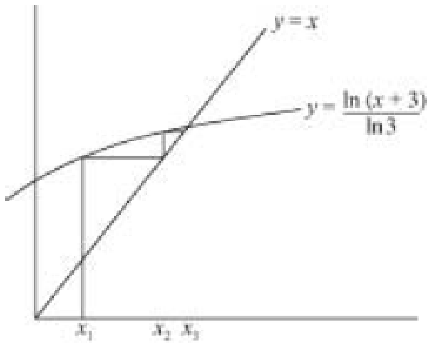
$\therefore \alpha$ lies between $x = 0.5$ and $x = 1.5$
$1.5 < \alpha < 0.5$

2

Commentary

In a number of questions on this paper candidates were required to show that a given answer was true. This was so that candidates could proceed with later parts of the question, BUT 2 enough steps need to be shown as to justify the result.
Candidates were able to arrive at the given answer with many shortcuts and were, in consequence, penalised.
This script has a missing line og log manipulation that was needed for full marks.

Mark Scheme

4(a)	y			
	x_0 1 3	B1		x values PI
	x_1 1.25 3.948(2)	B1		(4 +) y values correct
	x_2 1.5 5.196(2)			
	x_3 1.75 6.838(5)			
	x_4 2 9			
	$A=\frac{1}{3}\times\frac{1}{4}(3+4\times 3.9482+2\times 5.1962$ $+4\times 6.8385+9)$	M1		Simpson's rule
	$= 5.46$	A1	4	CAO
(b)(i)	$\mathrm{f}(x)=3^x-x-3$			
	$\left.\begin{array}{l}\mathrm{f}(0.5)=-1.77\\ \mathrm{f}(1.5)=0.696\end{array}\right\}$ change of sign $\therefore$ root	M1A1	2	
(ii)	$3^x=x+3$			
	$\ln 3^x=\ln(x+3)$	M1		correct use of logs
	$x\ln 3=\ln(x+3)$			
	$x=\frac{\ln(x+3)}{\ln 3}$	A1	2	correct with no mistakes; AG
(iii)	$x_1=0.5$			
	$(x_2=1.14)$	M1		
	$x_3=1.29=1.3$	A1	2	CAO
(iv)		M1		staircase
		A1	2	x_2, x_3 correct and labelled on x-axis
	Total		**12**	

Question 5b(ii)

(b) (i) Find $fg(x)$. *(1 mark)*

Student Response

ii. $fg(x) = 1$

$\sqrt{\frac{1}{x} - 2} = 1$ M1 attempt squaring

$\frac{1}{x} - 2 = \pm 1$ A0

$\frac{1}{x} = 3$, $\frac{1}{x} = 1$ A0

$x = \frac{1}{3}$ or $x = 1$ extra solutions

Commentary

A surprising number of candidates made this mistake when squaring the equation. They had obviously been drilled that a square root produces 2 answers and applied the same principle to squaring.
This leads to 2 solutions; the correct one and one extra spurious solution, hence this candidate gained the method mark for squaring but lost both accuracy marks.

Mark Scheme

5(a)	$f(x) \geq 0$	allow $y \geq 0$	M1 A1	2	> 0 or $f \geq 0$ or ≥ 0
(b)(i)	$\sqrt{\frac{1}{x} - 2}$		B1	1	
(ii)	$\frac{1}{x} - 2 = 1$		M1		squaring their (b)(i) in an equation
	$\frac{1}{x} = 3$	OE	A1		
	$x = \frac{1}{3}$		A1	3	CSO
(c)	$y = \sqrt{x - 2}$				
	$y^2 = x - 2$		M1		attempt to isolate; condone 1 slip
	$x^2 = y - 2$		M1		reverse $x \Leftrightarrow y$
	$y = x^2 + 2$		A1	3	
		Total		9	

Question 6

6 (a) Use integration by parts to find $\int xe^{5x}\,dx$. *(4 marks)*

(b) (i) Use the substitution $u = \sqrt{x}$ to show that

$$\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx = \int \frac{2}{1+u}\,du$$ *(2 marks)*

(ii) Find the exact value of $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx$. *(3 marks)*

Student Response

Leave blank

6) a) $\int x e^{5x}$

$u = e^{5x}$ $\qquad \frac{dv}{dx} = x$

M0

$\frac{du}{dx} = 5e^{5x}$ $\qquad$ A0 $v = 1$

$\frac{dy}{dx} = uv - \int v \frac{du}{dx}\,dx$

$\frac{dy}{dx} = e^{5x} \times (1) - \int (1) \times 5e^{5x}\,dx$

$\frac{dy}{dx} = e^{5x} - \int 5e^{5x}\,dx$

$\frac{dy}{dx} = e^{5x} - 5(\frac{1}{5})e^{5x} + c$ ~~dx~~

$5 \times (\frac{1}{5}) = 1$

$\frac{dy}{dx} = e^{5x} - e^{5x} + c$

C

Commentary

Calculus is the main topic on this module and all basic differentiating and integrating techniques must be understood.
When using integration by parts candidates must realise that at the outset they MUST integrate one term AND differentiate the other.
Some candidates chose the wrong term to integrate BUT they still scored the method mark. Obviously accuracy marks would be unlikely to be obtained as they were going down a dead-end, but they would have had the opportunity to recover without penalty.

Mark Scheme

6(a)	$\int x e^{5x} dx$			
	$u = x \quad dv = e^{5x}$	M1		integrate one term, differentiate one term
	$du = 1 \quad v = \frac{1}{5}e^{5x}$	A1		
	$\int = \frac{1}{5}xe^{5x} - \int \frac{1}{5}e^{5x}dx$	A1		
	$= \frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} (+c)$	A1	4	
(b)(i)	$u = x^{\frac{1}{2}}$			
	$du = \frac{1}{2}x^{-\frac{1}{2}} dx$	M1		
	$\int = \int \frac{1}{1+u} \times 2du$	A1	2	correct with no errors; AG
(ii)	$\int_1^9 dx = \int_1^3 \frac{2}{1+u}du$	m1		correct limits used in correct expression, ignoring k
	$= \left[2\ln(1+u)\right]_1^3$	M1		for $k\ln(1+u)$
	$= 2\ln 4 - 2\ln 2$ $(= \ln 4)$	A1	3	ISW OE
	Total		**9**	

Question 7

7 (a) A curve has equation $y = (x^2 - 3)e^x$.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Find $\frac{d^2y}{dx^2}$. *(2 marks)*

(b) (i) Find the x-coordinate of each of the stationary points of the curve. *(4 marks)*

(ii) Using your answer to part (a)(ii), determine the nature of each of the stationary points. *(2 marks)*

Student Response

b i) at stat point $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = (x^2-3)e^x + (2x)e^x = 0$ M0

$= (x^2-3)\ln x + (2x)\ln x = 0$ 0

$x^2\ln x - 3\ln x + 2x\ln x = 0$ X

$x^2\ln x + 2x\ln x - 3\ln x = 0$

$(x \quad + 3)(\quad - 1)$ M0 A0

Leave blank

0

Commentary

Two of the main topics on this module are calculus and working with natural logs.

This question brought both topics in one question.

This script had the correct answer for the first derivative and knew that for turning points the gradient had to be zero.

Also the candidate knew that the exponential function had to be dealt with. Many candidates were unsure as to how to proceed and used logs without realising the implications.

This solution showed a lack of understanding of questions involving natural logs and their inverses.

Mark Scheme

7(a)(i)	$y=\left(x^2-3\right)e^x$			
	$\frac{dy}{dx}=\left(x^2-3\right)e^x+2xe^x$	M1 A1	 2	product rule
(ii)	$\frac{d^2y}{dx^2}=\left(x^2-3\right)e^x+2xe^x+2xe^x+2e^x$	M1 A1	 2	product rule from their $\frac{dy}{dx}$
(b)(i)	$\frac{dy}{dx}=0$			
	$\Rightarrow e^x\left(x^2+2x-3\right)=0$	M1		$e^x f(x)=0$ from $\frac{dy}{dx}=0$
	$e^x(x+3)(x-1)=0$	m1		attempt at factorising or use of formula
	$\therefore x=-3, 1$	A1 A1	 4	first correct solution second correct solution, and no others SC No working shown: $x=-3$ B2, $x=1$ B2
(ii)	$x=-3$ $y''=-4e^x$ max (-0.2)	M1		Condone slip
	$x=1$ $y''=4e^x$ min (10.9)	A1	2	
	Total		**10**	

Question 8

8 (a) Write down $\int \sec^2 x \, dx$. *(1 mark)*

(b) Given that $y = \dfrac{\cos x}{\sin x}$, use the quotient rule to show that $\dfrac{dy}{dx} = -\operatorname{cosec}^2 x$. *(4 marks)*

(c) Prove the identity $(\tan x + \cot x)^2 = \sec^2 x + \operatorname{cosec}^2 x$. *(3 marks)*

(d) Hence find $\int_{0.5}^{1} (\tan x + \cot x)^2 \, dx$, giving your answer to two significant figures. *(4 marks)*

Student Response

b) $y = \dfrac{\cos x}{\sin x}$ $\begin{matrix} u \\ v \end{matrix}$ $u = \cos x$ $v = \sin x$

$u' = -\sin x$ $v' = \cos x$

$\dfrac{dy}{dx} = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \cot^2 x$ M1

A1

$= -1 - \cot^2 x$

$\cancel{= 1 + \cot^2 x}$ $s + c = \frac{1}{s}$ cosec

$\cot^2 x = -1$ A0 incorrect use of trig identities

$1 + \cot^2 x = -\operatorname{cosec}^2 x$ X AG A0

2

Commentary

In a number of questions on this paper candidates were required to show that a given answer was true. This was so that candidates could proceed with later parts of the question, BUT 2 wrongs do not make a right!

Candidates were able to arrive at the given answer by spurious methods!

This script starts the question correctly with the quotient rule but then, by not knowing the trig identities correctly, falls by the wayside with some very poor algebra.

Centres must stress to candidates that it is highly likely that one of the 2 trig identities stated in the specification for this module will be tested.

Mark Scheme

8(a)	$\tan x \ (+c)$	B1	1	
(b)	$f(x)=\dfrac{\cos x}{\sin x}$			
	$f'(x)=\dfrac{-\sin^2 x-\cos^2 x}{\sin^2 x}$	M1 A1		quotient rule $\dfrac{\pm\sin^2 x\pm\cos^2 x}{\sin^2 x}$
	$=\dfrac{-1}{\sin^2 x}$	A1		use of $\sin^2 x+\cos^2 x=1$
	$=-\text{cosec}^2 x$	A1	4	AG CSO **Special cases** $f(x)=\dfrac{\cot x}{1}$ $f'(x)=\dfrac{1\times-\text{cosec}^2 x-\cot x\times 0}{1^2}$ M1 $=-\text{cosec}^2 x$ A1 (max 2/4) **Or** $f(x)=\dfrac{1}{\tan x}$ $f'(x)=\dfrac{\tan x\times 0-1\times\sec^2 x}{\tan^2 x}$ M1 A1 $=\dfrac{-\sec^2 x}{\tan^2 x}$ $=\dfrac{-1}{\sin^2 x}=-\text{cosec}^2$ A1 (max 3/4)
(c)	LHS $=\tan^2 x+\cot^2 x+2\tan x\cot x$	M1		expanding
	$=\tan^2 x+1+\cot^2 x+1$	M1		correct use of trig identities
	$=\sec^2 x+\text{cosec}^2 x$ $=$ RHS	A1	3	CSO
(d)	$\int(\tan x+\cot x)^2\,dx=\int\sec^2 x+\text{cosec}^2 x\,dx$	M1		use of identity
	$=[\tan x-\cot x]_{0.5}^{1}$	M1 A1		$\pm\tan x\pm\cot x$ OE
	$=0.9153--1.2842$			
	$=2.2$	A1	4	AWRT
	Total		**12**	