

Teacher Support Materials Maths GCE

Paper Reference MS03

1 As part of an investigation into the starting salaries of graduates in a European country, the following information was collected.

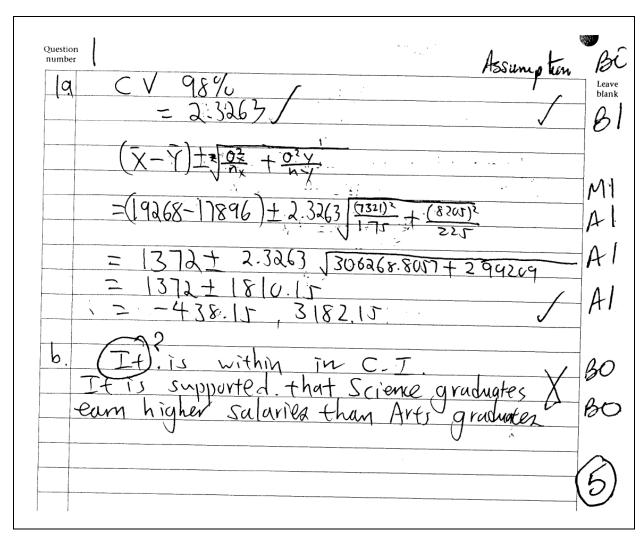
		Starting salary (€)			
	Sample size	Sample mean Sample standard devi			
Science graduates	175	19 268	7321		
Arts graduates	225	17 896	8205		

- (a) Stating a necessary assumption about the samples, construct a 98% confidence interval for the difference between the mean starting salary of science graduates and that of arts graduates.

 (6 marks)
- (b) What can be concluded from your confidence interval?

(2 marks)

Student Response



The candidate clearly knew the method required to determine the confidence interval correctly. However the candidate appeared unaware of the necessary assumption and also that when a (98%) confidence interval includes the value zero then it can concluded that there is no evidence (at the 2% significance level) of a difference between the two population means.

Candidates must be aware of assumptions and be able to make deductions.

Q	Solution	Marks	Total	Comments
1(a)	Samples are independent or random	B1		
	98% $\Rightarrow z = 2.3263$	В1		AWFW 2.32 to 2.33
	CI for $\mu_1 - \mu_2$ is:			
	$(\overline{x}_S - \overline{x}_A) \pm z \times \sqrt{\frac{s_S^2}{n_S} + \frac{s_A}{n_A}}$	M1		Form Allow: sigmas, A&B or 1&2 and n -
	$(n_s - n_A) = 2 - \sqrt{n_s - n_A}$	A1		Correct
	(19268-17896)			
	$\pm 2.3263 \times \sqrt{\frac{7321^2}{175} + \frac{8205^2}{225}}$	A1√		$\int \text{ on } z \text{ only}$ $s_p = 7830 \text{ to } 7850$
	ie 1372 ± (1805 to 1820)			1372 ± (1830 to 1845)
	or (-450 to -430, 3170 to 3200)	A1	6	AWFW
	(100 10 10 10 10 10 10 10 10 10 10 10 10			
(b)	Confidence interval includes zero so (at 5% level)	B1√` ↑dep↑		✓ on CI; OE
	Mean starting salaries may be equal	B1√	2	√ on CI; OE

2 A hill-top monument can be visited by one of three routes: road, funicular railway or cable car. The percentages of visitors using these routes are 25, 35 and 40 respectively.

The age distribution, in percentages, of visitors using **each** route is shown in the table. For example, 15 per cent of visitors using the road were under 18.

		Percentage of visitors using				
		Road	Funicular railway	Cable car		
	Under 18	15	25	10		
Age (years)	18 to 64	80	60	55		
	Over 64	5	15	35		

Calculate the probability that a randomly selected visitor:

(a) who used the road is aged 18 or over;

(1 mark)

(b) is aged between 18 and 64;

(3 marks)

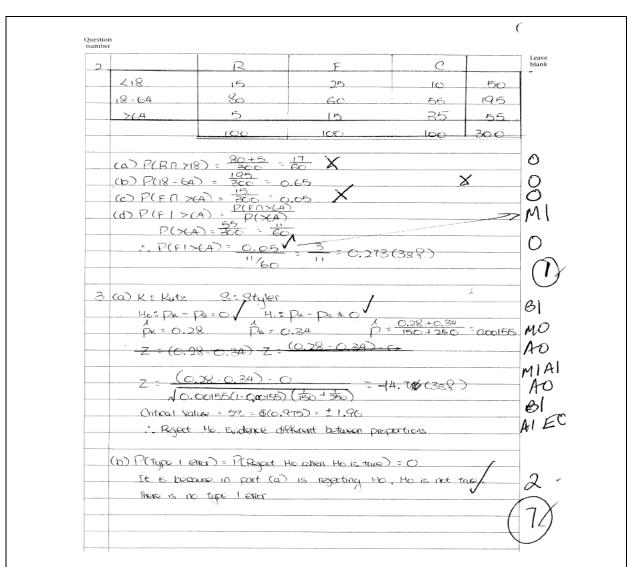
(c) used the funicular railway and is aged over 64;

(2 marks)

(d) used the funicular railway, given that the visitor is aged over 64.

(5 marks)

Student response



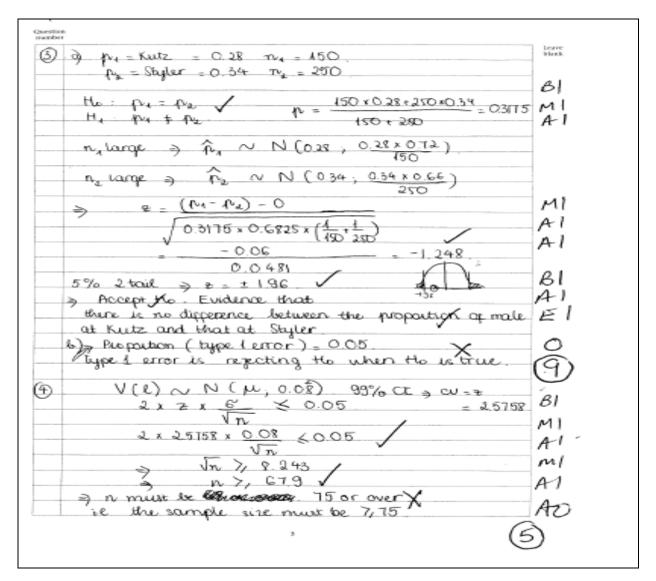
The candidate appeared not to have read the question carefully and so missed the aid in explanation contained in the second paragraph. As a result, the candidate has just treated the given information as a simple 2-way frequency table as would be examined in MS/SS1B.

Candidates must read questions carefully.

Q	Solution	Marks	Total	Comments
2(a)	$P(\ge 18 \mid Road) = 0.85$	B1	1	CAO; OE; not 85
(b)	P(18 to 64) = P(Route) × P(18 to 64 Route) =	M1		Use of 3 possibilities, each the product of 2 probabilities
	$(0.25 \times 0.80) + (0.60 \times 0.35) + (0.55 \times 0.40)$	A1		At least 1 term correct
	= 0.20 + 0.21 + 0.22 = 0.63	A1	3	CAO; OE
(c)	$P(FR \cap >64) = P(FR) \times P(>64 FR)$			
	= 0.35 × 0.15	B1		Correct expression
	= 0.052 to 0.053	B1	2	AWFW (0.0525)
(d)	$P(FR \mid >64) = \frac{(c)}{P(>64)} =$	M1 M1		$\frac{\text{answer(c)}}{\sum (3\times 2) \text{ probabilities}}$
	$\frac{0.0525}{(0.25\times0.05)+(0.35\times0.15)+(0.40\times0.35)}$	A1		At least 2 terms correct
	$= \frac{0.0525}{0.0125 + 0.0525 + 0.1400} = \frac{0.0525}{0.205}$	A1		CAO
	$= 0.256 \text{ or } \frac{21}{82}$	A1	5	AWRT/CAO; OE
	Total		11	

- 3 Kutz and Styler are two unisex hair salons. An analysis of a random sample of 150 customers at Kutz shows that 28 per cent are male. An analysis of an independent random sample of 250 customers at Styler shows that 34 per cent are male.
 - (a) Test, at the 5% level of significance, the hypothesis that there is no difference between the proportion of male customers at Kutz and that at Styler. (9 marks)
 - (b) State, with a reason, the probability of making a Type I error in the test in part (a) if, in fact, the actual difference between the two proportions is 0.05. (2 marks)

Student Response



The candidate has scored full marks in part (a) but the conclusion in context is very close to being too definitive. In part (b), the candidate has simply stated the definition of a Type I error and not applied it to the given context where H_0 was false.

Conclusions to hypothesis tests must not be definitive but should be qualified (some/strong evidence) or quantified (by the significance level) and quoted definitions without reference to the context rarely score marks.

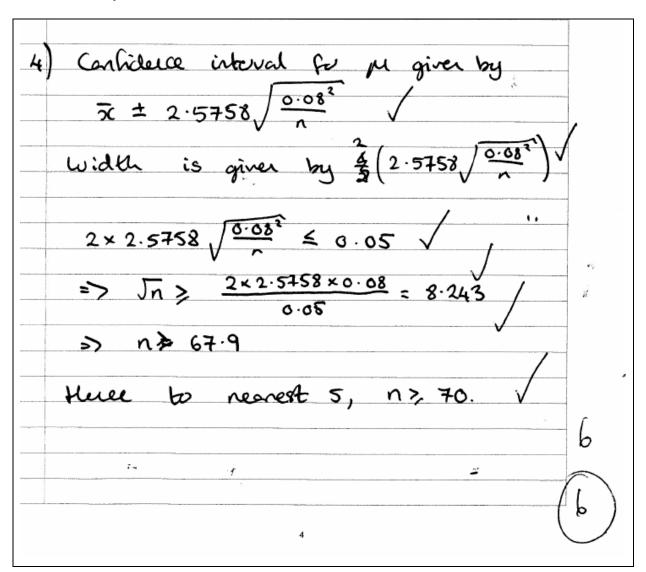
Q	Solution	Marks	Total	Comments
3(a)	$\begin{aligned} \mathbf{H}_0: \ p_{\mathbf{K}} &= \ p_{\mathbf{S}} \\ \mathbf{H}_1: \ p_{\mathbf{K}} &\neq \ p_{\mathbf{S}} \end{aligned}$	B1		Both; OE; allow A&B or 1&2
	$\begin{array}{c c} SL & \alpha = 0.05 \\ CV & z = 1.96 \end{array}$	B1		CAO
	$\hat{p} = \frac{(150 \times 0.28) + (250 \times 0.34)}{400}$	M1		Used
	$= \frac{127}{400} \text{ or } 0.317 \text{ to } 0.318$	A1		CAO/AWFW (0.3175)
	$z = \frac{(\hat{p}_{K} - \hat{p}_{S}) - 0}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_{K}} + \frac{1}{n_{S}}\right)}}$	M1		Used; accept unpooled denominator
	$ z = \frac{ 0.28 - 0.34 }{\sqrt{0.3175 \times 0.6825 \left(\frac{1}{150} + \frac{1}{250}\right)}}$	Al√		$ \checkmark $ on \hat{p} ; accept no pooling
	= 1.24 to 1.25	A1		AWFW; 1.26 to 1.27
	Thus accept H_0 as $ z < 1.96$	Al√		\checkmark on z and CV with same sign
	Thus no evidence, at 5% level, of a difference between two proportions of male customers in two salons	E1√	9	\nearrow on z and CV with same sign In context and qualified
(b)	Zero since	В1		CAO
	Cannot make a Type I error when H_0 is false	В1	2	OE
	Total		11	

4 A machine is used to fill 5-litre plastic containers with vinegar. The volume, in litres, of vinegar in a container filled by the machine may be assumed to be normally distributed with mean μ and standard deviation 0.08.

A quality control inspector requires a 99% confidence interval for μ to be constructed such that it has a width of at most 0.05 litres.

Calculate, to the nearest 5, the sample size necessary in order to achieve the inspector's requirement. (6 marks)

Student Response



Commentary

The candidate has used knowledge of the form of a confidence interval to deduce an expression for the width (including multiplier of 2) and then solved the resulting inequality (equality would have sufficed) for n.

Those candidates who tried to remember the formula for n sometimes made errors.

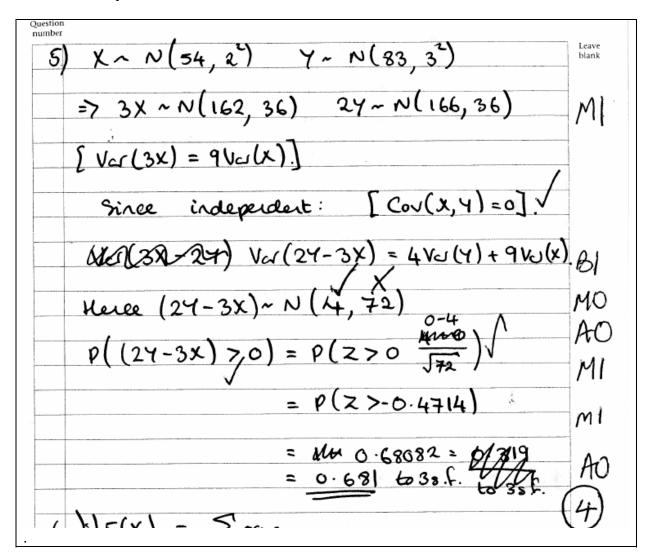
03 (cont)			ı	
Q	Solution	Marks	Total	Comments
4	$98\% \implies z = 2.5758$	B1		AWFW 2.57 to 2.58
	CI width is $2 \times \frac{z\sigma}{\sqrt{n}}$	M1		Used; allow $\frac{z\sigma}{\sqrt{n}}$
	Thus $2 \times \frac{2.5758 \times 0.08}{\sqrt{n}} = 0.05$	A1√		OE; ✓ on z; allow no '2 ×'
	Thus $\sqrt{n} = 8.24256$	m1		Solving for \sqrt{n} or n
	Thus $n = 67.9 \implies 68$	A1√		AWRT; $$ on z
	Thus, to nearest 5, $n = 70$	A1	6	CAO
	Total		6	

5 The duration, X minutes, of a timetabled 1-hour lesson may be assumed to be normally distributed with mean 54 and standard deviation 2.

The duration, Y minutes, of a timetabled $1\frac{1}{2}$ -hour lesson may be assumed to be normally distributed with mean 83 and standard deviation 3.

Assuming the durations of lessons to be independent, determine the probability that the total duration of a random sample of three 1-hour lessons is less than the total duration of a random sample of two $1\frac{1}{2}$ -hour lessons. (7 marks)

Student Response



Commentary

The candidate has used an expression involving 3X and 2Y rather than $\sum_{1}^{3} X$ and $\sum_{1}^{2} Y$. As the variance of the difference is then 72 rather than 30 (mean is the same), the final answer is incorrect.

Candidates must be aware of the difference is use between nX and $\sum_{i=1}^{n} X$.

5	$D = \sum_{i=1}^{3} X_i - \sum_{i=1}^{2} Y_i$ or $D' = \sum_{i=1}^{2} Y_i - \sum_{i=1}^{3} X_i$	M1		Used or implied
	have means $\mu = 162 - 166 = -4$ $\mu = 166 - 162 = +4$	В1		CAO either
	and variance $\sigma^2 = (3 \times 2^2) + (2 \times 3^2) = 12 + 18$ = 30	M1 A1		Use of $[a \times Var(Z)]$; implied CAO
	$P\left(\sum_{i=1}^{3} X_{i} < \sum_{i=1}^{2} Y_{i}\right) =$ $P(D < 0) \text{ or } P(D' > 0) =$	M1		Used or implied
	$P\left(Z > \frac{0 - (-4)}{\sqrt{30}}\right) \text{ or } P\left(Z > \frac{0 - (+4)}{\sqrt{30}}\right) =$	m1		Standardising 0 using μ and $\sqrt{\sigma^2}$
	P(Z < +0.73) or $P(Z > -0.73) =$			
	0.767 to 0.768 Total	A1	7 7	AWFW

Question 6 (a)(i) & (b)

- **6** (a) The random variable X has a binomial distribution with parameters n and p.
 - (i) Prove that E(X) = np.

(4 marks)

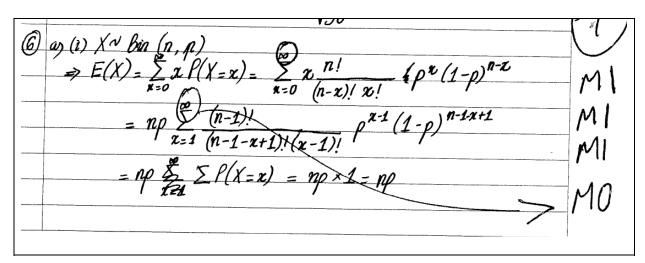
(ii) Given that $E(X^2) - E(X) = n(n-1)p^2$, show that Var(X) = np(1-p).

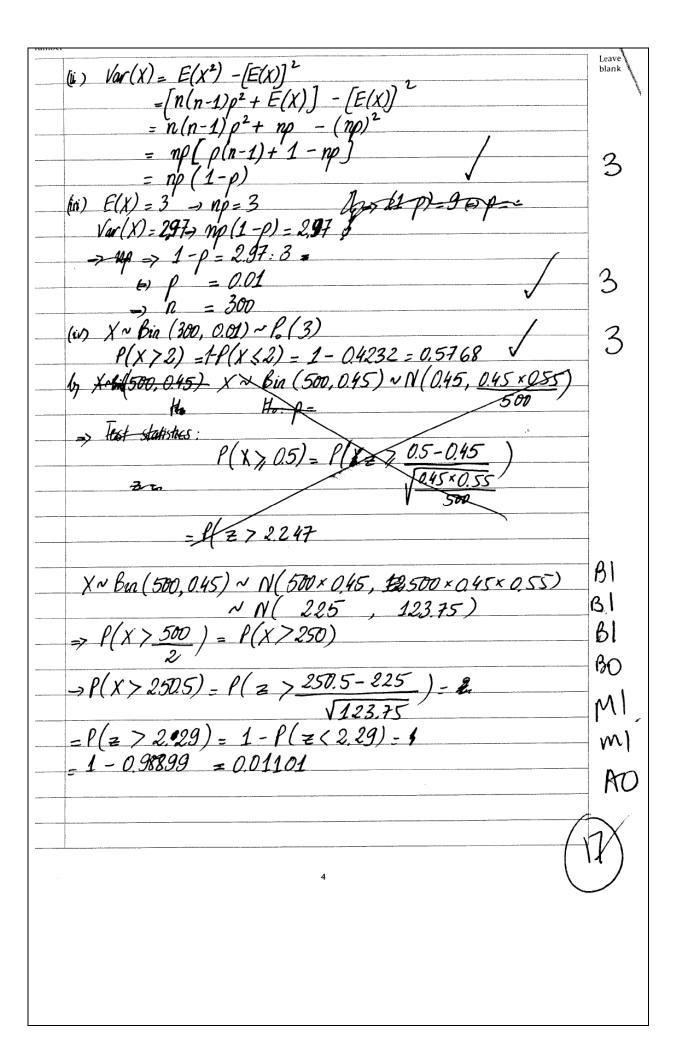
(3 marks

- (iii) Given that X is found to have a mean of 3 and a variance of 2.97, find values for n and p. (3 marks)
- (iv) Hence use a distributional approximation to estimate P(X > 2). (3 marks)
- (b) Dressher is a nationwide chain of stores selling women's clothes. It claims that the probability that a customer who buys clothes from its stores uses a Dressher store card is 0.45.

Assuming this claim to be correct, use a distributional approximation to estimate the probability that, in a random sample of 500 customers who buy clothes from Dressher stores, at least half of them use a Dressher store card. (7 marks)

Student Response (contd on next page)





In part (a)(i), the candidate clearly knows the principles of a method for proving that E(X) = np but has not produced a fully convincing derivation. In part (b), the use of an incorrect continuity correction has produced an inaccurate final answer. Otherwise a correct solution to the question.

Proofs require a derivation that is fully convincing and care must be taken in applying correctly continuity corrections.

Mark Scheme

Q	Solution	Marks	Total	Comments
6(a)(i)	$E(X) = \sum_{x=0}^{n} x \times {n \choose x} p^{x} (1-p)^{n-x}$	M1		Use of $\sum x \times P(X = x)$
	$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^{x} (1-p)^{n-x}$	M1		Expansion of ${}^{n}C_{x}$; cancelling of x (Ignore limits)
	$= np \times \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$	M1		Factors of <i>n</i> and <i>p</i> (Ignore limits)
	$= np \times \sum P(X = x) B(n-1, p) = np$	M1	4	AG; must be convincing

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)			13	
(b)	$Y \sim B(500, 0.45)$			
	or	-		-
	$Y \sim \text{(normal)}$ with mean $\mu = 225$	B1		PI
	and			
	variance $\sigma^2 = 123.75$			AWFW 123 to 124
	or standard deviation $\sigma = 11.124$	B1		AWFW 11.05 to 11.15
	standard deviation 0 – 11.124			AWFW 11.03 to 11.13
	(At least) half \Rightarrow (\geq) 250	В1		CAO
	$P(Y_B \ge 250) = P(Y_N > 249.5) =$	B1		CAO
	$r(I_B \ge 250) - r(I_N > 249.5) -$	B1		CAO
	$P\left(Z > \frac{249.5 - 225}{\sqrt{123.75}}\right) =$	M1		Standardising 249.5, 250 or 250.5 with
	$\sqrt{123.75}$	1411		c's μ and $\sqrt{\sigma^2}$
	P(Z > 2.20) = 1 - P(Z < 2.20)	m1		Area change
		1111		7 Hea change
	= 0.0138 to 0.014	A1	7	
	Note:			
	Use of $\frac{0.5-0.45}{\sqrt{0.000495}} \Rightarrow \text{max of 5 marks}$			Use of distribution of \hat{p}
				Ose of distribution of p
	Use of $\frac{0.499-0.45}{\sqrt{0.000495}} \Rightarrow \text{max of } 7 \text{ marks}$			Use of distribution of \hat{p}
	¥ 0.000+33		•••	with continuity correction
	Total		20	

7 In a town, the total number, R, of houses sold during a week by estate agents may be modelled by a Poisson distribution with a mean of 13.

A new housing development is completed in the town. During the first week in which houses on this development are offered for sale by the developer, the estate agents sell a total of 10 houses.

- (a) Using the 10% level of significance, investigate whether the offer for sale of houses by the developer has resulted in a reduction in the mean value of R. (6 marks)
- (b) Determine, for your test in part (a), the critical region for *R*. (2 marks)
- (c) Assuming that the offer for sale of houses on the new housing development has reduced the mean value of R to 6.5, determine, for a test at the 10% level of significance, the probability of a Type II error. (4 marks)

Student Response

7 (a) Ho: N= B /, RUB(B)	Bi
H.: X < B.	Bl
the obsertue value of the sell after the new	
development 5 (0 house for the first week.	MO
7.5. 3 = 10-13 = -0.832. Normal X	AO
For 10% 1 tail test, w = -1.2816.	A 1
Accept to foundance that those is	MI
-1286 us of reduction in the man falue	Al
configion for R in part (as to the	
The transfer of judging whother we accept to	
be reject to, evidence that there is a reduction	
of the mean Also, cot significant land also	
represent that the probability of making type I	0
recor is o.l.	•;
	åı
e). Type I error is beginned to whome it is the	BI
PERPORTED PLACEPH HO 1265)	MI
2 = obs - 13 = -1,2816 (10% text)	1,11
obs = 8.38. if obs > 8.38, we accept the	ml
P(Accept 40) = P(X>8.88 R=6.5).	
= P(5 > 8.39 - 6.2)	AO
= p(2 > 0.737) = 1 - p(2 < 0.74)	_
=1-0.77035-6.230	7
6	1)

. The candidate has used the normal approximation for Po(13) in part (a) and for Po(6.5) in part (c). In neither case can the mean be reasonably considered as large, particularly as cumulative probabilities for both are given in Table 2 of the supplied booklet. The answer to part (b) does not give a critical region for R although the value 8.38 used in part (c) does suggest a value but again using the normal approximation.

For binomial and Poisson distributions, candidates should not opt for approximations when exact tabled values are available in the supplied booklet unless advised so to do in the question.

Candidates cannot in general expect to have marks awarded in a part of a question for an implied answer in a later part of the question.

Q	Solution	Marks	Total	Comments	
7(a)	$H_0: \lambda = 13$	B1		CAO; OE	
	$H_1: \lambda < 13$	B1		CAO; OE	
	$P(R \le 10 \mid Po(13))$	M1		Used or implied	
	= 0.2517	A1		AWFW 0.251 to 0.252	
	Prob of 0.2517 > 0.10 (10%)	M1		Comparison of prob with 0.10	
	z = -0.83 to -0.70 > -1.28	IVII		Comparison of z with -1.28	
	Thus no evidence, at 10% level, of a	A1√	6	\checkmark on probability or z	
	reduction in the mean value of R	AI√	0	In 'context' and qualified	
(b)	Require $P(R \le r \mid Po(13)) \approx 0.10$	M1		Stated or implied	
	Critical Region is $R \le 8$ or $R < 9$	A1	2	Accept $R = 8$	
			_	May be scored in (a)	
(c)	Require P(accept H ₀ H ₀ false)	В1		OE; PI	
	$= P(R > 8 \mid Po(6.5))$	M1		Use of Po(6.5)	
	$= 1 - P(R \le 8 \mid Po(6.5))$	m1			
	= 1 - 0.7916				
	= 0.208 to 0.209	A1	4	AWFW	(0.2084
	Total		12		
	TOTAL		75		