ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics

PRACTICE PAPERS

ADVANCED SUBSIDIARY MATHEMATICS (5361)
ADVANCED SUBSIDIARY PURE MATHEMATICS (5366)
ADVANCED SUBSIDIARY FURTHER MATHEMATICS (5371)

ADVANCED MATHEMATICS (6361)
ADVANCED PURE MATHEMATICS (6366)
ADVANCED FURTHER MATHEMATICS (6371)

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Introduction

These practice papers are additional to the Specimen Units and Mark Schemes for the AQA GCE Mathematics specification (6360). The Specimen Units booklet, which contains an example of each question paper for the specification, is available from the AQA Publications Department and can also be downloaded from the AQA website (www.aqa.org.uk)

This booklet of practice papers contains a further question paper and mark scheme for selected units. Practice papers are included for: MPC1 because a non-calculator paper has not recently formed part of AS and A Level Mathematics; MPC2 because the equivalent paper in the previous specifications was problematical; MFP1 because the combination of subject content is different to any module of the previous specifications; MS03, MS04, MM03, MM04 and MM05 because there will be fewer , if any, past papers for the first cohorts of students taking these units.

Live papers are subject to many quality control checks before examinations to ensure that they are technically correct, within the specification and at the right level of demand. These practice papers have not been subject to the same degree of scrutiny. They are provided mainly to demonstrate the range of questions that could appear in a particular unit, rather than to illustrate the level of demand.

Abbreviations used in the Mark Schemes

M	Method mark for any acceptable method, even though numerical errors may occur. A method mark is not awarded until the stage referred to in the scheme is reached. Once awarded, a method mark cannot be lost. Method marks are not divisible when more than one is allocated, i.e. M2 can only result in the award of 2 or 0 marks.
m	Dependent method mark. A method mark which is only awarded if a previous M or m mark has been awarded. Where necessary the circumstances are specified in the scheme.
A	Accuracy mark. A mark which is awarded for accurate use of a correct method. An accuracy mark is dependent on all relevant M or m marks being gained.
B	Accuracy mark which is independent of any M or m mark.
E	Explanation mark. A mark for a response requiring explanation or comment by the candidate. This mark can be independent of, or dependent on, previous marks being gained. The circumstances are specified in the scheme.
ft or √	Follow through from candidate's previous answer. Follow-through marks may be given where at least one previous result which would have gained an A, B or E mark has been incorrect. The candidate's work is marked as though that previous result were correct. These marks are dependent on all relevant correct methods being used. Exact circumstances are specified if necessary in the scheme.
cao	Correct answer only. The accuracy mark depends on completely correct working to that stage. An exception is that answers given in the question paper can be used without loss of cao marks even if the candidate has not succeeded in obtaining the given answer. An accuracy mark is usually cao unless specified otherwise.
cso	Mark(s) can only be awarded if the specified method is used.
awfw	Anything which falls within the acceptable stated range of the answer.
awrt	Anything which rounds or truncates to the stated answer.
ag	Answer given (i.e. printed in the question paper). Beware faked solutions. However, a printed answer may be used in a later section of a question without penalty.
oe	Or equivalent. There are obvious alternative acceptable answers which can be given equivalent credit. Details are specified in the scheme if necessary.
sc	Special case. Where a particular solution given by candidates needs a different mark scheme to enable appropriate credit to be given.

General Certificate of Education
Practice paper
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1
Dateline

In addition to this paper you will require:

- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

MPC1

Answer **all** questions.

1 Express each of the following in the form $p+q\sqrt{5}$, where p and q are integers.

(a) $\left(4-\sqrt{5}\right)\left(3+2\sqrt{5}\right)$; *(3 marks)*

(b) $\dfrac{22}{4-\sqrt{5}}$. *(3 marks)*

2 (a) Express x^2+4x+7 in the form $(x+p)^2+q$, where p and q are integers. *(2 marks)*

(b) Hence describe geometrically the transformation which maps the graph of $y=x^2$ onto the graph of $y=x^2+4x+7$. *(3 marks)*

3 The points A and B have coordinates $(1,\ 6)$ and $(7,-2)$ respectively.

(a) Find the length of AB. *(2 marks)*

(b) Show that the line AB has equation $4x+3y=k$, stating the value of the constant k. *(3 marks)*

(c) The line AC is perpendicular to the line AB . Find the equation of AC. *(3 marks)*

4 Two numbers x and y are such that $2x+y=12$.

The product P is formed by multiplying the first number by the square of the second number, so that $P = xy^2$.

(a) Show that $P=4x^3-48x^2+144x$. *(2 marks)*

(b) Find the two values of x for which $\dfrac{\mathrm{d}P}{\mathrm{d}x}=0$. *(5 marks)*

(c) The values of x and y must both be positive.

(i) Show that there is only one value of x for which P is stationary. *(1 mark)*

(ii) Find the value of $\dfrac{\mathrm{d}^2P}{\mathrm{d}x^2}$ at this stationary value and hence show that it gives a maximum value. *(3 marks)*

(iii) Find the maximum value of P. *(1 mark)*

5 The polynomial p(x) is given by

$$p(x) = (x+1)(x^2 - 4x + 5)$$

(a) Find the remainder when $p(x)$ is divided by $x - 2$. *(2 marks)*

(b) Express $p(x)$ in the form $x^3 + mx^2 + nx + 5$, stating the value of each of the integers m and n. *(2 marks)*

(c) Show that the equation $x^2 - 4x + 5 = 0$ has no real roots. *(2 marks)*

(d) Find the coordinates of the points where the curve with equation $y = (x+1)(x^2 - 4x + 5)$ intersects the coordinate axes. *(2 marks)*

6 A curve has equation $y = x^3 - 3x^2 - 24x - 18$.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) (i) Show that y is increasing when $x^2 - 2x - 8 > 0$. *(2 marks)*

(ii) Hence find the possible values of x for which y is increasing. *(3 marks)*

(c) Find an equation for the tangent to the curve at the point (−1, 2). *(3 marks)*

7 A circle with centre C has equation $x^2 + y^2 + 4x - 6y = 7$.

(a) (i) Find the coordinates of C . *(2 marks)*

(ii) Find the radius of the circle, leaving your answer in the form $k\sqrt{5}$, where k is an integer. *(3 marks)*

(b) The line with equation $y = mx - 3$ intersects the circle.

(i) Show that the x–coordinates of any points of intersection satisfy the equation

$$(1 + m^2)x^2 + 4(1 - 3m)x + 20 = 0$$

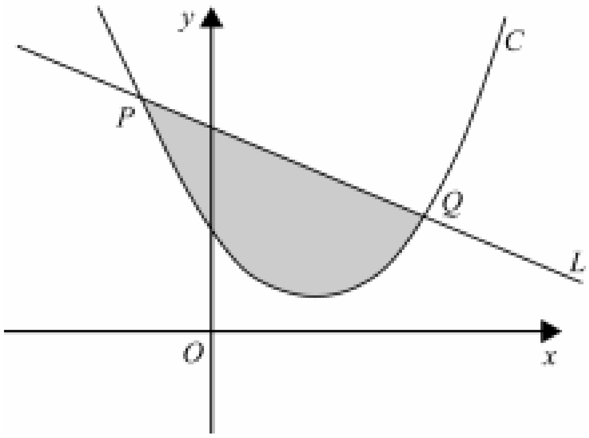
(3 marks)

(ii) Show that the quadratic equation $(1 + m^2)x^2 + 4(1 - 3m)x + 20 = 0$ has equal roots when $2m^2 - 3m - 2 = 0$. *(3 marks)*

(iii) Hence find the values of m for which the line is a tangent to the circle. *(2 marks)*

Turn over ►

8 The diagram shows a curve C and a line L.



The curve C has equation $y = 3x^2 - 5x + 3$ and the line L has equation $2x + y = 9$ and they intersect at the points P and Q.

(a) The point P has coordinates $(-1, 11)$. Find the coordinates of Q. *(4 marks)*

(b) (i) Find $\int (3x^2 - 5x + 3)\,dx$. *(3 marks)*

(ii) Find the area of the shaded region enclosed by the curve C and the line L. *(5 marks)*

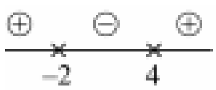
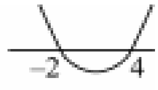
END OF QUESTIONS

Mathematics MPC1 Practice Paper

Question	Solution	Marks	Total	Comments
1(a)	$12+8\sqrt{5}-3\sqrt{5}-2\left(\sqrt{5}\right)^2$	M1		At least 3 terms
	$2\left(\sqrt{5}\right)^2=10$	B1		
	Answer = $2+5\sqrt{5}$	A1	3	
(b)	$\dfrac{22}{4-\sqrt{5}}\times\dfrac{4+\sqrt{5}}{4+\sqrt{5}}$	M1		Multiply top and bottom by conjugate
	$\left(4-\sqrt{5}\right)\left(4+\sqrt{5}\right)=11$	B1		
	Answer = $8+2\sqrt{5}$	A1	3	
	Total		**6**	
2(a)	$(x+2)^2$	B1		
	$+3$	B1	2	
(b)	Translation	M1		M1, A1, A0 for 'shift'
	$\begin{bmatrix}-2\\3\end{bmatrix}$	A1 A1	3	
	Total		**5**	
3(a)	$\text{AB}^2=(1-7)^2+(6+2)^2$	M1		
	$\text{AB}^2=100 \Rightarrow \text{AB}=10$	A1	2	
(b)	Gradient $=\dfrac{\Delta y}{\Delta x}=\dfrac{-8}{6}$	M1		
	$y-6=-\frac{4}{3}(x-1)$	A1√		
	$\Rightarrow 4x+3y=22$, *(k = 22)*	A1	3	Must be in this form with correct gradient
(c)	Use of $m_1m_2=-1$	M1		
	Gradient $AC=\frac{3}{4}$	A1√		√ from their gradient
	$y-6=\frac{3}{4}(x-1)$	A1	3	oe e.g. $4y-3x=21$
	Total		**8**	

Question	Solution	Marks	Total	Comments
4(a)	$y = 12 - 2x \Rightarrow P = x(12-2x)^2$	M1		
	$P = x(144 - 48x + 4x^2)$			
	$= 4x^3 - 48x^2 + 144x$	A1	2	ag
(b)	$\frac{dP}{dx} = 12x^2 - 96x + 144$	M1		Decrease power by 1
		A1		One term correct
		A1		All correct
	$\frac{dP}{dx} = 0 \quad \Rightarrow 12(x-6)(x-2)$	M1		Attempt to factorise / solve
	$x = 2,\ x = 6$	A1	5	
(c)(i)	$x = 6 \Rightarrow y = 0$ rejected			
	$\Rightarrow x = 2$ is only value	E1	1	
	$y = 8$			
(ii)	$\frac{d^2P}{dx^2} = 24x - 96$	M1		
	When $x = 2,\ \frac{d^2P}{dx^2} = -48$	A1√		ft their stationary value
	$\frac{d^2P}{dx^2} < 0 \Rightarrow$ Maximum	E1	3	
(iii)	Max $P = 2 \times 64 = 128$	B1	1	cso
	Total		**12**	
5(a)	$p(2) = (2+1)(2^2 - 8 + 5)$	M1		
	Remainder = 3	A1	2	Must have statement
(b)	$p(x) = x^3 - 4x^2 + 5x + x^2 - 4x + 5$	M1		Multiplying out
	$= x^3 - 3x^2 + x + 5$	A1	2	$m = -3,\ n = 1$
(c)	Discriminant = $16 - 20 = -4$	M1		Or $(x-2)^2 = -1$
	$< 0 \Rightarrow$ no real roots	A1	2	
(d)	$(-1, 0)$ and	B1		
	$(0, 5)$	B1	2	
	Total		**8**	

MPC1 (cont)

Question	Solution	Marks	Total	Comments
6(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 6x - 24$	M1		Reduce power by 1
		A1		One term correct
		A1	3	All correct
(b)(i)	Their Writing their $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$	M1		
	$\Rightarrow 3(x^2 - 2x - 8) > 0$			
	$\Rightarrow x^2 - 2x - 8 > 0$	A1	2	ag
(ii)	Critical values $x = 4, -2$	B1		 
	Use of sign diagram / sketch	M1		
	$x > 4, \quad x < -2$	A1	3	
(c)	When $x = -1, \; \frac{\mathrm{d}y}{\mathrm{d}x} = 3 + 6 - 24$	M1		
	$= -15$	A1		
	Tangent has equation $y - 2 = -15(x+1)$	A1	3	oe e.g. $15x + y + 13 = 0$
	Total		**11**	
7(a)(i)	$(x+2)^2 + (y-3)^2$	M1		Completing square attempted or one coordinate correct
	Centre $(-2, 3)$	A1	2	
(ii)	$r^2 = 4 + 9 + 7$	M1		Good attempt
	$= 20$	A1		
	$\Rightarrow r = 2\sqrt{5}$	A1	3	
(b)(i)	$x^2 + (mx-3)^2 + 4x - 6(mx-3) = 7$	M1		
	$x^2 + (m^2x^2 - 6mx + 9) + 4x$ $- 6mx + 18 = 7$	M1		Multiplied out, condone one sign slip
	$(1+m^2)x^2 + 4(1-3m)x + 20 = 0$	A1	3	ag
(b)(ii)	Equal roots $b^2 - 4ac = 0$			
	$16(1-3m)^2 - 80(1+m^2) = 0$	M1		
	$1 - 6m + 9m^2 - 5 - 5m^2 = 0$	m1		Multiplied out and attempt to simplify
	$4m^2 - 6m - 4 = 0$			
	$\Rightarrow 2m^2 - 3m - 2$	A1	3	ag
(iii)	$(2m+1)(m-2) = 0$	M1		Attempt to solve / factorise
	$m = 2$ or $m = -\frac{1}{2}$	A1	2	
	Total		**13**	

Question	Solution	Marks	Total	Comments
8(a)	$9-2x=3x^2-5x+3$	M1		Substitution of $y=9-2x$
	$3x^2-3x-6=0$			
	$3(x+1)(x-2)=0$	m1		Attempt to solve / factorise
	$x=-1,\ x=2$	A1		Both
	$\Rightarrow Q\ (2,\ 5)$	A1	4	
(b)(i)	$x^3-\dfrac{5x^2}{2}+3x\ (+c)$	M1		Raise power by 1
		A1		One term correct
		A1	3	All correct
(ii)	Use of their x_Q and -1	M1		
	$=[8-10+6]-[-1-\frac{5}{2}-3]$			
	$=10\frac{1}{2}$	A1		
	Area trapezium $=\frac{1}{2}(11+5)\times3=24$	B1√		ft their y_Q and x_Q
	Trapezium − Integral	M1		
	$=13\frac{1}{2}$	A1	5	cso
	Total		**12**	
	TOTAL		**75**	

General Certificate of Education
Practice paper
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Dateline

In addition to this paper you will require: • an 8-page answer book. • the **blue** AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

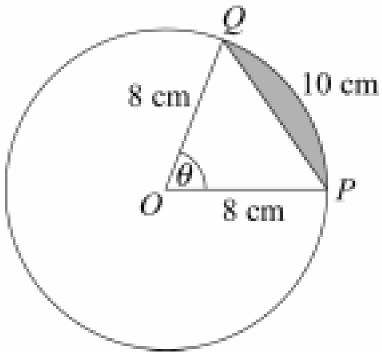
Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

MPC2

Answer **all** questions.

1 The diagram shows a circle with centre O and radius 8 cm. The angle between the radii OP and OQ is θ radians.



The length of the minor arc PQ is 10 cm.

(a) Show that $\theta = 1.25$. *(2 marks)*

(b) (i) Calculate the area of the triangle OPQ to the nearest cm^2. *(2 marks)*

(ii) Calculate the area of the minor sector OPQ to the nearest cm^2. *(2 marks)*

(iii) Hence find the area of the shaded segment to the nearest cm^2. *(1 mark)*

(c) Calculate the length of the side PQ of the triangle OPQ to the nearest mm. *(3 marks)*

2 (a) Using the binomial expansion, or otherwise, express $(1+2x)^4$ in the form

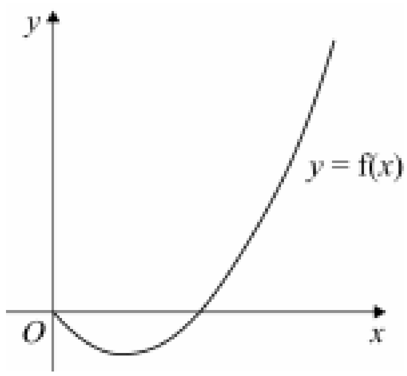
$$1 + ax + bx^2 + 32x^3 + 16x^4$$ where a and b are integers. *(3 marks)*

(b) In the expansion of $(2+x)^{10}$, the coefficient of x^9 is k. Find the value of k. *(2 marks)*

(c) Find the coefficient of x^{13} in the expansion of $(1+2x)^4(2+x)^{10}$. *(3 marks)*

3 The diagram shows the graph of $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 5x\sqrt{x} - 3x$$



(a) Write $x\sqrt{x}$ in the form x^k, where k is a constant. *(1 mark)*

(b) Differentiate $\mathrm{f}(x)$ to find $\mathrm{f}'(x)$. *(3 marks)*

(c) Show that, at the stationary point on the graph, $x = 0.16$. *(3 marks)*

(d) Show that the gradient of the curve at the point P where $x = 1$ is $\frac{9}{2}$. *(1 mark)*

(e) Find an equation of the normal to the curve at the point P. *(4 marks)*

4 (a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximation for

$$\int_1^3 \frac{1}{x^3 + 3}\,\mathrm{d}x$$

giving your answer to three significant figures. *(4 marks)*

(ii) Comment on how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

(b) The curve $y = \dfrac{1}{x^3 + 3}$ is translated by the vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ to give the curve with equation $y = \mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$.

(Do not simplify your answer.) *(2 marks)*

Turn over ►

5 The sum to infinity of a geometric series is three times the first term of the series. The first term of the series is a.

(a) Show that the common ratio of the geometric series is $\frac{2}{3}$. *(3 marks)*

(b) The third term of the geometric series is 81.

(i) Find the sixth term of the series. *(2 marks)*

(ii) Find the value of a as a fraction. *(2 marks)*

(iii) Hence show that $\log_{10} a = 6\log_{10} 3 - 2\log_{10} 2$. *(2 marks)*

6 (a) Explain briefly why $\log_5 125 = 3$. *(1 mark)*

(b) Find the value of:

(i) $\log_5(125^2)$; *(1 mark)*

(ii) $\log_5 \sqrt{125}$; *(1 mark)*

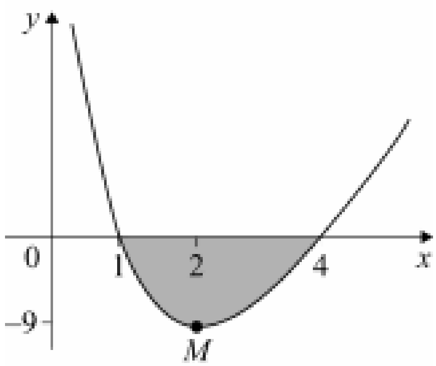
(iii) $\log_5\left(\frac{1}{\sqrt{125}}\right)$. *(1 mark)*

(c) Solve the equation $\log_5(125x) = 4$. *(2 marks)*

7 A curve is defined, for $x>0$, by the equation

$$y = \frac{x^4 - 17x^2 + 16}{x^2}$$

The curve intersects the x-axis at $x = 1$ and $x = 4$ as shown in the diagram below.



(a) Express $\dfrac{x^4 - 17x^2 + 16}{x^2}$ in the form $x^p - 17 + 16x^q$, where p and q are integers. *(2 marks)*

(b) (i) Find $\displaystyle\int \frac{x^4 - 17x^2 + 16}{x^2}\,\mathrm{d}x$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the x-axis. *(3 marks)*

(c) The point $M\,(2,–9)$ is the minimum point of the curve. Find the value of $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ at the point M. *(3 marks)*

8 (a) Sketch the graph of $y = \tan\ x$ for $0^\circ \le x \le 360^\circ$ *(3 marks)*

(b) Describe the single transformation by which the curve with equation $y = \tan 2x$ can be obtained from the curve $y = \tan x$. *(2 marks)*

(c) (i) Express the equation $5\sin 2x = 4\cos 2x$ in the form $\tan 2x = k$ where k is a constant. *(2 marks)*

(ii) Hence find all solutions of the equation $5\sin 2x = 4\cos 2x$ in the interval $0^\circ \le x \le 360^\circ$, giving your answers to the nearest 0.1°.

(No credit will be given for simply reading values from a graph.) *(5 marks)*

END OF QUESTIONS

Mathematics MPC2 Practice Paper

Question	Solution	Marks	Total	Comments
1(a)	$r\theta = 10$	M1		
	$\theta = 10/8 = 1.25$	A1	2	ag cso
(b)(i)	Appropriate use of $\sin\theta$	M1		Condone omission of units throughout
	Triangle area = 32sin1.25 = 30.36 = 30 cm^2	A1	2	
(ii)	Area of sector = ½ $r^2\theta$	M1		
	….. = 0.5 × 64 × 1.25 = 40 cm^2	A1	2	
(iii)	Segment area = 40 – 30.36... = 9.63 = 10 cm^2	A1√	1	Dep on previous two Ms
(c)	$PQ^2 = 8^2 + 8^2 - 2\times 8\times 8\cos 1.25$	M1		oe
	…… = 128 – 40.36….	m1		
	$PQ^2 = 87.63\ldots \Rightarrow PQ = 9.36\ldots = 9.4$ cm	A1	3	
	Total		**10**	
2(a)	$(1+2x)^4 = (1)^4 + 4(1)^3(2x) + 6(1^2)(2x)^2 + 4(1)(2x)^3 + (2x)^4$	M1		Any valid complete method for full expansion
	$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$	A1		Even terms; accept $a = 8$
		A1	3	Odd terms; accept $b = 24$
(b)	x^9 term is $\binom{10}{9}2x^9 = k\,x^9$	M1		
	$k = 20$	A1	2	
(c)	x^{13} terms from $32x^3(px^{10})$ and $16x^4(kx^9)$	M1		
	Coefficient of x^{13} is $32 + 16k = 352$	A2√	3	ft on cand's value of k
				sc if M0 award B1 for either $32(x^{13})$ or for $16k(x^{13})$
	Total		**8**	

Question	Solution	Marks	Total	Comments
3(a)	$x\sqrt{x} = x^{\frac{3}{2}}$	B1	1	Accept $k = 1.5$
(b)	$\text{f}(x) = 5x\sqrt{x} - 3x \Rightarrow \text{f}'(x) = 5\left(\frac{3}{2}x^{\frac{1}{2}}\right) - 3$	M1		
		A1√		
		A1	3	
(c)	For st. pt. $\text{f}'(x) = 0$ when $15x^{\frac{1}{2}} = 6$	m1		
	$x^{\frac{1}{2}} = 0.4$	A1		
	$x = 0.4^2 = 0.16$. At st. pt. $x = 0.16$	A1	3	ag
(d)	Gradient = $\text{f}'(1) = \frac{15}{2} - 3 = \frac{9}{2}$	A1	1	ag cso
(e)	When $x = 1$, $y = 2$	B1		For $y = 2$
	Gradient of normal = $-2/9$	M1		$m \times m' = -1$
	Eqn of normal $y - 2 = -\frac{2}{9}(x - 1)$	M1		oe $y - "2" = m(x - 1)$
		A1	4	Award at 1st correct form
	Total		**12**	
4(a)(i)	$h = 0.5$	B1		
	Integral = $h/2\{\ldots\}$	M1		At least 3 terms correct (accept 2dp)
	$\{\ldots\} = \left[\frac{1}{4} + \frac{1}{30} + 2\left(\frac{8}{51} + \frac{1}{11} + \frac{8}{149}\right)\right]$	A1		Five terms, at least four correct (exactly or to 3dp)
	Integral = 0.222	A1	4	cao
(ii)	Increase the number of ordinates	E1	1	
(b)	$\text{f}(x) = \frac{1}{(x-2)^3 + 3} + 1$	B2	2	Award B1 if either part of the translation is correct
	Total		**7**	

MPC2 (cont)

Question	Solution	Marks	Total	Comments
5(a)	$\frac{a}{1-r}=3a$	M1		
	$\Rightarrow 1-r=\frac{a}{3a}$	A1		
	$\Rightarrow r=\frac{2}{3}$	A1	3	ag cso
(b)(i)	6th term = $81r^3$	M1		
	= 24	A1	2	
(ii)	$ar^2 = 81$	M1		
	$a = 81\times\frac{9}{4}=\frac{729}{4}$	A1	2	
(iii)	$\log_{10} a = \log_{10} 729 - \log_{10} 4$	M1		
	$= \log_{10} 3^6 - \log_{10} 2^2 = 6\log_{10} 3 - 2\log_{10} 2$	A1	2	ag cso
	Total		**9**	
6(a)	$5^3 = 125$ so $\log_5 125 = 3$	E1	1	
(b)(i)	$\log_5(125^2) = 6$	B1	1	
(ii)	$\log_5 \sqrt{125} = 1.5$	B1	1	
(iii)	$\log_5\left(\frac{1}{\sqrt{125}}\right) = -1.5$	B1√	1	If not – 1.5, ft (ii)
(c)	Use of $\log kx = \log k + \log x$	M1		Or $125x = 5^4$
	$x = 5$	A1	2	
	Total		**6**	

Question	Solution	Marks	Total	Comments
7(a)	$\dfrac{x^4-17x^2+16}{x^2} = x^2-17+16x^{-2}$	M1 A1	2	Two of 3 terms correct Accept $p = 2$, $q = -2$
(b)(i)	$\dfrac{x^3}{3}-17x-16x^{-1}$ $(+c)$	M1 A1√ A1	3	Index raised by 1 (any term) One term correct ft p, q. All correct.
(ii)	$\displaystyle\int_1^4 \frac{x^4-17x^2+16}{x^2}\,\mathrm{d}x =$	B1		
	$\left[\dfrac{(4)^3}{3}-17(4)-\dfrac{16}{4}\right]-\left[\dfrac{1}{3}-17-16\right] =$	M1		F(4) – F(1)
	-18 Area is 18 (Integral negative as region is below x-axis)	A1	3	
(c)	$y = x^2-17+16x^{-2}$			
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x-32x^{-3}$	B1√		ft p and a negative q
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2+96x^{-4}$	B1√		ft if equivalent difficulty
	$= 8$ when $x = 2$	B1√	3	ft provided there is a negative power in $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$
	Total		**11**	

MPC2 (cont)

Question	Solution	Marks	Total	Comments
8(a)	Correct shape from *O* to 90°	M1		
	Complete graph for $0^\circ \le x \le 360^\circ$	A1		
	Correct scaling on x-axis $0^\circ \le x \le 360^\circ$	A1	3	
(b)	Stretch in x-direction	M1		
	scale factor $\frac{1}{2}$	A1	2	
(c)(i)	$\frac{\sin\theta}{\cos\theta} = \tan\theta$	M1		Stated or used
	$5\sin 2x = 4\cos 2x \Rightarrow \tan 2x = 0.8$	A1	2	oe Accept $k = 0.8$
				ft wrong k in all part (c)(ii)
(ii)	$\tan^{-1} k = 38.6598\ldots\ (=\alpha)$	M1		$\tan^{-1}k$
	$\{2x=\}\quad 180 + \alpha$;	m1		For $360-\alpha$ (or $x=90+\alpha/2$)
	$\{2x=\}\quad 360+\alpha$; $360 + [180+\alpha]$	m1		For either $360+\alpha$ or for $360 + [180+\alpha]$ **oe** for x
	$2x=38.65..; 218.65..; 398.65..; 578.65..$			
	$x = 19.3°;\ 109.3°;\ 199.3°;\ 289.3°$	A2	5	If not A2, give A1 for at least two correct or three correct but given to nearest degree. Accept greater accuracy answers. Ignore extra values outside the given interval. Extra values inside interval lose A mark(s).
	Total		**12**	
	TOTAL		**75**	

General Certificate of Education
Practice paper
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Dateline

In addition to this paper you will require:
• an 8-page answer book.
• the **blue** AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

MFP1

Answer **all** questions.

1 The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$$

(a) Calculate the matrices:

(i) **AB** ; *(2 marks)*

(ii) **ABC** . *(2 marks)*

(b) Describe the geometrical transformation represented by the matrix **AB**. *(2 marks)*

2 The complex number z is equal to $x + \mathrm{i}y$, where x and y are real numbers.

(a) Given that z^* is the conjugate of z, expand $(1 - \mathrm{i})z^*$ in terms of x and y. *(2 marks)*

(b) Given that

$$2(z - 1) = (1 - \mathrm{i})z^*$$

find the value of the complex number z. *(4 marks)*

3 (a) The quadratic equation $2x^2 - 6x + 1 = 0$ has roots α and β.

Write down the numerical values of:

(i) $\alpha\beta$; *(1 mark)*

(ii) $\alpha + \beta$. *(1 mark)*

(b) Another quadratic equation has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. Find the numerical values of:

(i) $\frac{1}{\alpha} \times \frac{1}{\beta}$; *(1 mark)*

(ii) $\frac{1}{\alpha} + \frac{1}{\beta}$. *(2 marks)*

(c) Hence, or otherwise, find the quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, writing your answer in the form $x^2 + px + q = 0$. *(2 marks)*

4 Given that $f(x) = x^4 - 1$:

(a) write down the value of $f(-1)$; *(1 mark)*

(b) show that $f(-1 + h) = -4h + 6h^2 - 4h^3 + h^4$; *(3 marks)*

(c) hence find the value of $f'(-1)$. *(2 marks)*

5 (a) Use the identity

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

to show that

$$\sum_{r=1}^{n} (r^3 - 1) = \tfrac{1}{4}n(n-1)(n^2 + 3n + 4)$$ *(4 marks)*

(b) Hence show that $\sum_{r=1}^{9} (r^3 - 1)$ is divisible by 7 . *(2 marks)*

6 A curve satisfies the differential equation $\frac{dy}{dx} = \sqrt{9 - x^2}$.

Starting at the point (0, 3) on the curve, use a step-by-step method with a step length of 0.5 to estimate the value of y at $x = 1$, giving your answer to two decimal places. *(5 marks)*

7 (a) Write down the exact values of $\sin\frac{\pi}{3}$, $\cos\frac{\pi}{3}$ and $\tan\frac{\pi}{3}$. *(3 marks)*

(b) Find the general solutions of the following equations, giving all solutions in terms of π:

(i) $2\sin\theta = \sqrt{3}$, *(5 marks)*

(ii) $2\sin(\theta - \frac{\pi}{3}) = \sqrt{3}$. *(3 marks)*

Turn over ▸

8 A curve has equation $y = \dfrac{x^2}{x^2 + 3x + 3}$.

(a) Write down the equation of the horizontal asymptote to the curve. *(1 mark)*

(b) (i) Prove that, for all real values of x, y satisfies the inequality $0 \leqslant y \leqslant 4$. *(6 marks)*

(ii) Hence find the coordinates of the turning points on the curve. *(3 marks)*

(c) Given that there are no vertical asymptotes, sketch the curve. *(3 marks)*

9 (a) Sketch the ellipse C which has equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

showing the coordinates of the points where the ellipse intersects the axes. *(4 marks)*

(b) Describe a sequence of geometrical transformations which would transform the unit circle

$$x^2 + y^2 = 1$$

into the ellipse C. *(4 marks)*

(c) Show that, if the line L which has equation

$$8x + 9y = 30$$

intersects the ellipse C, then the x-coordinates of the points of intersection must satisfy the quadratic equation

$$25x^2 - 120x + 144 = 0$$ *(5 marks)*

(d) By considering the discriminant of this quadratic, or otherwise, determine whether L is:

a tangent to C,

a line intersecting C in two distinct points, or

a line which does not intersect C. *(2 marks)*

END OF QUESTIONS

Mathematics MFP1 Practice Paper

Question	Solution	Marks	Total	Comments
1(a)(i)	**AB** = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$	M1A1	2	M1 for two correct entries
(ii)	**ABC** = $\begin{bmatrix} 30 & 20 \\ 10 & 0 \end{bmatrix}$	M1 A1F	2	ditto ft wrong answer to (i)
(b)	Enlargement ...	M1		
	... with scale factor 10	A1	2	
	Total		**6**	
2(a)	$z^* = x - \mathrm{i}y$	M1		
	$(1-\mathrm{i})z^* = x - \mathrm{i}y - \mathrm{i}x - y$	A1	2	oe; $\mathrm{i}^2 = -1$ must be used
(b)	Equating $2(x + \mathrm{i}y - 1)$ to above Equating R and I parts Solving sim equations $x = 3$, $y = -1$ (so $z = 3 - \mathrm{i}$)	M1 m1 m1 A1	4	
	Total		**6**	
3(a)(i)	$\alpha\beta = \frac{1}{2}$	B1	1	
(ii)	$\alpha + \beta = 3$	B1	1	
(b)(i)	$\left(\dfrac{1}{\alpha}\right)\left(\dfrac{1}{\beta}\right) = 2$	B1F	1	ft wrong answer to (a)(i)
(ii)	$\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta} = 6$	M1A1F	2	ft wrong answers in (a)
(c)	Equation is $x^2 - 6x + 2 = 0$	M1A1F	2	ft wrong answers in (b)
	Total		**7**	
4(a)	$\mathrm{f}(-1) = 0$	B1	1	
(b)	$(-1 + h)^4 = 1 - 4h + 6h^2 - 4h^3 + h^4$	M1A1		M1 for two correct terms
	Hence result	A1	3	ag convincingly shown
(c)	$\dfrac{\mathrm{f}(-1+h) - \mathrm{f}(-1)}{(-1+h)-(-1)} = -4$ + terms in h	M1		
	So $\mathrm{f}'(-1) = -4$	A1	2	
	Total		**6**	
5(a)	$(n + 1)^2 = n^2 + 2n + 1$	B1		
	$\Sigma(r^3 - 1) = (\Sigma r^3) - n$	M1		
	... $= \frac{1}{4}n(n^3 + 2n^2 + n - 4)$	A1		oe
	Hence result	A1	4	ag convincingly shown
(b)	$n = 9 \Rightarrow \frac{1}{4}(n^2 + 3n + 4) = 28$	M1		
	... $\Rightarrow$ expression = 7(4×9×8), Hence result	A1	2	ag convincingly shown
	Total		**6**	

MFP1 (cont)

Question	Solution	Marks	Total	Comments
6	$x = 0 \Rightarrow y' = 3$	M1		
	$\delta y \approx 3\delta x = 1.5$	m1		
	$x = 0.5 \Rightarrow y \approx 3 + 1.5 = 4.5$	A1		
	... and $y' \approx \sqrt{8.75} \approx 2.958$	m1		
	$x = 1 \Rightarrow y \approx 4.5 + (2.958)(0.5)$ ... ≈ 5.98	A1F	5	ft error in y(0.5)
	Total		**5**	
7(a)	$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}, \cos\frac{\pi}{3} = \frac{1}{2}, \tan\frac{\pi}{3} = \sqrt{3}$	B1 × 3	3	Allow 0.5 for cosine
(b)(i)	One solution is $\frac{\pi}{3}$	B1		
	Another is $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$	M1A1		oe
	Gen soln is $\frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi$	M1A1F	5	oe; ft wrong solutions
(ii)	Add $\frac{\pi}{3}$ to all solutions	M1		
	GS is $\frac{2\pi}{3} + 2n\pi, \pi + 2n\pi$	A1A1F	3	oe; ft small error
	Total		**11**	
8(a)	Asymptote is $y = 1$	B1	1	
(b)(i)	$y = k \Rightarrow x^2 = k(x^2 + 3x + 3)$	M1		
	ie $0 = (k - 1)x^2 + 3kx + 3k$	A1		
	Real roots if $9k^2 - 12k(k - 1) \geq 0$	m1		
	ie if $-3k^2 + 12k \geq 0$	A1F		ft error in coefficients
	ie if $k(4 - k) \geq 0$	m1		
	ie if $0 \leq k \leq 4$	A1	6	ag convincingly shown
(ii)	Horiz tangents $y = 0$ and $y = 4$	B1		
	$y = 0 \Rightarrow x = 0$	B1		
	$y = 4 \Rightarrow x = -2$	B1	3	
(c)	Curve (generally correct shape)	M1		
	Approaching $y = 1$ as $x \to \pm\infty$	A1		
	Max and min pts correctly shown	A1	3	
	Total		**13**	
9(a)	Ellipse symmetrical about axes	M1A1		
	$(\pm 3, 0), (0, \pm 2)$ indicated	A1A1	4	Allow labels on sketch
(b)	Stretch parallel to x-axis ...	M1		
	... with scale factor 3	A1		
	Stretch parallel to y-axis ...	M1		
	... with scale factor 2	A1F	4	A1 for SFs $\frac{1}{3}, \frac{1}{2}$
(c)	$y = \dfrac{30 - 8x}{9}$	B1		
	$\frac{1}{9}x^2 + \frac{1}{4}\left(\dfrac{30 - 8x}{9}\right)^2 = 1$	M1A1		oe
	$9x^2 + (15 - 4x)^2 = 81$	m1		oe
	Hence result	A1	5	ag convincingly found
(d)	$\Delta = 120^2 - 4(25)(144) = 0$	B1		
	So L is a tangent to C	B1	2	
	Total		**15**	
	TOTAL		**75**	

General Certificate of Education
Practice paper
Advanced Level Examination

MATHEMATICS **MS03**
Unit Statistics 3

Dateline

In addition to this paper you will require:
• an 8-page answer book;
• the **blue** AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS03.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

MS03

Answer **all** questions.

1 A machine is used to fill bags with compost. The weight, X kilograms, in a bag filled by this machine can be modelled by a normal distribution with mean μ and standard deviation 0.125.

An inspector wishes to calculate a 95% confidence interval for μ with a width of approximately 0.05 kilograms.

Calculate, to the nearest 10, the sample size necessary. *(5 marks)*

2 A fruit grower, who suspects that apples of Variety D weigh, on average more than apples of Variety E, obtains the following information on weights, in grams, of apples.

Variety	Sample size	Sample mean	Sample standard deviation
D	65	202.3	10.4
E	55	197.2	11.8

Investigate, at the 2% level of significance, the fruit grower's suspicion. *(6 marks)*

3 The random variable X has a binomial distribution with parameters n and p.

(a) Prove that $\mathrm{E}(X) = np$. *(4 marks)*

(b) Given that $\mathrm{E}(X(X-1)) = n(n-1)p^2$, show that $\mathrm{Var}(X) = np(1-p)$. *(3 marks)*

4 (a) In a certain population of animals, 42 per cent are male and 1.6 per cent are carriers of disease G. A random sample of 500 animals is selected from the population.

Using an appropriate approximation to a binomial distribution, estimate the probability that:

(i) fewer than 5 animals in the sample are carriers of disease G; *(3 marks)*

(ii) more than 200 but fewer than 225 animals in the sample are male. *(6 marks)*

(b) A random sample of 400 animals is selected from a second population of animals and is seen to contain 192 males.

Calculate an approximate 99% confidence interval for the proportion of males in this population. *(5 marks)*

(c) It is claimed that there is no difference between the proportions of males in the two populations.

State, giving a reason, whether or not you agree with this claim *(2 marks)*

5 A Passenger Transport Executive (PTE) carries out a survey of the commuting habits of city centre workers.

The PTE discovers that 40% of city centre workers travel by bus, 25% travel by train and the remainder use private vehicles.

Of those who travel by bus, 65% have a journey of less than 5 miles and 30% have a journey of between 5 and 10 miles.

Of those who travel by train, 25% have a journey of between 5 and 10 miles and 60% have a journey of more than 10 miles.

Of those who use private vehicles, 15% travel less than 5 miles and the same percentage travel more than 10 miles.

A city centre worker is selected at random. Determine the probability that the worker:

(a) has a journey of between 5 and 10 miles; *(3 marks)*

(b) travels by bus or has a journey of between 5 and 10 miles; *(3 marks)*

(c) uses a private vehicle, given that the worker travels between 5 and 10 miles; *(3 marks)*

(d) travels by train, given that the worker travels more than 10 miles. *(4 marks)*

TURN OVER FOR THE NEXT QUESTION

Turn over ►

6 (a) The random variables X and Y are such that:

$$E(X) = 2 \qquad E(X^2) = 13 \qquad E(Y) = 3 \qquad E(Y^2) = 25 \qquad E(XY) = 12$$

Calculate values for:

(i) Var(X) and Var(Y); *(2 marks)*

(ii) Cov(X, Y) and ρ ; *(3 marks)*

(iii) Var($6X - 5Y$). *(3 marks)*

(b) At a particular university, all first year students are required to visit a Registry Desk and a Finance Desk as part of the enrolment process.

The times, R seconds, at the Registry Desk are normally distributed with mean 220 and standard deviation 20.

The times, F seconds, at the Finance Desk are independent of those at the Registry Desk and are normally distributed with mean 175 and standard deviation 40.

Determine the probability that a first year student spends:

(i) a **total** of less than 5 minutes at the Registry and Finance desks; *(5 marks)*

(ii) more time at the Registry Desk than at the Finance desk. *(4 marks)*

7 A MOT testing station for cars has, for many weeks, placed an advertisement in a local free newspaper, offering half-price MOT tests on Fridays.

The number, X, of cars tested at the station on a Friday may be modelled by a Poisson distribution with a mean of 8.

In an effort to increase business on Fridays, the testing station replaces its advertisement in the local free newspaper with an advertisement in the local evening newspaper.

(a) In the first week following the change of newspaper for the advertisement, the station tests 10 cars on the Friday.

(i) Using the 5% level of significance, investigate whether the change of newspaper for the advertisement has resulted in more MOT business for the station on Fridays. *(5 marks)*

(ii) Determine, for your test in part (a)(i), the critical region for X. *(2 marks)*

(b) Assuming that the change of newspaper for the advertisement has resulted in an increase to 10 in the mean number of cars tested by the station on Fridays, determine, for a test at the 5% level of significance, the probability of a Type II error. *(3 marks)*

(c) State the implications of your answer to part (b). *(1 mark)*

END OF QUESTIONS

Mathematics MS03 Practice Paper

Question	Solution	Marks	Total	Comments
1	CI width = $2\times\frac{z\times\sigma}{\sqrt{n}}$	M1		use of; allow $\frac{z\times\sigma}{\sqrt{n}}$
	For 95%, $z = 1.96$	B1		cao
	Thus $2\times\frac{1.96\times0.125}{\sqrt{n}}=0.05$	A1√		or equivalent √ on z-value only
	Thus $n=\left(\frac{2\times1.96\times0.125}{0.05}\right)^2 = 96.04$	m1		solving for n
	Thus, to nearest 10, $n = 100$		**5**	cao
	Total		**5**	
2	H_0: $\mu_D = \mu_E$ H_1: $\mu_D > \mu_E$	B1		both; or equivalent
	SL $\alpha = 0.02$ CV $z = 2.0537$			awfw 2.05 to 2.06
	$z=\frac{(\bar{x}_D-\bar{x}_E)}{\sqrt{\frac{s_D^2}{n_D}+\frac{s_E^2}{n_E}}}$ or $z=\frac{(\bar{x}_D-\bar{x}_E)}{\sqrt{s_p^2\left(\frac{1}{n_D}+\frac{1}{n_E}\right)}}$	M1		use of; $s_p^2=\frac{14441.2}{118}$
	$=\frac{202.3-197.2}{\sqrt{\frac{10.4^2}{65}+\frac{11.8^2}{55}}}$ or $\frac{202.3-197.2}{\sqrt{122.38\left(\frac{1}{65}+\frac{1}{55}\right)}}$	A1√		√ if s_p^2 used
	= 2.48 to 2.52	A1		awfw
	Thus, at 2% level of significance, evidence to support the fruit grower's suspicion	A1√	**6**	√ on z-value and CV
	Total		**6**	

Question	Solution	Marks	Total	Comments
3(a)	$\text{E}(X) = \sum_{x=0}^{n} x \times \binom{n}{x} p^x (1-p)^{n-x} =$			use of $\text{E}(X) = \sum x \times \text{P}(X = x)$
	$\sum_{x=1}^{n} x \times \left(\frac{n!}{x! \times (n\text{-}x)!} \right) p^x (1-p)^{n-x} =$	m1		summation from 1 and expansion of $\binom{n}{x}$
	$np \times \sum_{x=1}^{n} \left(\frac{(n-1)!}{(x-1)! \times (n\text{-}x)!} \right) p^{x-1} (1-p)^{n-x}$ =	A1		factor of np, change n to $(n-1)$, x to $(x-1)$ and p^x to p^{x-1} to give fully correct expression
	$np \times \sum (\text{terms of } \text{B}(n-1, p)) = np \times 1$ $= np$	m1	**4**	ag
(b)	$\text{Var}(X) = \text{E}(X^2) - \mu^2 =$	M1		use of
	$\text{E}(X(X-1)) + \text{E}(X) - \mu^2 =$	m1		expression for $\text{E}(X^2)$ and
	$n(n-1)p^2 + np - (np)^2 =$			substitutions
	$n^2p^2 - np^2 + np - n^2p^2 =$			correct deduction
	$np(1-p)$		**3**	ag
	Total		**7**	

Question	Solution	Marks	Total	Comments
4(a)(i)	$n = 500$ and $p = 0.016$ (1.6%) so Poisson approximation with parameter/mean, $\lambda = 8$	M1 A1		may be implied cao
	$P(X < 5) = P(X \leq 4) = 0.0996$	A1	**3**	awfw 0.099 to 0.1
(ii)	$n = 500$ and $p = 0.42$ (42%) so Normal approximation with mean $\mu = 210$ and variance $\sigma^2 = 121.8$	M1 A1		may be implied cao both
	$P(200 < X_B < 225) = P(200.5 < X_N < 224.5)$			both continuity corrections correct
	$= P\left(Z < \frac{224.5 - 210}{\sqrt{121.8}}\right) - P\left(Z < \frac{200.5 - 210}{\sqrt{121.8}}\right)$	M1		standardising
	$= P(Z < 1.31) - P(Z < -0.86) =$			
	$= \Phi(1.31) - (1 - \Phi(0.86))$	m1		area difference
	$= 0.90490 - (1 - 0.80511) = 0.710$	A1	**6**	awfw 0.710 to 0.711
(b)	$\hat{p} = \frac{192}{400} = 0.48$	B1		cao
	99% implies $z = 2.5758$			awfw 2.57 to 2.58
	CI: $\hat{p} \pm z \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	M1		use of
	ie $0.48 \pm 2.5758 \times \sqrt{\frac{0.48 \times 0.52}{400}}$	A1√		√ on $\hat{p}$ and z only
	(0.416, 0.544)	A1	**5**	awrt
(c)	Agree with claim as	B1√		√ on part (b)
	0.42 belongs to CI	E1√	**2**	√ on part (b)
	Total		**16**	

Question	Solution	Marks	Total	Comments
5 (a)	P(5 to 10) = P(B ∩ 5-10) + P(T ∩ 5–10) + P(PV ∩ 5–10) =			≥ 1 term involving ∩
	$0.40 \times 0.30 + 0.25 \times 0.25 + 0.35 \times 0.70 =$	A1		≥ 2 terms
	0.4275	A1	3	awfw 0.427 to 0.428
(b)	P(B ∪ 5–10) = P(B) + P(T ∩ 5–10) + P(PV ∩ 5–10) =	M1		P(B) both
	$0.40 + 0.25 \times 0.25 + 0.35 \times 0.70 =$ 0.7075	A1	3	awfw 0.707 to 0.708
(c)	$P(PV \mid 5–10) = \frac{P(PV \cap 5-10)}{P(5-10)} =$	M1		use of conditional probability in parts (c) or (d)
	0.245/0.4275 =			√ on part (a)
	0.573	A1	3	awrt
(d)	$P(T \mid >10) = \frac{P(T \cap >10)}{P(>10)} =$			
	$\frac{0.25 \times 0.60}{0.40 \times 0.05 + 0.25 \times 0.60 + 0.35 \times 0.15} =$	A1 A1		
	0.15/0.2225 =			√ on expression providing < 1
	0.674	A1	4	awrt
	Total		**13**	

Question	Solution	Marks	Total	Comments
6(a)(i)	$\text{Var}(X) = 13 - 2^2 = 9$			cao
	$\text{Var}(Y) = 25 - 3^2 = 16$		**2**	cao
(ii)	$\text{Cov}(X, Y) = 12 - 2 \times 3 = 6$			cao
	$\rho = \dfrac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \times \text{Var}(Y)}} = \dfrac{6}{\sqrt{9 \times 16}} =$	M1		use of
	0.5	A1	**3**	cao
(iii)	$\text{Var}(6X - 5Y) = 6^2 \times \text{Var}(X) + 5^2 \times \text{Var}(Y) - 2 \times 6 \times 5 \times \text{Cov}(X, Y) =$	M1		use of correct form
	$36 \times 9 + 25 \times 16 - 60 \times 6 =$	A1√		√ on parts (a)(i) and (ii)
	364	A1	**3**	cao
(b)	$R \sim \text{N}(220, 20^2)$ and $F \sim \text{N}(175, 40^2)$			
(i)	$T = (R + L)$ has mean = 395	B1		cao
	and variance = 2000	B1		cao; sd = 44.7 awrt
	$\text{P}(T < 5 \times 60) = \text{P}\left(Z < \dfrac{300 - 395}{\sqrt{2000}}\right) =$	M1		standardising 300
	$\text{P}(Z < -2.12) = 1 - \Phi(2.12) =$	m1		area change
	0.0165 to 0.0170	A1	**5**	awfw
(ii)	$D = (R - L)$ has mean = 45	M1		use of difference
	and variance = 2000	A1		cao both; sd = 44.7 awrt
	$\text{P}(D > 0) = \text{P}\left(Z > \dfrac{0 - 45}{\sqrt{2000}}\right) =$	M1		standardising 0
	$\text{P}(Z > -1.01) = \Phi(1.01) =$			
	0.841 to 0.844	A1	**4**	awfw
	Total		**17**	

MS03 (cont)

Question	Solution	Marks	Total	Comments
7(a)(i)	H_0: $\lambda = 8$ H_1: $\lambda > 8$	B1		both
	$P(X \geq 10 \mid \lambda = 8) =$	M1		attempt at
	$1 - P(X \leq 9 \mid \lambda = 8) =$	m1		for tables or calculation
	$1 - 0.7166 = 0.283$ to 0.284 > 0.05 (5%)	A1		awfw
	Thus, at 5% level of significance, no evidence of more MOT business on Fridays	A1√	5	√ on probability with 5%
(ii)	Require $P(X \geq x \mid \lambda = 8) \; = / < \; 0.05$ or Require $P(X \leq (x-1) \mid \lambda = 8) \; = / > \; 0.95$	M1		may be implied
	From tables $x - 1 = 13$ Thus critical region is $X \geq 14$		2	cao; can be scored in part (a)(i)
(b)	Type II error = P(accept H_0 | H_1 true) =			
	$P(X$ not in CR $\mid \lambda = 10) =$	M1		use of
	$P(X \leq 13 \mid \lambda = 10) =$	A1√		√ on part (a)(ii)
	0.864 to 0.865	A1	3	awfw
(c)	Test is very poor at detecting increase to 10 in mean number of cars tested	A1√	1	√ on part (b)
	Total		**11**	
	TOTAL		**75**	

General Certificate of Education
Practice paper
Advanced Level Examination

MATHEMATICS **MS04**
Unit Statistics 4

Dateline

In addition to this paper you will require:
- an 8-page answer book.
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS04.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

MS04

Answer **all** questions.

1 A school bus travels the same route each morning. The time, T minutes, from its first stop to the school is recorded on each of a random sample of 40 mornings.

The recorded times then gave

$$\sum (t - \bar{t})^2 = 643.5$$

where $\bar{t}$ denotes the sample mean.

(a) Stating the necessary distributional assumption, construct a 95% confidence interval for the standard deviation of the morning journey time of the bus. *(7 marks)*

(b) Hence comment on the claim that the standard deviation of the morning journey time of the bus is 5 minutes. *(2 marks)*

2 The time, D days, between successive accidents at a factory can be modelled by an exponential distribution with mean 16.

(a) Write down the numerical value for the standard deviation of D. *(1 mark)*

(b) Calculate the probability that the time between successive accidents at the factory is more than 20 days. *(3 marks)*

(c) Given that there are no accidents during a 20-day period, determine the probability that there are no accidents during the next 20 days. Justify your answer. *(3 marks)*

3 Twelve babies, paired according to birth weight, were used to compare an enriched formula baby food with a standard formula baby food. The weight gains, in grams, over a fixed period of time were as follows.

Pair	1	2	3	4	5	6
Enriched formula	3600	2950	3345	3760	4310	3075
Standard formula	3140	3100	2810	4030	3770	2630

(a) Assuming differences to be normally distributed, determine a 95% confidence interval for the mean difference in weight gain between babies fed on enriched formula baby food and those fed on standard formula baby food. *(8 marks)*

(b) State, with a justification, what conclusion may be inferred from your confidence interval. *(2 marks)*

4 The number of calls per hour to a telephone hotline, during the period 9 am to 4 pm on weekdays, is recorded with the following results.

Period	9-10	10-11	11-12	12-1	1-2	2-3	3-4
Number of calls	132	151	143	129	117	134	125

Test, at the 10% level of significance, the hypothesis that the number of calls per hour during the period 9 am to 4 pm on weekdays follows a rectangular distribution. *(8 marks)*

5 A random observation, X_1, is taken from a population with mean μ and variance $\sigma_1^{\,2}$. A random observation X_2, is taken from a population, also with a mean μ, but with variance $\sigma_2^{\,2}$.

(a) An unbiased estimator for μ is $U = aX_1 + bX_2$, where a and b are constants.

Show that $a + b = 1$. *(3 marks)*

(b) Show that:

$$\text{Var}(U) = a^2\left(\sigma_1^{\,2} + \sigma_2^{\,2}\right) - 2a\sigma_2^{\,2} + \sigma_2^{\,2}$$ *(3 marks)*

(c) Deduce that if U has minimum variance then:

$$a = \frac{\sigma_2^{\,2}}{\sigma_1^{\,2} + \sigma_2^{\,2}} \qquad \text{and} \qquad b = \frac{\sigma_1^{\,2}}{\sigma_1^{\,2} + \sigma_2^{\,2}}$$ *(5 marks)*

6 The random variable X follows the probability distribution

$$\text{P}(X = r) = \begin{cases} q^{r-1}p & \text{for } r = 1,2,3... \\ 0 & \text{otherwise} \end{cases}$$

where $q = 1 - p$.

(a) Prove that:

(i) $\text{E}(X) = \dfrac{1}{p}$; *(3 marks)*

(ii) $\text{Var}(X) = \dfrac{q}{p^2}$. *(5 marks)*

(b) A fair die is rolled until a six is obtained. Given that no six is obtained in the first three rolls, calculate the probability that a six is obtained in the next three rolls. *(4 marks)*

Turn over ►

7 A zoologist discovers two colonies of lizards on neighbouring islands, *A* and *B,* in the Pacific. She traps a small number of these lizards on each island and measures their lengths with the following results.

Island *A* lengths (*x* cm)	20.8	21.9	19.8	20.5	21.7	19.5	21.3
Island *B* lengths (*y* cm)	22.3	23.4	20.0	23.6	24.1	22.9	

Assuming that these are random samples from normal populations, test, at the 5% significance level, the hypothesis that:

(a) the population variances are equal; *(9 marks)*

(b) the population means are equal. *(9 marks)*

END OF QUESTIONS

Question	Solution	Marks	Total	Comments
1(a)	Assumption: T ~ Normal	B1		
	C.I. for σ^2 : $\dfrac{\sum(t-\bar{t})^2}{\chi^2(\text{U})}$ to $\dfrac{\sum(\text{t - }\bar{\text{t}})^2}{\chi^2(\text{L})}$	M1		Use of, or equivalent $(s^2 = 16\cdot 5)$
	$\upsilon = 39$	B1		cao
	$95\% \Rightarrow 0.025$ and 0.975 so values are 23.654 and 58.120	B1		both, awrt 23.6/7 and 58.1
	$\therefore$ C.I. for σ^2 is $\dfrac{643.5}{58.120}$ to $\dfrac{643.5}{23.654}$	A1√		√ on χ^2 values
	$(11.1,\ 27.2)$	A1√		awrt, √ on χ^2 values - may be implied
	$\therefore$ C.I. for σ is : $(3.3, 5.2)$	A1	7	square root
(b)	$5 \in$ C.I.	B1√		
	$\therefore$ Accept that S.D. is 5	B1√	2	
	Total		**9**	
2(a)	$D \sim \text{Exp}(16) \Rightarrow \sigma = 16$	B1	1	cao
(b)	Exponential with $\dfrac{1}{\lambda} = 16$	M1		use of associated pdf or df
	$\text{P}(D > 20) = \left[-\text{e}^{-\frac{d}{16}}\right]_{20}^{\infty}$ or $1-\left(1-\text{e}^{-\frac{20}{16}}\right)$	A1		either
	$\text{e}^{-1.25} = 0.286$ or 0.287	A1	3	awfw
(c)	$\text{P}(D_2 > 20 \mid D_1 > 20) = \dfrac{\text{P}(D_1 + D_2 > 40)}{\text{P}(D_1 > 20)}$	M1		use of conditional probability
		A1		correct expression
	$= \dfrac{\text{e}^{-2.5}}{\text{e}^{-1.25}}$			
	$= \text{e}^{-1.25}$	A1√		√ if calculated answer = (b)
	$= 0.286$ to 0.287		3	justification of equality $\Rightarrow 3$ if (b) correct $\Rightarrow 2$ if (b) incorrect
	Total		**7**	

Question	Solution	Marks	Total	Comment
3(a)	d: 460 150 535 270 540 445	M1		
	$\sum d = 1560 \quad \sum d^2 = 1082850$	M1		
	$\bar{d} = 260$	B1		cao
	$v = 112875 \qquad s_d^2 = 135450$	A1		awrt
	$s_d = 368.035$			
	$t(5, 0.975) = 2.571$	B1		cao
	C.I. is $260 \pm 2.571\sqrt{\frac{135450}{6}}$	M1 A1√		√ on $\bar{d}$, s_d^2 and t
	ie 260 ± 386			
	ie $(-126,\ 646)$	A1	8	awrt
(b)	No evidence of a difference in true mean weight gains	B1√		√ on C.I. (accept other statistical alternatives)
	Interval includes zero	E1√	2	√ on C.I. (accept other statistical justifications)
	Total		**10**	
4	H_0 : number is constant			
	H_1 : number is not constant	B1		at least H_0
	SL $\alpha = 0.10$ DF $v = 7 - 1 = 6$	B1		cao
	CV $\chi^2 = 10.645$	B1		awfw 10.6 to 10.7
	Mean per hour $= \frac{\sum \text{calls}}{7}$	M1		use of
	$= \frac{931}{7} = 133$	A1		cao
	$\chi^2 = \sum \frac{(O - E)^2}{E}$	M1		use of
	$\frac{1}{133}\sum (O - 133)^2 = 5.73$	A1		awfw 5.72 to 5.74
	Thus insufficient evidence, at 10% level, to suggest that number per hour is not constant	A1√	8	√ on χ^2 and upper CV
	Total		**8**	
5(a)	$E(aX_1 + bX_2) = aE(X_1) + bE(X_2)$			
	$= a\mu + b\mu$			
	$= (a + b)\mu$	M1		
	but $E(aX_1 + bX_2) = \mu$	M1		
	$\therefore a + b = 1$	A1	3	

Question	Solution	Marks	Total	Comment
(b)	$\text{Var}(\text{U}) = \text{Var}(aX_1 + bX_2)$			
	$= a^2\text{Var}(X_1) + b^2\text{Var}(X_2)$	M1		
	$= a^2\sigma_1{}^2 + (1-a)^2\sigma_2^2$	M1		
	$= a^2(\sigma_1^2 + \sigma_2^2) - 2a\sigma_2^2 + \sigma_2^2$	A1	3	ag (sufficient working seen)
(c)	$\frac{\text{dV}}{\text{d}a} = 2a(\sigma_1^2 + \sigma_2^2) - 2\sigma_2^2 \quad = \ 0$	M1 A1		
	$a = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$, $b = \frac{1 - \sigma_2^2}{\sigma_1^2 + \sigma_2^2} = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$	M1 A1 A1	5	ag (sufficient working seen)
	Total		**11**	
6(a)(i)	$\text{E}(X) = 1p + 2qp + 3q^2p + ...$	M1		
	$= p(1 + 2q + 3q^2 + ...)$	A1		
	$= p(1 - q)^{-2}$			
	$= \frac{p}{p^2} = \frac{1}{p}$	A1	3	ag (sufficient working seen)
(ii)	$\text{E}(X^2) = 1p + 4qp + 9q^2p + 16q^3p + ...$ $= p + 3qp + 6q^2p + 10q^3p + ...$	M1		
	$+ qp + 3q^2p + 6q^3p + ...$	A1		
	$= p(1 + 3q + 6q^2 + 10q^3 + ...)$ $+ qp(1 + 3q + 6q^2 + ...)$ $= p(1-q)^{-3} + qp(1-q)^{-3}$	M1		
	$= \frac{1}{p^2} + \frac{q}{p^2}$	A1		
	$\text{var}(X) = \frac{1}{p^2} + \frac{q}{p^2} - \frac{1}{p^2} = \frac{q}{p^2}$	A1	5	ag (intermediate stage required)
(b)	$\frac{\text{P}(4) + \text{P}(5) + \text{P}(6)}{\{1 - \text{P}(1) + \text{P}(2) + \text{P}(3)\}}$	M1		Conditional probability
	$= \frac{\left(\frac{5}{6}\right)^3\left(\frac{1}{6} + \frac{5}{36} + \frac{25}{216}\right)}{\left\{1 - \left(\frac{1}{6} + \frac{5}{36} + \frac{25}{216}\right)\right.}$	A1 A1		Numerator Denominator
	$= \frac{\frac{125}{216} \times \frac{91}{216}}{\frac{125}{216}} = \frac{91}{216}$	A1	4	oe Accept 0.421
	Total		**12**	

Question	Solution	Marks	Total	Comments
7(a)	$H_0 : \sigma_x^2 = \sigma_y^2$	B1		both
	$H_1 : \sigma_x^2 \neq \sigma_y^2$			
	$s_x{}^2 = \frac{3029 \cdot 37}{6} - \frac{145 \cdot 5^2}{6 \times 7}$	M1		B3 if found from calculator
	$= 0 \cdot 8414 \ (0 \cdot 841)$	A1		B2 one correct
	$s_y{}^2 = \frac{3107 \cdot 03}{5} - \frac{136 \cdot 3^2}{5 \times 6}$	A1		B1 for values $\div n$
	$= 2 \cdot 1497 \ (2 \cdot 15)$			
	$F_{calc} = \frac{s_y{}^2}{s_x{}^2} = \frac{2 \cdot 1497}{0 \cdot 8414} = 2 \cdot 55$	M1 A1√		use of awfw $2 \cdot 55$ to $2 \cdot 56$ √ on variances
	DF $\nu_1 = 5 \quad \nu_2 = 6$	B1		both
	$F_{crit} = 5 \cdot 988$	B1		accept 5.60
	$2 \cdot 55 < 5 \cdot 988$			
	Do not reject H_0.			
	It is reasonable to believe that the variances are equal.	A1√	9	√ on F_{calc} and CV
(b)	$H_0 : \mu_x = \mu_y$	B1		both
	$H_1 : \mu_x \neq \mu_y$			
	$\bar{x} - \bar{y} = \frac{145 \cdot 5}{7} - \frac{136 \cdot 3}{6} = -1 \cdot 93$	B1		or $\bar{y} - \bar{x} = 1.93$ cao
	$s_p^2 = \frac{6 \times 0 \cdot 8414 + 5 \times 2 \cdot 1497}{7 + 6 - 2}$	M1		
	$= 1 \cdot 436$	A1		awrt
	DF $\nu = 11$	B1		cao
	$t_{crit} = 2 \cdot 201 \ (2 \cdot 20)$	B1		cao
	$t_{calc} = \frac{(-)1 \cdot 93}{\sqrt{1 \cdot 436\left(\frac{1}{7} + \frac{1}{6}\right)}}$	M1		
	$= (-)2 \cdot 89 \ (2 \cdot 90)$	A1		cao
	Reject H_0. The evidence suggests that that the means are different	A1√	9	√ on t_{calc} and CV
	Total		**18**	
	TOTAL		**75**	

General Certificate of Education
Practice paper
Advanced Level Examination

MATHEMATICS **MM03**
Unit Mechanics 3

In addition to this paper you will require:
• an 8-page answer book;
• the AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.
-

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.
-

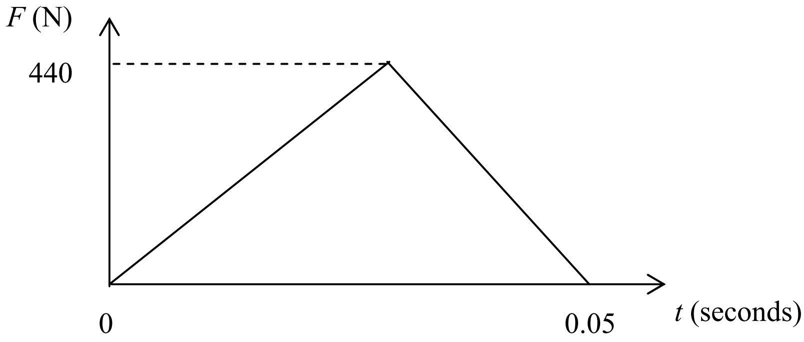
Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet

MM03

Answer **all** questions.

1 A particle has mass 2 kg and moves in a straight line on a smooth horizontal surface. The particle strikes a vertical barrier that is perpendicular to its path and rebounds. The graph below shows how the magnitude of the force on the particle varies while it is in contact with the barrier.



(a) Calculate the magnitude of the impulse on the ball. *(2 marks)*

(b) The ball rebounds at a speed of 3 m s^{-1}. Find the speed of the ball when it hit the barrier. *(3 marks)*

(c) Find the coefficient of restitution between the ball and the barrier. *(1 mark)*

2 The magnitude of the resistance force on a moving body is to be modelled as having magnitude kv^n, where v is the speed of the body and k and n are constants.

(a) If $n = 2$, find the dimensions of k. *(3 marks)*

(b) If the dimensions of k are $\mathrm{ML}^{-\frac{1}{2}}\mathrm{T}^{q}$, find n and q. *(5 marks)*

3 Two particles, A and B, are moving towards each other along a straight, horizontal line. Particle A has mass 13 kg and speed 5 m s^{-1}. Particle B has mass 7 kg and speed 3 m s^{-1}. The coefficient of restitution between the two particles is 0.4. The two particles collide.

(a) Show that the speed of B after the collision is 4.28 m s^{-1}. *(6 marks)*

(b) Find the speed of A after the collision. *(2 marks)*

(c) State, giving a reason for your answer, which of the two particles changes direction as a result of the collision. *(1 mark)*

(d) Calculate the magnitude of the impulse on B during the collision. *(2 marks)*

4 A ball is hit from a point O on a horizontal surface. It initially moves with speed 14 m s^{-1} at an angle α above the horizontal. At time t the horizontal displacement of the ball from O is x metres and the vertical displacement is y metres. Assume that the only force acting on the ball after it has been thrown is gravity.

(a) Show that $y = x\tan\alpha - \dfrac{x^2}{40}\left(1+\tan^2\alpha\right)$. *(5 marks)*

(b) A vertical wall is 10 metres from O. The ball hits the wall at a height of 4 metres. Find the two possible values of α. *(6 marks)*

5 In this question, the unit vectors **i** and **j** are directed east and north respectively.

A ship is initially at the origin and it moves with a constant velocity of $(3\mathbf{i}+8\mathbf{j})\,\mathrm{m\,s^{-1}}$. A boat is initially 2400 m north of the ship and it moves with a constant velocity of $5(a\mathbf{i}+b\mathbf{j})\,\mathrm{m\,s^{-1}}$ where a and b are constants such that $a^2+b^2=1$.

(a) Find the velocity of the boat relative to the ship. *(2 marks)*

(b) Find a and b so that the boat intercepts the ship in the shortest possible time. *(6 marks)*

(c) Find the bearing on which the boat should head to intercept the ship in the shortest possible time. *(2 marks)*

(d) Find the distance of the ship from the origin when the interception takes place. *(5 marks)*

6 A slope is inclined at an angle of 20° below the horizontal. A ball is projected at a speed of 30 m s^{-1} from the slope at an angle of 40° above the slope. The ball moves in a plane that contains the line of greatest slope of the plane.

(a) Find the time of flight of the ball, given that it moves down the slope. *(5 marks)*

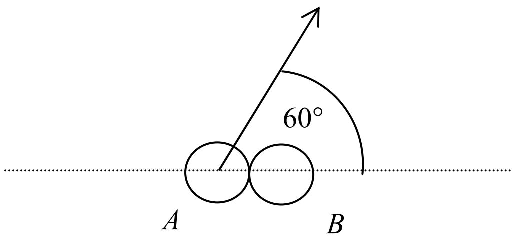
(b) Find the range of the ball. *(4 marks)*

(c) Find the speed of the ball when it hits the slope, giving your answer correct to 2 significant figures. *(4 marks)*

TURN OVER FOR THE NEXT QUESTION

Turn over ►

7 Two smooth spheres, *A* and *B*, have mass *m* and 2*m* respectively. Sphere *A* is moving with a constant velocity of 5 m s^{-1} when it collides with sphere *B*, which was at rest. The velocity of *A* was at an angle of 60° to the line of centres of the sphere when the collision took place. The coefficient of restitution between the two sphere is $\frac{1}{2}$.



(a) Show that the speed of *B* after the collision is $\frac{5}{4}$ m s^{-1}. *(7 marks)*

(b) Find the speed of *A* after the collision. *(4 marks)*

END OF QUESTIONS

Mathematics MM03 Practice paper

Question	Solution	Marks	Total	Comments
1(a)	$I=\frac{1}{2}\times 0.05\times 440=11\text{ N s}$	M1A1	2	
(b)	$11=2\times 3-2(-u)$	M1A1		
	$u=\frac{5}{2}\text{ m s}^{-1}$	A1	3	
(c)	$\frac{5}{2}=3e$			
	$e=\frac{5}{6}$	B1	1	
	Total		**6**	
2(a)	$\text{MLT}^{-2}=[k]\text{L}^2\text{T}^{-2}$	M1A1		
	$[k]=\text{ML}^{-1}$	A1	3	
(b)	$\text{MLT}^{-2}=\text{ML}^{-\frac{1}{2}}\text{T}^q\text{L}^n\text{T}^{-n}$	M1		
	$1=-\frac{1}{2}+n$	M1		
	$n=\frac{3}{2}$	A1		
	$-2=q-n$	M1		
	$q=-\frac{1}{2}$	A1	5	
	Total		**8**	
3(a)	$13\times 5+7\times(-3)=13v_A+7v_B$			
	$44=13v_A+7v_B$	M1A1		
	$v_A-v_B=-0.4(5-(-3))$			
	$v_A-v_B=-3.2$	M1A1		
	$v_A=v_B-3.2$			
	$44=13(v_B-3.2)+7v_B$	M1		
	$v_B=\frac{85.6}{20}=4.28$	A1	6	
(b)	$v_A=4.28-3.2=1.08$	M1A1	2	
(c)	B, as the sign of the velocity changes during the collision.	B1	1	
(d)	$I=7\times 4.28-7\times(-3)=50.96\text{ N s}$	M1A1	2	
	Total		**11**	

Question	Solution	Marks	Total	Comments
4(a)	$x = 14\cos\alpha t$			
	$t = \dfrac{x}{14\cos\alpha}$	M1		
	$y = 14\sin\alpha t - \dfrac{1}{2}gt^2$	M1A1		
	$= 14\sin\alpha \times \dfrac{x}{14\cos\alpha} - \dfrac{1}{2}\times 9.8\left(\dfrac{x}{14\cos\alpha}\right)^2$	M1		
	$= x\tan\alpha - \dfrac{9.8x^2}{2\times 14^2}\left(\sec^2\alpha\right)$			
	$= x\tan\alpha - \dfrac{x^2}{40}\left(1+\tan^2\alpha\right)$	A1	5	
(b)	$4 = 10\tan\alpha - \dfrac{10^2}{40}(1+\tan^2\alpha)$	M1A1		
	$2.5\tan^2\alpha - 10\tan\alpha + 6.5 = 0$	A1		
	$\tan\alpha = 0.817$ or 3.183	M1A1		
	$\alpha = 39.2°$ or $72.6°$	A1	6	
	Total		**11**	
5(a)	$\mathbf{v}_{BS} = 5(a\mathbf{i} + b\mathbf{j}) - (3\mathbf{i} + 8\mathbf{j})$			
	$= (5a-3)\mathbf{i} + (5b-8)\mathbf{j}$	M1A1	2	
(b)	$5a - 3 = 0$	M1		
	$a = \dfrac{3}{5}$	A1		
	$\left(\dfrac{3}{5}\right)^2 + b^2 = 1$	M1A1		
	$b = \dfrac{\pm 4}{5}$	A1		
	Require $b = \dfrac{-4}{5}$ to give maximum velocity south.	A1	6	
(c)	$90 + \tan^{-1}\left(\dfrac{4}{3}\right) = 143°$	M1A1	2	
(d)	$2400 = 12t$	M1		
	$t = \dfrac{2400}{12} = 200$ s	A1		
	$\mathbf{r} = (3\mathbf{i} + 8\mathbf{j}) \times 200 = 600\mathbf{i} + 1600\mathbf{j}$	M1		
	$\lvert\mathbf{r}\rvert = \sqrt{600^2 + 1600^2} = 1710$ m	M1A1	5	
	Total		**15**	

MM03 (cont)

Question	Solution	Marks	Total	Comments
6(a)	$y = 30\sin 40° t - 4.9\cos 20° t^2$ $0 = 30\sin 40° t - 4.9\cos 20° t^2$ $t = 0$ or $t = \frac{30\sin 40°}{4.9\cos 20°} = 4.188\text{ s}$	M1A1 A1 M1 A1	5	
(b)	$x = 30\cos 40° \times 4.188 + 4.9\sin 20° \times 4.188^2$ $= 126\text{ m}$	M1A1 A1 A1	4	
(c)	$v_x = 30\cos 40° + 9.8\sin 20° \times 4.188$ $= 37.02$ $v_y = 30\sin 40° - 9.8\cos 20° \times 4.188$ $= -19.28$ $v = \sqrt{37.02^2 + (-19.28)^2} = 42\text{ m s}^{-1}$	M1A1 A1 A1	4	
	Total		**13**	
7(a)	$5m\cos 60° = m \times v_A \cos\alpha + 2mv_B$ $\frac{5}{2} = v_A \cos\alpha + 2v_B$ $v_A \cos\alpha - v_B = -\frac{1}{2}(5\cos 60°)$ $v_A \cos\alpha = v_B - \frac{5}{4}$ $\frac{5}{2} = 3v_B - \frac{5}{4}$ $v_B = \frac{15}{12} = \frac{5}{4}$	M1 A1 M1A1 M1A1 A1	7	
(b)	$v_A \sin\alpha = 5\sin 30° = \frac{5\sqrt{3}}{2}$ $v_A \cos\alpha = \frac{5}{4} - \frac{5}{4} = 0$ $v_A = \frac{5\sqrt{3}}{2} = 4.33\text{ m s}^{-1}$	B1 M1 M1A1	4	
	Total		**11**	
	TOTAL		**75**	

General Certificate of Education
Practice paper
Advanced Level Examination

MATHEMATICS **MM04**
Unit Mechanics 4

In addition to this paper you will require:
• an 8-page answer book;
• the AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM04.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.
-

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.
-

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet

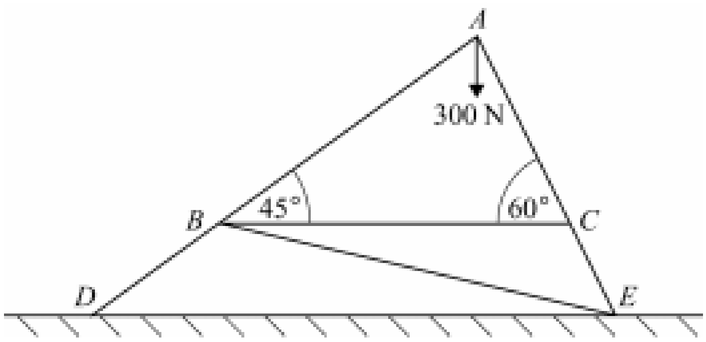
MM04

Answer **all** questions.

1 A force, **P**, 3**i** – 6**j** + 8**k**, acts at *B* on a light rod *AB*. *A* is at the point whose co-ordinates are (5, 4, 9) and *B* is at the point whose co-ordinates are (2, 5, -3). The three unit vectors **i, j** and **k** are mutually perpendicular and in the direction of the *x, y* and *z* axes.

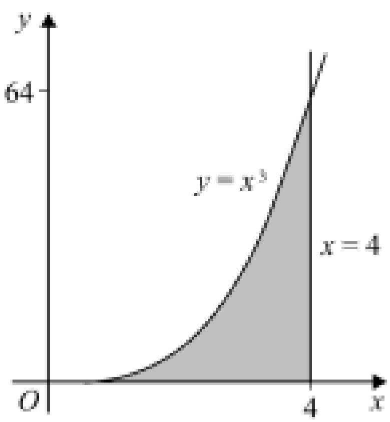
Find the moment of the force **P** about the point *A*. *(4 marks)*

2 A framework is composed of seven light, inextensible, smoothly jointed rods, *AB, AC, BD, BE, CE, DE* and *BC* as shown in the diagram below.



The framework stands, in a vertical plane, on rough horizontal ground. A load of 300 N is hung at *A*. The framework remains in equilibrium with *BC* horizontal. By considering the forces acting at *A*, find the forces in each of the rods *AB* and *AC*. *(7 marks)*

3 A uniform lamina is bounded by the curve $y = x^3$, the line $x = 4$ and the x-axis.



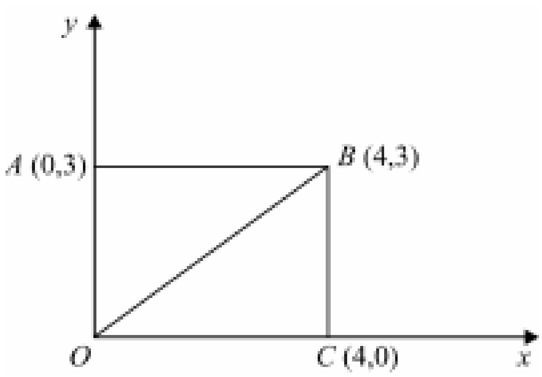
(a) Find the area of the lamina. *(2 marks)*

(b) Use integration to show that the x-coordinate of the centre of mass of the lamina is 3.2 . *(4 marks)*

(c) Find the y-coordinate of the centre of mass of the lamina. *(4 marks)*

(d) The lamina is suspended in equilibrium from its right-angled corner. Find the angle between the longer of the two straight sides of the lamina and the vertical. *(4 marks)*

4 The points *O, A, B* and *C* have co-ordinates (0, 0), (0, 3), (4, 3) and (4, 0) respectively.



A clockwise couple of magnitude 19 N m acts in the plane together with forces of magnitudes 5 N, 6 N, 4 N, 7 N, and 5 N acting along *OA, BA, CB, OC* and *OB* respectively.

(a) Show that the resultant of this system of forces and the couple is of magnitude 13 N. *(5 marks)*

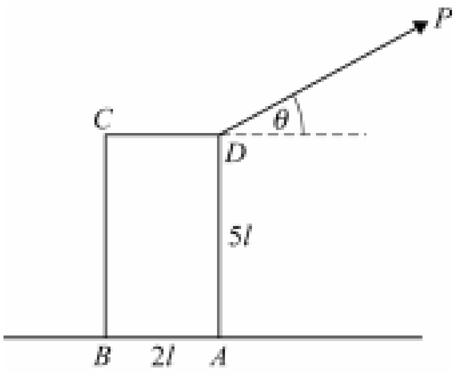
(b) (i) Show that the line of action of the resultant cuts the y–axis at (0, –3) . *(5 marks)*

(ii) Find the equation of the line of action of the resultant. *(3 marks)*

Turn over ►

5 A uniform solid cuboid of mass M is placed on a rough horizontal floor. The cuboid has a square base of side $2l$ and a height of $5l$.

A force, P, which is gradually increasing, is applied to the mid point of, and perpendicular to, a top edge.



This force acts as shown in the diagram where $ABCD$ is a vertical cross section through the centre of mass of the cuboid.

The force, P, makes an angle θ with the horizontal.

The coefficient of friction between the block and the rough horizontal floor is μ.

(a) Show that the block is on the point of sliding when

$$P = \frac{\mu M g}{\cos\theta + \mu \sin\theta}$$

(6 marks)

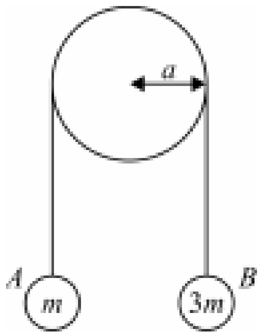
(b) Find P when the block is on the point of toppling. *(4 marks)*

(c) Given that $\tan\theta = \frac{1}{7}$, find an inequality that μ must satisfy if the block slides before it topples. *(5 marks)*

6 A uniform circular disc of radius a can rotate freely in a vertical plane about a fixed horizontal axis through its centre and perpendicular to its plane. The moment of inertia of the disc about this axis is $5ma^2$.

A light inextensible string passes over the rough rim of the disc and two particles A and B, of masses m and $3m$ respectively, are attached to its ends.

Assume that in the subsequent motion the string does not slip around the disc.



Initially the system is at rest with the particles hanging freely in equilibrium. The system is then released and after time t the wheel has turned through an angle θ.

In the subsequent motion, the particle at A remains below the disc and no slipping occurs between the string and the disc.

(a) Explain why the speed of the particles is $a\dot{\theta}$. *(1 mark)*

(b) By conservation of energy, or otherwise, show that

$$a\dot{\theta}^2 = \tfrac{4}{9}g\theta$$ *(8 marks)*

7 A body is composed of a uniform wire and three particles. The wire of length $6l$ and mass $3m$ is bent to form an equilateral triangle ABC. The three particles of masses $2m$, $4m$ and $4m$ are fixed at the vertices A, B and C respectively. The body can rotate in a vertical plane about a horizontal axis through A perpendicular to the triangle.

(a) Show that the moment of inertia of the section BC of the wire about the axis is $\tfrac{10}{3}ml^2$. *(4 marks)*

(b) Hence show that the moment of inertia of the body about the axis is $38ml^2$. *(4 marks)*

(c) The body is released from rest with BC horizontal and above A. Find the maximum angular velocity of the body in the subsequent motion. *(5 marks)*

END OF QUESTIONS

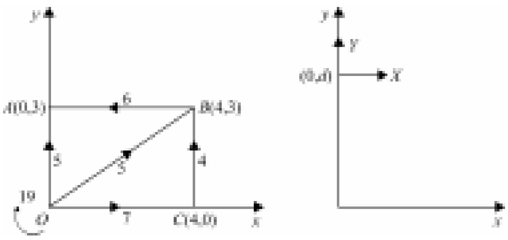
Mathematics MM04 Practice paper

Question	Solution	Marks	Total	Comments
1	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} =$			
	$= \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \\ 9 \end{pmatrix}$			
	$= \begin{pmatrix} -3 \\ 1 \\ -12 \end{pmatrix}$	B1		
	Moment is $\mathbf{r} \times \mathbf{F}$			
	$= \begin{pmatrix} -3 \\ 1 \\ -12 \end{pmatrix} \times \begin{pmatrix} 3 \\ -6 \\ 8 \end{pmatrix}$			
	$= \begin{vmatrix} i & j & k \\ -3 & 1 & -12 \\ 3 & -6 & 8 \end{vmatrix}$	M1 A1		
	$= -64\mathbf{i} - 12\mathbf{j} + 15\mathbf{k}$	A1		
	Total		**4**	
2	Resolve vertically at *A*	M1		
	$T_{AB}\cos 45 + T_{AC}\cos 30 = 300$	A1		
	Resolve horizontally at A	M1		
	$T_{AB}\sin 45 = T_{AC}\sin 30$	A1		
	$\frac{T_{AC}}{2}(1+\sqrt{3}) = 300$	M1		
	$T_{AC} = \frac{600}{1+\sqrt{3}} = 220$ N	A1		Accept 219.6
	$T_{AB} = \frac{T_{AC}}{\sqrt{2}}$			
	$T_{AB} = \frac{300\sqrt{2}}{1+\sqrt{3}} = 155$ N	A1		
	Total		**7**	

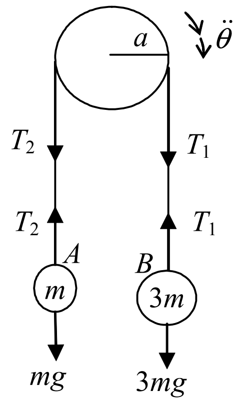
MM04 (cont)

Question	Solution	Marks	Total	Comments
3(a)	$\int_0^4 x^3 \mathrm{d}x = \left[\frac{x^4}{4}\right]_0^4 = 64$	M1		Integrating x^3
		A1	2	cao
(b)	$64\bar{x} = \int_0^4 x^4 \mathrm{d}x = \left[\frac{x^5}{5}\right]_0^4$	M1		Integrating x^4
		A1		Obtaining $\frac{x^5}{5}$
	$64\bar{x} = \frac{1024}{5}$	M1		Finding $\bar{x}$
	$\bar{x} = 3.2$	A1	4	Correct answer from correct working
(c)	$64\bar{y} = \int_0^4 \frac{1}{2} x^6 \mathrm{d}x = \left[\frac{x^7}{14}\right]_0^4$	M1		Integrating $\frac{1}{2}x^6$
		A1		Obtaining $\frac{x^7}{14}$
	$64\bar{y} = \frac{16384}{14}$	M1		Finding $\bar{y}$
	$\bar{y} = 18.3$	A1	4	Correct answer (awrt 18.3)
(d)	$\tan\theta = \frac{4 - 3.2}{18.286}$	M1		Using tan with a fraction
		A1		Numerator
	$\theta = 2.51°$	A1√		ft Denominator
		A1√	4	ft Angle
	Total		**14**	

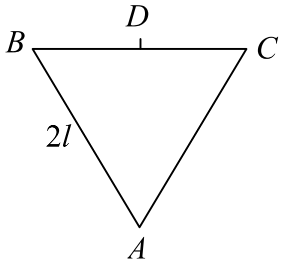
MM04 (cont)

Question	Solution	Marks	Total	Comments
4				
(a)	$X = 7 - 6 + 5\cos\theta$	M1A1		Correct at this stage
	$= 1 + 5 \times \frac{4}{5}$			
	$= 5$			
	$Y = 4 + 5 + 5\sin\theta$	M1		
	$= 9 + 5 \times \frac{3}{5}$			
	$= 12$	A1		(both X and Y correct)
	$\therefore$ Resultant $= \sqrt{5^2 + 12^2}$			
	$= 13$	A1	5	cao
(b)(i)	$Xd = -4 \times 4 - 6 \times 3 + 19$	M1A1		(for Xd)
	$5d = -15$	A1		1st 2 terms RHS
	$d = -3$	A1		(+ 19)
	$\therefore$ line cuts axis at $(0, -3)$	A1	5	cao
(ii)	Gradient line of action + $\frac{Y}{X} = \frac{12}{5}$	M1 A1F		ft from (a)
	$\therefore y = \frac{12}{5}x - 3$	A1F	3	
	(or any acceptable equivalent **e.g $5y = 12x - 15$ etc)**			
	Total		**13**	

MM04 (cont)

Question	Solution	Marks	Total	Comments
5(a)	Resolve vertically $R = Mg - P\sin\theta$	M1		
		A1		
	Resolve horizontally $F = P\cos\theta$	B1		
	On point of sliding $F = \mu R$	B1		
	$\mu(Mg - P\sin\theta) = P\cos\theta$	M1		
	$P = \dfrac{\mu Mg}{\cos\theta + \mu\sin\theta}$	A1	6	
(b)	Taking moments about A	M1		
	$5l \times P\cos\theta = l \times W$	A1		
		A1		
	$P = \dfrac{Mg}{5\cos\theta}$	B1	4	
(c)	If slides before it topples,			
	$\dfrac{\mu Mg}{\cos\theta + \mu\sin\theta} < \dfrac{Mg}{5\cos\theta}$	M1		
	$5\mu\cos\theta < \cos\theta + \mu\sin\theta$	M1		
	$5\mu < 1 + \mu\tan\theta$	A1		
	$\tan\theta = \frac{1}{7}$, $5\mu < 1 + \frac{1}{7}\mu$	M1		
	$\mu < \frac{7}{34}$	A1	5	
	Total		**15**	
6(a)	Since string does not slip, speed of particle is the same as the speed of the rim of the disc			
	$\therefore$ speed of particle is $a\dot{\theta}$	B1	1	
(b)				
	By conservation of energy			
	$\frac{1}{2}.3mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}.5ma^2\dot{\theta}^2$	M1 A1A1		
	$= 3mga\theta - mga\theta$	A1		
	Using $v = a\dot{\theta},\ x = a\theta$			
	$\Rightarrow 2ma^2\dot{\theta}^2 + \frac{5}{2}ma^2\dot{\theta}^2 = 2mga\theta$	M1A1		
	$\frac{9}{2}a\dot{\theta}^2 = 2g\theta$			
	$\therefore a\dot{\theta}^2 = \frac{4}{9}g\theta$	A1	8	
	Total		**9**	

MM04 (cont)

Question	Solution	Marks	Total	Comments
7(a)				
	Each of *AB, BC, AC* has mass *m* and length $2l$			
	M of I of *BC* about axis through *D* is $\frac{1}{3}ml^2$	B1		
	$AD = \sqrt{3}l$	B1		
	By parallel axis theorem, M of I of *BC* about axis through *A* is $\frac{1}{3}ml^2 + m\left(\sqrt{3}l\right)^2$	M1		
	$= \frac{10}{3}ml^2$	A1	4	
(b)	M of I of *AB* about axis through *A* is $\frac{4}{3}ml^2$ M of I of *AC* is also $\frac{4}{3}ml^2$	B1		for either
	M of I of system is $\frac{4}{3}ml^2 + \frac{4}{3}ml^2 + \frac{10}{3}ml^2$ $+ 4m(2l)^2 + 4m(2l)^2$	B1 M1		3 rods or 2 particles all 5 parts
	$= 38ml^2$	A1	4	
(c)	Using conservation of energy			
	$\frac{1}{2}.38ml^2\omega^2 = 4m.2\sqrt{3}gl + 4m2\sqrt{3}gl$ $+ m\sqrt{3}gl + m\sqrt{3}gl + m2\sqrt{3}gl$	M1 A1 A1		A1 left, A1 right
	$19ml^2\omega^2 = 20\sqrt{3}gl$	m1		dependent on first M1
	$\omega = \sqrt{\frac{20\sqrt{3}g}{19l}}$	A1	5	
	Total		**13**	
	TOTAL		**75**	

General Certificate of Education
Practice paper
Advanced Level Examination

MATHEMATICS **MM05**
Unit Mechanics 5

In addition to this paper you will require:
• an 8-page answer book;
• the AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM05.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8$ m s $^{-2}$, unless stated otherwise.
-

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.
-

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet

MM05

1 A particle moves with simple harmonic motion on a straight line between two points A and B which are 0.4 metres apart. The maximum speed of the particle is 10 m s^{-1}

(a) Show that the period of the motion is $\frac{\pi}{25}$ seconds. *(4 marks)*

(b) Find the speed of the particle when it is 0.04 metres from A. *(3 marks)*

(c) The distance, s , of the particle from A at time t is given by

$$s = p - q\cos(\omega t)$$

where ω, p and q are constants.

(i) State the values of ω and q. *(2 marks)*

(ii) When $t = 0$, the particle is at A. Find the value of p. *(2 marks)*

2 The polar coordinates of a particle at time t are

$$r = 2t^3 + 4 \quad \text{and} \quad \theta = 24\sin\left(\frac{t\pi}{4}\right).$$

Find the radial and transverse components of:

(a) the velocity of the particle when $t = 2$; *(5 marks)*

(b) the acceleration of the particle when $t = 2$. *(5 marks)*

3 A hailstone falls vertically under gravity through still air. As it falls, water vapour from the surrounding still air condenses on the hailstone causing its mass to increase. The hailstone is modelled as a uniform sphere, and at time t it has mass m and radius r.

(a) Given that $\frac{dr}{dt} = \lambda r$, where λ is a positive constant, show that

$$\frac{dm}{dt} = 3\lambda m$$ *(4 marks)*

(b) Assume that the only external force acting on the hailstone is gravity. If the speed of the hailstone at time t is v, show that

$$\frac{dv}{dt} = g - 3\lambda v$$ *(4 marks)*

(c) If the initial speed of the hailstone is u, show that

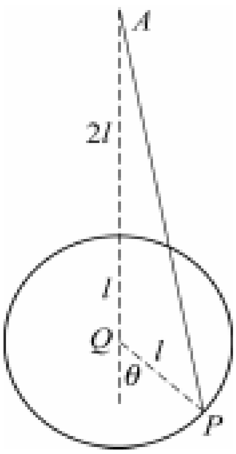
$$v = \frac{g}{3\lambda} - \frac{(g - 3\lambda u)e^{-3\lambda t}}{3\lambda}$$ *(7 marks)*

(d) Hence show that the limiting value of v is $\frac{g}{3\lambda}$. *(2 marks)*

4 A particle, P, of mass 3m can move freely around a smooth circular ring of radius l and centre Q. The circular ring is in a vertical plane.

The particle is attached by a light elastic string, of natural length $2l$ and modulus of elasticity $4mg$, to a fixed point A, where A is a vertical distance $3l$ above Q.

The radius PQ makes an angle θ with the downward vertical.



(a) Show that the potential energy , V, of the system may be given by

$$V = mgl(\sqrt{10 + 6\cos\theta} - 2)^2 - 3mgl\cos\theta \ + \text{ constant}$$ *(5 marks)*

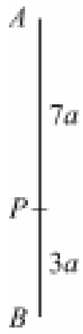
(b) Show that

$$\frac{dV}{d\theta} = \frac{3mgl\sin\theta}{\sqrt{10 + 6\cos\theta}}\left(4 - \sqrt{10 + 6\cos\theta}\right)$$ *(6 marks)*

(c) Hence find the values of θ for which the system is in equilibrium. *(3 marks)*

Turn over ►

5 A spring of natural length $4a$ and modulus λ has one end attached to a fixed support A, and a particle P of mass m is attached to its other end. Another spring of natural length $2a$ and modulus $4mg$ has one end attached to P and the other end attached to a fixed support B which is situated at a distance of $10a$ vertically below A. The system is in equilibrium in a vertical line with the upper spring stretched to a length of $7a$ and the lower spring stretched to a length $3a$ as shown in the diagram.



(a) Show that $\lambda = 4mg$. *(4 marks)*

(b) At time $t = 0$, the particle is lowered to a distance $\frac{a}{2}$ below its equilibrium position and released from rest. The subsequent motion of P is subject to a resistance of magnitude $\frac{1}{5}mk\,v$, where

$k^2 = \frac{6g}{a}$ and v is the speed of the particle at time t.

(i) Given that x is the downward displacement of P from its equilibrium position at time t, show that

$$10\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 5k^2x = 0$$ *(6 marks)*

(ii) Hence find x in terms of a, k and t. *(10 marks)*

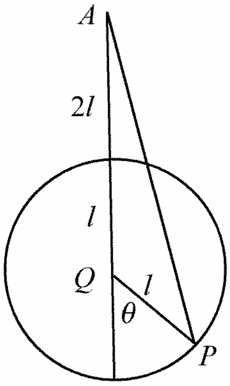
(iii) Is the damping of the motion of the particle light, critical or heavy? Give a reason for your answer. *(3 marks)*

END OF QUESTIONS

Question	Solution	Marks	Total	Comments
1(a)	$a = 0.2$	B1		Stating amplitude
	$0.2\omega = 10$	M1		Using $v = a\omega$
	$\omega = 50$	A1		Correct value of ω
	$P = \frac{2\pi}{50} = \frac{\pi}{25}$	A1	4	Correct period from correct working
(b)	$v = 50\sqrt{0.2^2 - 0.16^2}$	M1		Using $x = 0.16$ in SHM formula
		A1		Correct substitution of all values
	$= 6\,\text{m}\,\text{s}^{-1}$	A1	3	Correct speed
(c)(i)	$\omega = 50$, $q = 0.2$	B1		Correct ω
		B1	2	Correct q
(ii)	$0 = p - 0.2\cos 0$	M1		Using $s = 0$
	$p = 0.2$	A1	2	Correct p
	Total		**11**	
2(a)	$\dot{r} = 6t^2$	B1		
	$\dot{\theta} = 6\pi\cos\frac{t\pi}{4}$	B1		
	$\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\,\hat{\boldsymbol{\theta}}$	M1		
	when $t = 2$, $r = 20$			
	$\dot{r} = 24$, $\dot{\theta} = 0$	A1		
	$\therefore$ Components of velocity are 24 radially, 0 transversely	A1	5	
(b)	$\ddot{r} = 12t$	B1		
	$\ddot{\theta} = -\frac{3\pi^2}{2}\sin\frac{t\pi}{4}$	B1		
	when $t = 2$, $\ddot{r} = 24$, $\ddot{\theta} = -\frac{3}{2}\pi^2$	A1		
	$\ddot{\mathbf{r}} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathbf{r}} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\hat{\boldsymbol{\theta}}$	M1		
	Components of acceleration are 24 radially, $-30\pi^2$ transversely	A1	5	
	Total		**10**	

Question	Solution	Marks	Total	Comments
3 (a)	$\frac{dm}{dt}=\frac{dm}{dr}\cdot\frac{dr}{dt}$	M1		
	$=4\pi r^2\rho\times\lambda r$	B1 A1F		Using $m=\frac{4}{3}\pi r^3\rho$
	$=4\pi\rho\lambda\times\frac{3m}{4\pi\rho}$ $=3m\lambda$	A1	4	cao
(b)	Change in momentum=impulse of external force	B1		
	$(m+\delta m)(v+\delta v)-mv=(m+\delta m)g\delta t$	M1		
	As $\delta t\to 0$ $mv+m\delta v+v\delta m-mv=mg\delta t$	A1		
	$m\frac{dv}{dt}+v\frac{dm}{dt}=mg$			
	$m\frac{dv}{dt}+3mv\lambda=mg$			
	$\frac{dv}{dt}=g-3\lambda v$	A1	4	cao
	Alternative to part (b)			
	Change in momentum = Impulse	(B1)		
	$\frac{d}{dt}(mv)=mg$	(M1)		
	$m\frac{dv}{dt}+v\frac{dm}{dt}=mg$	(A1)		
	$\frac{dv}{dt}=g-3\lambda v$	(A1)	(4)	
(c)	$\int\frac{dv}{g-3\lambda v}=\int dt$	M1		
	$-\frac{1}{3\lambda}\ln(g-3\lambda v)=t+c$	A1		For ln form
	$t=0$, $v=u$ $c=-\frac{1}{3\lambda}\ln(g-3\lambda u)$	m1 A1F		
	$t=\frac{1}{3\lambda}\ln\frac{g-3\lambda u}{g-3\lambda v}$	m1 A1		Attempting ln form
	$v=\frac{g}{3\lambda}-\frac{g-3\lambda u}{3\lambda}e^{-3\lambda t}$	A1	7	cao
(d)	As $t\to\infty$, $e^{-3\lambda t}\to 0$	M1		
	Therefore $v\to\frac{g}{3\lambda}$	A1	2	Printed result
	Total		**17**	

MM05 (cont)

Question	Solution	Marks	Total	Comments
4 (a)				
	$AP^2 = (3l)^2 + l^2 + 2.3l.\,l\cos\theta$	M1		(use of cosine rule)
	$= 10l^2 + 6l^2\cos\theta$			
	$\therefore$ Extension is $l\sqrt{10+6\cos\theta} - 2l$	A1		
	EPE is $\dfrac{\lambda x^2}{2l}$	M1		dep
	$= \dfrac{4mgl}{4}\left\{(10+6\cos\theta)^{\frac{1}{2}} - 2\right\}^2$	A1		
	P.E. of particle below Q is $-3mgl\cos\theta$			
	$\therefore V = mgl\left(\sqrt{10+6\cos\theta} - 2\right)^2 - 3mgl\cos\theta$	A1	5	
(b)	$\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = mgl.2.\dfrac{1}{2}\cdot\dfrac{-6\sin\theta}{\sqrt{10+6\cos\theta}}\left(\sqrt{10+6\cos\theta} - 2\right)$ $+ 3mgl\sin\theta$	M1 A1 B1		B1 $(3mgl\sin\theta)$
	$= -mgl6\sin\theta + mgl\dfrac{12\sin\theta}{\sqrt{10+6\cos\theta}} + 3mgl\sin\theta$	M1		
	$= 12mgl\dfrac{\sin\theta}{\sqrt{10+6\cos\theta}} - 3mgl\sin\theta$	M1		
	$= \dfrac{3mgl\sin\theta}{\sqrt{10+6\cos\theta}}\left(4 - \sqrt{10+6\cos\theta}\right)$	A1	6	
(c)	When in equilibrium, $\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = 0$	B1		
	$\therefore \sin\theta = 0$ or $4 - \sqrt{10+6\cos\theta} = 0$			
	$16 = 10 + 6\cos\theta$			
	$\cos\theta = 1$	M1		
	$\therefore \theta = 0,\ \pi$ (or 2π)	A1	3	
	Total		**14**	

Question	Solution	Marks	Total	Comments
5 (a)	$T_{AP} = \dfrac{\lambda.3a}{4a} = \dfrac{3}{4}\lambda$ $T_{PB} = 4mg.\dfrac{a}{2a} = 2mg$	B1		Either
	Using $F = ma$ vertically $mg + T_{PB} = T_{AP}$	M1		
	$\therefore mg + 2mg = \dfrac{3}{4}\lambda$	A1		
	$\lambda = 4mg$	A1	4	
(b) (i)	When particle is moved a distance x below the equilibrium position, forces acting on it are			
	mg, $T_{AP} = \dfrac{\lambda.(3a+x)}{4a} = \dfrac{mg(3a+x)}{a}$,	M1		
	$T_{PB} = 4mg.\dfrac{(a-x)}{2a} = \dfrac{2mg}{a}(a-x)$			
	and resistance $\frac{1}{5}mk\dot{x}$	M1		All four forces
	[forces 2 and 4 are upwards] Using $F = ma$ vertically downwards			
	$m\ddot{x} = mg + T_{PB} - T_{AP} - \frac{1}{5}mk\dot{x}$	m1		Dependent on both M1 above
	$m\ddot{x} =$ $mg + \dfrac{2mg}{a}(a-x) - \dfrac{mg(3a+x)}{a} - \frac{1}{5}mk\dot{x}$	A1		
	$\ddot{x} - g - \dfrac{2g}{a}(a-x) + \dfrac{g(3a+x)}{a} + \frac{1}{5}k\dot{x} = 0$			
	$\ddot{x} + \frac{1}{5}k\dot{x} + \dfrac{3gx}{a} = 0$	A1		
	$10\dfrac{d^2x}{dt^2} + 2k\dfrac{dx}{dt} + 5k^2x = 0$	A1	6	

MM05 (cont)

Question	Solution	Marks	Total	Comments
(b) (ii)	Substituting $x = Ae^{nt}$, $10n^2 + 2kn + 5k^2 = 0$			
	$n = \dfrac{-2k \pm \sqrt{4k^2 - 200k^2}}{20}$	M1		
	$= \frac{1}{10}(-k \pm 7k\mathrm{i})$	A1		
	$x = \mathrm{e}^{-\frac{k}{10}t}(A\cos\frac{7}{10}kt + B\sin\frac{7}{10}kt)$	M1 A1√		
	When $t = 0$, $x = \dfrac{a}{2}$, $A = \dfrac{a}{2}$	B1		
	Differentiating			
	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = -\dfrac{k}{10}\mathrm{e}^{-\frac{k}{10}t}(A\cos\frac{7}{10}kt + B\sin\frac{7}{10}kt)$			
	$+ \ \mathrm{e}^{-\frac{k}{10}t}(-\frac{7}{10}kA\sin\frac{7}{10}kt + \frac{7}{10}kB\cos\frac{7}{10}kt)$	M1 A1√ A1√		
	When $t = 0$, $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 0$, $\quad 0 = -\dfrac{k}{10}A + \dfrac{7}{10}kB$	M1		
	$B = \dfrac{a}{14}$			
	$x = \dfrac{a}{14}\mathrm{e}^{-\frac{k}{10}t}(7\cos\frac{7}{10}kt + \sin\frac{7}{10}kt)$	A1	10	
(iii)	*The damping is light damping* since the motion is oscillating with the amplitude reducing to zero	B1 B1 B1	3	
	Total		**23**	
	TOTAL		**75**	