Teacher Support Materials 2008

Maths GCE

Paper Reference MS2B

Question 1

It is thought that the incidence of asthma in children is associated with the volume of traffic in the area where they live.

Two surveys of children were conducted: one in an area where the volume of traffic was heavy and the other in an area where the volume of traffic was light.

For each area, the table shows the number of children in the survey who had asthma and the number who did not have asthma.

	Asthma	**No asthma**	**Total**
Heavy Traffic	52	58	110
Light Traffic	28	62	90
Total	80	120	200

(a) Use a χ^2 test, at the 5% level of significance, to determine whether the incidence of asthma in children is associated with the volume of traffic in the area where they live. (8 marks)

(b) Comment on the number of children in the survey who had asthma, given that they lived in an area where the volume of traffic was heavy. (1 mark)

Student Response

1. a.

O	E	$\frac{(\lvert O-E\rvert-0.5)^2}{E}$
52	44	1.278
28	36	1.563
58	66	0.852
62	54	1.042

Hypoth

4.735 ← test statistic

$V = (2-1)(2-1) = 1$ C.V. = 3.841

3.841 < 4.732 so H_1 is rejected at 5 % lev of significance so evidence suggests volume of traffic doesn't incidence of asthma.

b. more children ~~who~~ had asthma, given that they lived in an area with heavy traffic than expected.

SO (dep)

Commentary

Hypotheses not stated in part (a).
Wrong conclusion ‘No association’ stated in part (a) but candidate still thought that they were justified in stating ‘more than expected had asthma’ in part (b).

Mark scheme

<table>
<tr><td>1(a)</td><td>
<table>
<tr><th>O_i</th><th>E_i</th><th>$\lvert O_i - E_i \rvert - 0.5$</th><th>$(7.5)^2/E_i$</th></tr>
<tr><td>52</td><td>44</td><td>7.5</td><td>1.2784</td></tr>
<tr><td>58</td><td>66</td><td>7.5</td><td>0.8523</td></tr>
<tr><td>28</td><td>36</td><td>7.5</td><td>1.5625</td></tr>
<tr><td>62</td><td>54</td><td>7.5</td><td>1.0417</td></tr>
<tr><td></td><td></td><td></td><td>4.7349</td></tr>
</table>
</td><td>M1
M1
M1
A1</td><td></td><td>E attempted
Yates’ correction attempted
χ^2 attempted
Awfw 4.73 to 4.74</td></tr>
<tr><td></td><td>H_0: No association between incidence of asthma and volume of traffic
H_1: Association</td><td>B1</td><td></td><td>(at least H_0 stated correctly)</td></tr>
<tr><td></td><td>$\nu = 1$
$\chi^2_{crit} = 3.841 < 4.7349$</td><td>B1</td><td></td><td>Critical value</td></tr>
<tr><td></td><td>Reject H_0 at 5% level</td><td>A1ft</td><td></td><td></td></tr>
<tr><td></td><td>Evidence to suggest an association between the incidence of asthma in children and the volume of traffic where they live.</td><td>E1ft</td><td>8</td><td></td></tr>
<tr><td>(b)</td><td>More than expected had asthma</td><td>E1</td><td>1</td><td>Dep. ‘association’ in conclusion to part (a)</td></tr>
<tr><td></td><td>Total</td><td></td><td>9</td><td></td></tr>
</table>

Question 2

(a) The number of telephone calls, X, received per hour for Dr Able may be modelled by a Poisson distribution with mean 6.

Determine $\mathrm{P}(X = 8)$. (2 marks)

(b) The number of telephone calls, Y, received per hour for Dr Bracken may be modelled
by a Poisson distribution with mean λ and standard deviation 3.

(i) Write down the value of λ. (1 mark)

(ii) Determine $\mathrm{P}(Y > \lambda)$. (2 marks)

(c) **(i)** Assuming that X and Y are independent Poisson variables, write down the distribution of the **total** number of telephone calls received per hour for Dr Able and Dr Bracken. (1 mark)

(ii) Determine the probability that a total of at most 20 telephone calls will be received during any one-hour period. (1 mark)

(iii) The total number of telephone calls received during each of 6 one-hour periods is to be recorded. Calculate the probability that a total of at least 21 telephone calls will be received during exactly 4 of these one-hour periods. (3 marks)

Student response

Q2 $X \sim P_o(6)$

A) $P(X=8)$ using $P(X=x) = e^{-\lambda} \times \frac{\lambda^x}{x!}$

$\Rightarrow P(X=8) = e^{-6} \times \frac{6^8}{8!}$

$= 0.1032577\ldots$

$= \underline{0.103}$ (3sf)

B) $Y \sim P_o(9)$

i) $\lambda = 3^2$

$\underline{\lambda = 9}$

ii) $P(Y > \lambda)$

$= P(Y > 9)$

$= 1 - P(Y \leq 9)$

$= 1 - 0.5874$ (from tables)

$= \underline{0.4126}$

C) i) $X + Y = T$

$\Rightarrow T \sim P_o(6+9)$

$\Rightarrow \underline{T \sim P_o(15)}$

2

1

2

1

1	ii)	$P(T \leq 20)$ where $\lambda = 15$)
		$= 0.9170$ (from tables)
	iii)	$P(T \geq 21)$
1		$= 1 - P(T \leq 20)$
		$= 1 - 0.9170$
		$= 0.083$
		$\Rightarrow P(T \geq 21)$ on four of these one hour periods
M0		$= 0.083^4 \times 0.917^2$
A0		$= 0.0000399071 8 \ldots$
8		$= 0.0000399$ (3sf)

Commentary

Didn't use B(6, p) to work out solution in part (c)(iii).
Many in this part also did not realise that $P(T \text{ at least } 21) = 1 - P(T \text{ at most } 20)$.
Candidate Brendan Chadwick 7879 (centre: 43421) gained full marks on this question.

Mark Scheme

2(a)	$P(X = 8) = P(X \leq 8) - P(X \leq 7)$ $= 0.8472 - 0.7440$ $= 0.103$	M1 A1	 2	$P(X = 8) = \dfrac{e^{-6} \times 6^8}{8!}$
(b)(i)	$\lambda = 9$	B1	1	
(ii)	$P(X > 9) = 1 - P(X \leq 9)$ $= 1 - 0.5874 = 0.4126$	M1 A1ft	 2	 Awfw 0.412 to 0.413
(c)(i)	$T \sim \mathrm{Po}(15)$	B1ft	1	
(ii)	$P(T \leq 20) = 0.917$	B1ft	1	
(iii)	$P(T \text{ at least } 21) = 0.083$ $p = 15 \times (0.083)^4 (0.917)^2$ $= 0.000599$	B1ft M1 A1	 3	 For B(6, (iii)) used (awfw 0.0005978 – 0.0006)
	Total		**10**	

Question 3

Alan's company produces packets of crisps. The standard deviation of the weight of a packet of crisps is known to be 2.5 .

Alan believes that, due to the extra demand on the production line at a busy time of year, the mean weight of packets of crisps is not equal to the target weight of 34.5grams.

In an experiment set up to investigate Alan's belief, the weights of a random sample of 50 packets of crisps were recorded. The mean weight of this sample is 35.1 grams.

Investigate Alan's belief, at the 5% level of significance. (6 marks)

Student Response

Leave blank

3 $H_0 \quad \mu = 34.5$

$H_1 \quad \mu \neq 34.5$ 2tailed test B1

$\mu = 34.5$

$\bar{x} = 35.1$

$n = 50$

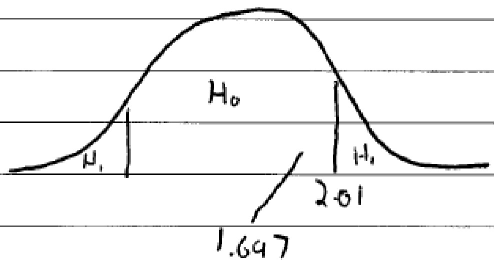
$\sigma = 2.5$

B0

At 97.5% $\quad v = 49 \quad t \approx 2.01$

Test statistic $= \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = 1.6971$ M1

A1



(t - used) A0

E0

Ho Accepted - Insuffsicient evidence to show that $\mu \neq 34.5$ 3

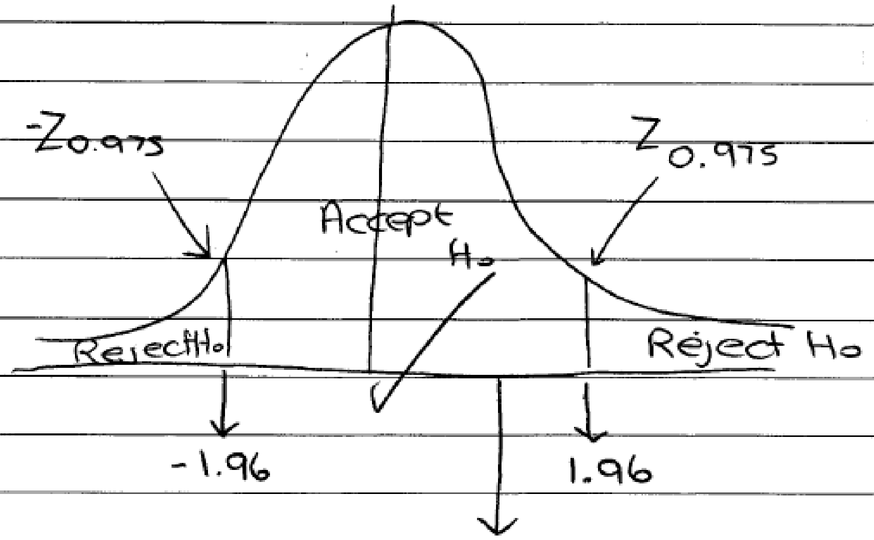
3. $\sigma = 2.5$ grams $\qquad$ H_0: Mean = 34.5

B0 t.w. = 34.5 g $\qquad$ H_1: Mean $\neq$ 34.5

$n = 50$

$\bar{x} = 35.1$

5% significance level



B1

M1

A1 $\dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{35.1 - 34.5}{2.5/\sqrt{50}} = 1.697$

A1 $-1.96 < 1.697 < 1.96$, so accept H_0,

there is evidence to suggest

E1 the mean weight is 34.5 grams.

5

Commentary

The candidate stated the Hypotheses incorrectly as H_0: 34.5 and H_1: $\neq 34.5$ or H_0: $\bar{x} = 34.5$ and H_1: $\bar{x} \neq 34.5$.

Since the population standard deviation, σ, is given, $z = \pm 1.96$ must be used and not $t = \pm 2.009$
Also, the comments in context were often too positive in nature.

3	$H_0: \mu = 34.5$ $H_1: \mu \neq 34.5$	B1		
	$z_{crit} = \pm 1.96$	B1ft		
	$z = \dfrac{35.1 - 34.5}{2.5/\sqrt{50}}$ = 1.70	M1 A1		(1.697)
	accept H_0	A1		
	Insufficient evidence, at 5% level of significance, to suggest that the mean weight has changed.	E1	**6**	Or…..to confirm Alan's belief
	Total		**6**	

Question 4

The delay, in hours, of certain flights from Australia may be modelled by the continuous random variable T, having probability density function

$$f(t) = \begin{cases} \frac{2}{15}t & 0 \le t \le 3 \\ 1 - \frac{1}{5}t & 3 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of f. (3 marks)

(b) Calculate:

(i) $P(T \le 2)$; (2 marks)

(ii) $P(2 < T < 4)$. (3 marks)

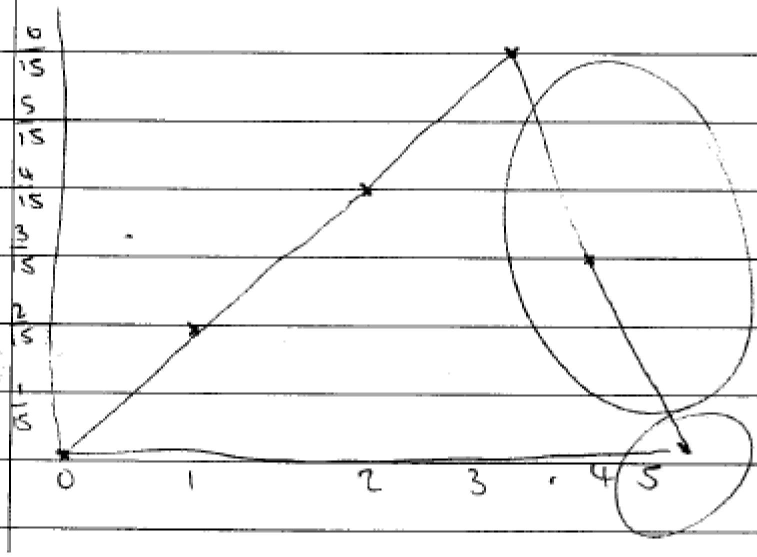
(c) Determine $E(T)$. (4 marks)

Student Response

4a)

t	f(t)
0	0
1	$\frac{2}{15}$
2	$\frac{4}{15}$
3	$^{6}/_{15}$
4	$^{3}/_{15}$
5	0

B1
B0
B1



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bi) $P(T \leq 2) = \frac{4}{15}$ ✓ 2

ii) $P(\cancel{T}\ 2 < T < 4) = \frac{1}{5} - \left(1 - \frac{4}{15}\right) = \frac{1}{5} - \frac{11}{15} = -\frac{8}{15}$ ✗ M0 A0 A0

c) $E(\cancel{x})$

T	f(t)	
0	0	0
1	2/15	2/15
2	4/15	8/15
3	6/15	18/15
4	3/15	12/15
5	0	0

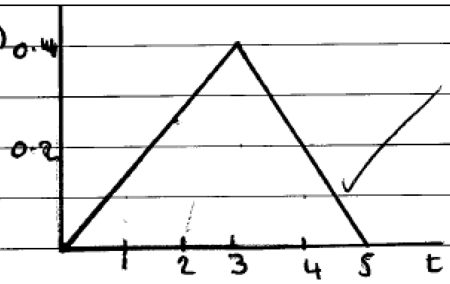
$E(T) = \frac{8}{3} = 2.667$

treated as discrete!!

M0
B0
B0
A0

4

4 a)



3

b) i) $P(T \leq 2) =$ ✗

M0

A0

M0 A0 A0 (ii)

M0 c) $E(T) = (0 \times 2/15) + (1 \times 2/15) + (3 \times 2/15) + \cancel{\ldots}(1 - 4/5) + (1 - 1) + (2 \times 2/15)$

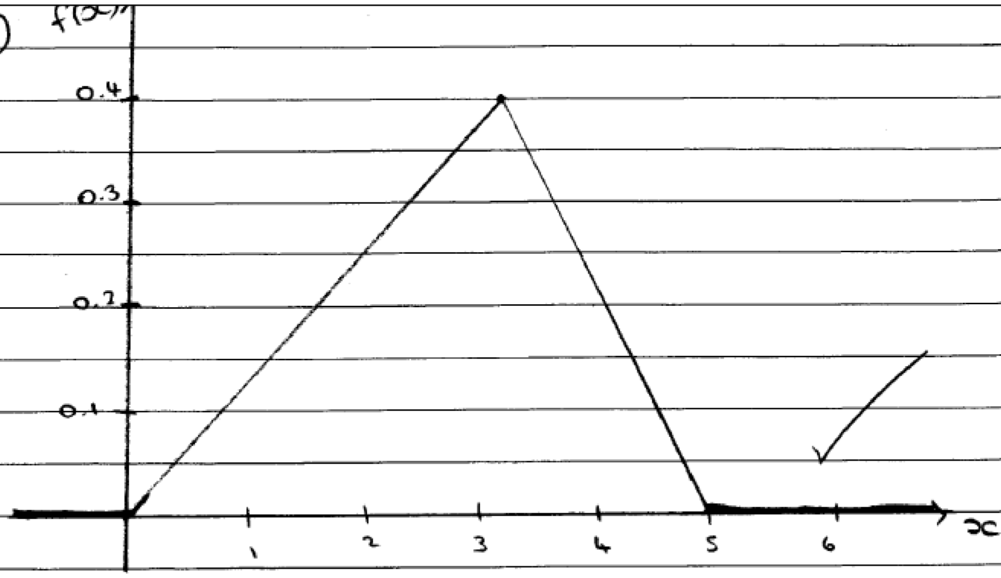
B0 B0 $= 0 + 2/15 + \frac{6}{15} + 4/15 + 1/5 + 0$ ✗

A0 $= 1$

3

Leave blank

4. a)



3

b)(i) $P(T \leq 2)$

$\int_0^2 \frac{2}{15}t = \left[\frac{2t^2}{30}\right]_0^2 = \frac{8}{30} = \frac{4}{15}$ 2

(ii) $P(2 < T < 4) = P(T \leq 3) - P(T \leq 2)$

$= \int_0^3 \frac{2}{15}t = \left[\frac{2t^2}{30}\right]_0^3 = \frac{18}{30} = \frac{9}{15}$ M0 A0

A0

$\frac{9}{15} - \frac{4}{15} = \frac{5}{15} = \frac{1}{3}$

c) $E(T)$

$\frac{9}{15} + \int_3^5 1 - \frac{1}{5}t \; dx$ M0

$\frac{9}{15} + \left[\cancel{t} - \frac{t^2}{10}\right]_3^5$ $= \frac{9}{5} + \left(5 - \frac{25}{10} - 3 - \frac{9}{10}\right)$ B0 B0

$= 0.4$ A0

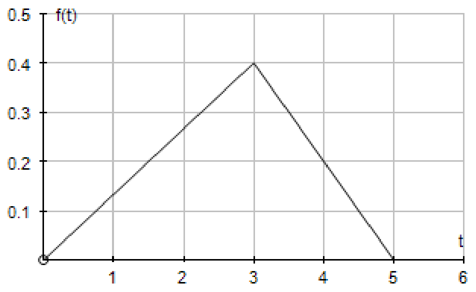
5

Commentary

Many candidates, in part(b)(ii), thought incorrectly that $\mathrm{P}(2 < T < 4) = \mathrm{P}(T \le 3) - \mathrm{P}(T \le 2)$.

Others, treated this as a discrete distribution throughout the question.

Mark Scheme

4(a)		B1 B1 B1	3	B1 line segment on 0 - 3 B1 line segment on 3 - 5 B1 scales (0.4 vertical; 0–5 horizontal)
(b)(i)	$\mathrm{P}(T \le 2) = \frac{1}{2} \times 2 \times \frac{4}{15}$	M1		
	$= \frac{4}{15}$	A1	2	(0.267)
(ii)	$\mathrm{P}(2 < T < 4)$			
	$= 1 - \left(\mathrm{P}(T < 2) + \mathrm{P}(T > 4)\right)$	M1		For $\mathrm{P}(T > 4) = \frac{1}{10}$
	$= 1 - \left(\frac{4}{15} + \frac{1}{2} \times \frac{1}{5}\right)$	A1		$\frac{1}{2} d\left[\left(f_1 + f_4\right) + 2 f_3\right]$
	$= 1 - \frac{4}{15} - \frac{1}{10}$			$f_2 = \frac{4}{15};\ f_4 = \frac{1}{5};\ f_3 = \frac{2}{5}$
	$= \frac{19}{30}$	A1	3	$d = 1$ (0.633)
(c)	$\mathrm{E}(T) = \int_0^3 \frac{2}{15} t^2 \, dt + \int_3^5 t\left(1 - \frac{1}{5} t\right) \mathrm{d}t$	M1		Both
	$= \left[\frac{2}{45} t^3\right]_0^3 + \left[\frac{1}{2} t^2 - \frac{1}{15} t^3\right]_3^5$	B1B1		
	$= \frac{6}{5} + \frac{25}{6} - \frac{27}{10}$			
	$= 2\frac{2}{3}$	A1	4	oe
	Total		**12**	

Question 5

The weight of fat in a digestive biscuit is known to be normally distributed.
Pat conducted an experiment in which she measured the weight of fat, *x* grams, in each of a random sample of 10 digestive biscuits, with the following results:

$$\sum x = 31.9 \quad \text{and} \quad \sum (x - \bar{x})^2 = 1.849$$

(a)(i) Construct a 99% confidence interval for the mean weight of fat in digestive biscuits. (5 mark

(ii) Comment on a claim that the mean weight of fat in digestive biscuits is 3.5 grams. (2 ma

(b) If 200 such 99% confidence intervals were constructed, how many would you expect **not** to contain the population mean? (1 ma

Student Response

5) a)i) $n = 10$ $\quad \bar{x} = 3.19$ $\quad \sigma = \sqrt{1.849} = 1.3597$

value = 2.5758

$c \cdot i = \bar{x} \pm \text{value}\,\dfrac{\sigma}{\sqrt{n}}$

$= 3.19 \pm 2.5758\,\dfrac{1.3597}{\sqrt{10}}$

$= 3.19 \pm 1.1075$

$\therefore$ ci = 2.082 to 4.298

B0 ii) 3.5 grams is inside the confidence interval
$\therefore$ the claim is a correct one

E0

b) samples not containing mean = 1% of 200
= 2

B1

Commentary

Many candidates couldn't calculate the correct value of s. They also used z-values (usually $z = 2.5758$) instead of the required t-value, $t = 3.250$.

5(a)(i)	$\bar{x} = 3.19$ and $s^2 = \frac{1.849}{9} = 0.2054$	B1		Both $(s = 0.453)$
	$t_9 = 3.250$	B1		
	99% Confidence Interval:			
	$3.19 \pm 3.250 \times \frac{\sqrt{0.2054}}{\sqrt{10}}$	M1		
	$= 3.19 \pm 0.4658$	A1ft		
	$= (2.72, 3.66)$	A1	5	(2.72 to 2.73; 3.65 to 3.66)
(ii)	Reasonable claim with 3.5 within the 99% confidence interval	B1 E1	2	Dep correct CI in (a)(i)
(b)	$0.01 \times 200 = 2$	B1	1	
	Total		**8**	

Question 6

The management of the Wellfit gym claims that the mean cholesterol level of those members who have held membership of the gym for more than one year is 3.8.

A local doctor believes that the management's claim is too low and investigates by measuring the cholesterol levels of a random sample of 7 such members of the Wellfit gym, with the following results:

4.2	4.3	3.9	3.8	3.6	4.8	4.1

Is there evidence, at the 5% level of significance, to justify the doctor's belief that the mean cholesterol level is greater than the management's claim?
State any assumption that you make. (8 marks)

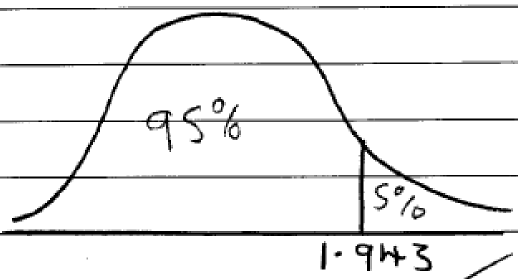
Student Response

6. $\mu = 3.8$ $\quad \bar{x} = 4.1$

$n = 7$ $\quad \sigma = 0.392$ (3sf)

$H_0: \mu = 3.8$ ✓

$H_1: \mu > 3.8$ One tailed test



$v = n - 1$

$= 6$ ✓

Test statistic: $\cancel{\mu}\ \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$\dfrac{4.1 - 3.8}{\frac{0.392}{\sqrt{7}}}$ $\quad 2.03 > 1.943$ ✓

$\therefore$ Reject H_0 ✓

6) $H_0: \mu = 3.8$ $\quad \bar{x} = 4.1$ ✓ $\quad n = 7$ — B0

$H_1: \mu < 3.8$ $\quad \sigma = 0.3915$ $\quad \therefore v = 6$

$\therefore$ critical value $= 1.943$ — B0

test statistic $= \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$ — B1

$= \dfrac{4.1 - 3.8}{\frac{0.3915}{\sqrt{7}}} = \dfrac{0.3}{0.14797} = 2.027$ — M1 A1

test statistic > critical value

$\therefore$ reject H_0 — A0

$\therefore$ the doctor's belief is correct, and the management's claim is false. This, however, is assuming that the ~~7~~ results are selected randomly and are independent from each other. — E0 B0

3

Commentary

The assumption asked for was often omitted or stated incorrectly. In the second example the candidate stated the Alternative hypothesis incorrectly. As for question 3, the hypotheses were often stated incorrectly.

Mark Scheme

6	$\bar{x} = 2.7 \quad s = 0.868$	B1		(both)
	$H_0: \mu = 3.8$ $H_1: \mu > 3.8$	B1		(both)
	$t = \dfrac{4.1 - 3.8}{0.392/\sqrt{7}} = 2.03$	M1 A1		(awfw 2.02 and 2.03)
	$t_{crit} = 1.943$	B1		
	Reject H_0	A1		
	Evidence at 5% level of significance to support the doctor's belief that the cholesterol level is higher than the management board's claim of 3.8.	E1		
	Cholesterol levels normally distributed	B1		
	Total		**8**	

Question 7

a) The number of text messages, N, sent by Peter each month on his mobile phone never exceeds 40.

When $0 \le N \le 10$ he is charged for 5 messages.
When $10 < N \le 20$ he is charged for 15 messages.
When $20 < N \le 30$ he is charged for 25 messages.
When $30 < N \le 40$ he is charged for 35 messages.

The number of text messages, *Y*, that Peter is charged for each month has the following probability distribution:

y	5	15	25	35
$\mathbf{P}(\boldsymbol{Y} = \boldsymbol{y})$	0.1	0.2	0.3	0.4

(i) Calculate the mean and standard deviation of *Y*. (4 marks)

(ii) The Goodtime phone company makes a total charge for text messages, *C* pence, each month given by:

$$C = 10Y + 5$$

Calculate $\mathrm{E}(C)$. (1 mark)

(b) The number of text messages, X, sent by Joanne each month on her mobile phone is such that:

$$\mathrm{E}(X) = 8.35 \qquad \text{and} \qquad \mathrm{E}(X^2) = 75.25$$

The Newtime phone company makes a total charge for text messages, *T* pence, each month given by

$$T = 0.4X + 250$$

Calculate $\mathrm{Var}(T)$. (4 marks)

Student Response

7a) i) $E(x) = (5\times0.1) + (15\times0.2) + (25\times0.3) + (35\times0.4)$

$= 0.5 + 3 + 7.5 + 14$

$= \underline{\underline{25}}$ ✓ 1

$E(x^2) = (25\times0.1) + (225\times0.2) + (625\times0.3) + (1225\times0.4)$

$= 2.5 + 45 + 187.5 + 490$

$= 723$ ✗ M1

Ac

$Var(X) = E(x^2) - [E(x)]^2$

$= 723 - 625$

$= 98$ A1

$\sigma = \sqrt{98}$

$= \underline{\underline{9.899}}\ (4\text{sf.})$

ii) ~~E~~ $C = 10y + 5$

$\Rightarrow E(c) = (55\times0.1) + (155\times0.2) + (255\times0.3) + (355\times0.4)$

$= 5.5 + 31 + 76.5 + 142$ ✓ 1

$= 255.$

b) $E(T) = 0.4(8.35) + 250$

$= 253.34$ ✓

B1 $E(T^2) = \cancel{0.4(8.35)+250}$

$T^2 = (0.4x + 250)(0.4x + 250)$

$= 0.16x^2 + 200x + 62500$

B1 $E(T^2) = \cancel{0.16(8.88}\ 0.16(75.25) + 200(8.35) + 62500$

$= 64182.04$ ✓

M1 $Var(T) = E(T^2) - E(T)^2$ ✓

A1 $= \cancel{64}\ 64182.04 - 64181.1556$

$= 0.8844$ ✓

8

Leave blank

7a) i) $E(Y) = (5\times0.1) + (15\times0.2) + (25\times0.3)$
$+ (35\times0.4)$.

$= 0.5 + 3 + 7.5 + 14$ 1
$= \underline{\underline{25}}$

$Var(Y) = E(Y^2) - [E(Y)]^2$.

$E(Y^2) = (25\times0.1) + (225\times0.2) + (625\times0.3)$ M1
$+ (1225\times0.4)$
$= 2.5 + 45 + 187.5 + 490$
$= 725$.

$\therefore Var(Y) = 725 - 25^2$ A1
$= \underline{\underline{100}}$.
AA A0

ii) $E(C) = 10\times E(Y) + 5$
$= 255$ pence. 1

b) $Var(X) = 75.25 - 8.35^2$
$= 5.5275$

4

$Var(T) = 0.4^2 \times Var(X)$
$= 0.4^2 \times 5.5275 = \underline{0.8844}$ 8

b) $E(T) = 0.4(8.35) + 250$
$= 253.34$ ✓

B1 $\cancel{E(T^2) = 0.4(...)+250}$

$T^2 = (0.4x + 250)(0.4x + 250)$

$= 0.16x^2 + 200x + 62500$

B1 $E(T^2) = \cancel{0.16(8.35)}\ 0.16(75.25) + 200(8.35) + 62500$
$= 64182.04$ ✓

M1 $Var(T) = E(T^2) - E(T)^2$ ✓

A1 $= \cancel{64}\ 64182.04 - 64181.1556$
$= 0.8844$ ✓

8

Commentary

A very well attempted question but some candidates (2019 Cand A), in part (a)(i) failed to evaluate the requested standard deviation, having correctly found the variance.

Some candidates, (1345 Cand b), in part (b) attempted to evaluate Var(T) by using $\mathrm{E}(T^2) - \mathrm{E}(T)^2$ but were unable to establish the correct value for $\mathrm{E}(T^2) = 64182.04$ having found $\mathrm{E}(T) = 253.34$ correctly. The easiest and most efficient way of doing this question is shown in the mark scheme.

7(a)(i)	$\mathrm{E}(Y) = \sum y\ \mathrm{P}(Y = y)$ $= 5\times0.1+15\times0.2+25\times0.3+35\times0.4$ $= 25$	B1		
	$\mathrm{Var}(Y) = \mathrm{E}(Y^2) - [\mathrm{E}(Y)]^2$ $= 725 - 25^2$ $= 100$	M1A1		cao
	Standard deviation $= 10$	A1ft	4	ft on Var(*Y*) > 0
(ii)	$C = 10Y + 5$ $\mathrm{E}(C) = 10\mathrm{E}(Y) + 5$ $= 10\times25+5$ $= 255$ pence	B1	1	oe
(b)(i)	$\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$ $= 75.25 - (8.35)^2$	M1		
	$= 75.25 - 69.7225$ $= 5.5275$	A1	2	Awfw 5.52 to 5.53
(ii)	$T = 0.4X + 250$ $\mathrm{Var}(T) = \mathrm{Var}(0.4X + 250)$ $= 0.4^2 \times \mathrm{Var}(X)$	M1		Var(*X*) > 0
	$= 0.16\times5.5275$ $= 0.8844$	A1	2	Awfw 0.884 to 0.885
	Total		**9**	

Question 8

The continuous random variable *X* has cumulative distribution function

$$\mathrm{F}(x)=\begin{cases}0 & x<-1\\ \dfrac{x+1}{k+1} & -1\le x\le k\\ 1 & x>k\end{cases}$$

where *k* is a positive constant.

(a) Find, in terms of *k*, an expression for $\mathrm{P}(X<0)$. (2 marks)

(b) Determine an expression, in terms of *k*, for the lower quartile, q_1. (3 marks)

(c) Show that the probability density function of *X* is defined by

$$\mathrm{f}(x)=\begin{cases}\dfrac{1}{k+1} & -1\le x\le k\\ 0 & \text{otherwise}\end{cases}$$

(2 marks)

(d) Given that $k=11$:

(i) sketch the graph of f; (2 marks)

(ii) determine $\mathrm{E}(X)$ and $\mathrm{Var}(X)$; (2 marks)

(iii) show that $\mathrm{P}(q_1<X<\mathrm{E}(X))=0.25$. (2 marks)

Student Response

8a) $\frac{0+1}{k+1} = \frac{1}{k+1} = P(X<0)$ ✓ 2

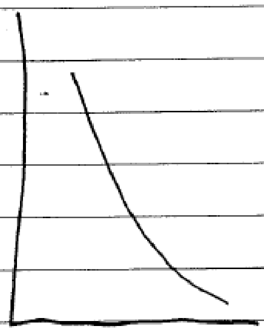
b) $\frac{x+1}{k+1} = 0.25$ $x+1 = 0.25(k+1)$ ✓ $x+1 = 0.25k + 0.25$ M1

$x + 0.75 = 0.25k$ A1

$\frac{x+0.75}{0.25} = k$ A0

c) $\frac{0+1}{k+1} = \frac{1}{k+1}$ * more req^d. M0 A0

d)i)



-1	0
0	1
1	½
2	1/3
3	1/4
4	1/5
5	1/6
6	1/7
7	1/8
8	1/9
9	1/10
10	1/11
11	1/12

B0

B0

ii) $E(x) = \int x\, fx = \int x\,(k+1)^{-1} = \left[\frac{x^2}{2}\ \frac{(k+1)^{-2}}{-2}\right]$ ✗

B0 $\text{var}(x) = \int x^2\, fx = \int x^2 (k+1)^{-1} = \left[\frac{x^3}{3}\ \frac{(k+1)^{-2}}{-2}\right] - \left[\frac{x^2}{2}\ \frac{(k+1)^{-2}}{-2}\right]$

B0 $\frac{x^3}{3} - \frac{x^2}{2} = \frac{2x^3}{6} - \frac{3x^2}{6}$

iii) ~~$\frac{x^3}{3} - \frac{x^2}{2}$~~

M0

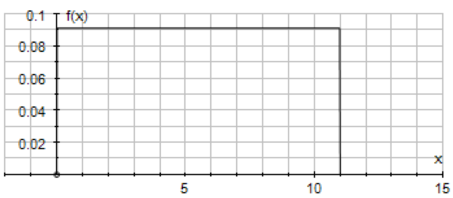
A0

4

Commentary

In part (b), many found an expression for k interms of x, instead of q_1 in terms of k.
Also many used calculus to find their answers to part (d)(ii) instead of the formulae stated in the booklet provided.

Mark Scheme

8(a)	$\mathrm{P}(X<0)=\mathrm{F}(0)$ $=\frac{1}{k+1}$	M1 A1	**2**	
(b)	$(q_1+1)\times\frac{1}{(k+1)}=\frac{1}{4}$ $q_1+1=\frac{1}{4}(k+1)$ $q_1=\frac{1}{4}(k+1)-1$	M1 A1 A1	**3**	Alternative (from a sketch) $q_1=-1+\frac{1}{4}(k+1)$ $q_1=\frac{1}{4}(k-3)$ oe
(c)	$\mathrm{f}(x)=\frac{d}{dx}(\mathrm{F}(x))$ $=\frac{1}{k+1}\times\frac{d}{dx}(x+1)$ $\left.\begin{array}{ll}=\frac{1}{k+1} & -1\le x\le k\\ =0 & \text{otherwise}\end{array}\right\}$	M1 A1	**2**	Use of $\frac{1}{k+1}$ clearly deduced AG
(d)(i)	$k=11$ $\Rightarrow \quad \mathrm{f}(x)=\begin{cases}\frac{1}{12} & -1\le x\le 11\\ 0 & \text{otherwise}\end{cases}$ Rectangular Distribution 	B1 B1	**2**	horizontal line on $[-1,11]$ at $\mathrm{f}=\frac{1}{12}$
(ii)	$\mathrm{E}(X)=\frac{1}{2}(-1+11)=5$ $\mathrm{Var}(X)=\frac{1}{12}(11--1)^2=12$	B1 B1	**2**	

(iii)	$\mathrm{P}\left(q_1 < X < \mathrm{E}(X)\right) = \mathrm{P}(2 < X < 5)$ $= (5-2) \times \frac{1}{12}$ $= \frac{1}{4}$	M1 A1	**2**	AG
			13	