

Maths 4th Edition

GCSE Foundation Student Book



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Ratio, speed and proportion

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Skill focus key

PS

- 飏 🛛 mathematical reasoning
- 😡 | communicate mathematically
 - problem-solving and making connections
- valuate and interpret

This chapter is going to show you:

- what a ratio is
- how to solve problems involving direct proportion
- how to compare prices of products
- how to calculate speed
- how to divide an amount according to a given ratio.

You should already know:

- multiplication tables up to 12 × 12
- · how to simplify fractions
- how to find a fraction of a quantity
- how to multiply and divide, with and without a calculator.

About this chapter

This chapter is about comparing pieces of information. You use fractions, decimals, percentages, ratio and proportion in your everyday life to help calculate quantities or to compare two or more pieces of information.

Shops and supermarkets always seem to have sales and offers. Every time you go shopping you see signs such as: '3 for 2', '2 for £3.00', 'Buy one, get one free', 'save 25%' and 'A third off today only!'. If you do not have a good grasp of fractions, ratio and proportion then you cannot appreciate if these offers are worth it. For example, which is the best bargain? '3 for the price of 2' or 'A third off the price of 1'. In fact they are the same.

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7.1 Ratio

This section will show you how to:

- simplify a ratio
- express a ratio as a fraction
- divide amounts according to ratios
- complete calculations from a given ratio and partial information.

ratio
simplify

A ratio is a way of comparing the sizes of two or more quantities.

A ratio can be expressed in a number of ways. For example, if you mix 5 centilitres of cordial with 20 centilitres of water, the ratio of the quantities is:

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	cordial	:	water	
which is:	5	:	20	
which simplifies to:	1	:	4	Dividing both sides by 5.

Hints and tips When you say a ratio, you do not say '5 colon 20' or '1 colon 4', you say '5 to 20' or '1 to 4'.

When you are comparing ratios, you may find it helpful to use a table.

So if the ratio is 5 : 20 you can summarise the quantities of cordial and water in a table like this one.

Cordial	5	1	2	4	10	25
Water	20	4	8	16	40	100

The value in each column is simply a multiplier or divisor of the value in a previous column.

This method is useful if you want to know how much cordial to mix with a litre (100 centilitres) of water. The last column shows that the answer is 25 centilitres.

Also, how much water would you need if you only have 15 centilitres of cordial? You can find out by adding the numbers in the first and fifth columns, so the answer is 60 centilitres.

Common units

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When working with a ratio involving different units, always convert them all to a **common unit**. You can only simplify ratio when the units of each quantity are the same, because the ratio itself has no units.

Once the units are the same, the ratio can be cancelled out and then the remaining ratio can be **simplified**.

For example, you must convert the ratio of 125 g to 2 kg to the ratio of 125 g to 2000 g, so that you can simplify it.

	125	:	2000
Divide both sides by 25.	5	:	80
Divide both sides by 5.	1	:	16

The ratio 125 : 2000 can be simplified to 1 : 16.

When a ratio has been simplified so that its parts do not have any common factors, it is in its **simplest form**.

Express 25 minutes : 1 hour as a ratio in its simplest form. Example The units must be the same, so change 1 hour into 60 minutes.

25 minutes : 1 hour = 25 minutes : 60 minutes

= 25 : 60 Cancel the units (minutes).

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= 5 : 12 Divide both sides by 5.

So, 25 minutes : 1 hour simplifies to 5 : 12.

Ratios as fractions

You can express ratios as fractions by using the total of the ratios as the denominator (bottom number) of each fraction. Then use the numbers in the ratio as the numerators. If the ratio is in its simplest form, the fractions will not cancel.

Always simplify a ratio before converting it to fractions.

- A garden is divided into lawn and shrubs in the ratio 3 : 2.
- What fraction of the garden is covered by: **a** lawn **b** shrubs?
- Example The denominator (bottom number) of the fraction comes from adding the number in the ratio (so 2 + 3 = 5).
 - a The lawn covers $\frac{3}{5}$ of the garden.
 - b The shrubs cover $\frac{2}{5}$ of the garden.

Exercise 7A

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	Express	each	ratio	in its	simp	lest for	n.
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a 6:18	b 16:24	c 20 to 50	d 25 to 40
e 15:10	f 28 to 12	g 0.5 to 3	h 2.5 to 1.5

Write each ratio of quantities in its simplest form.

Remember: Always express both parts in a common unit before you simplify.

a £5 to to 1 day	b 125 g to 300 g	c 34 kg to 30 kg
d 3 kg to 750 g	e 1 hour to 1 day	f 1.25 kg : 500 g
g 4 weeks : 14 days	h 465 mm : 3 m	

A length of wood is cut into two pieces in the ratio 3 : 7. What fraction of the original length is the longer piece?

Jack and Thomas find a bag of marbles. They share them in the ratio of their ages. Jack is 10 years old and Thomas is 15 years old. What fraction of the marbles did Jack get?

Dave and Sasha share a pizza in the ratio 2 : 3. They eat it all.

- **a** What fraction of the pizza did Dave eat?
- **b** What fraction of the pizza did Sasha eat?

Amy gets $\frac{2}{3}$ of a packet of sweets. Her sister Susan gets the rest. Work out the ratio of sweets that each sister gets. Write it in the form Amy : Susan.

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7 Ratio, speed and proportion

	7	a The recipe for a fruit punch is 1.25 litres of fruit crush to 6.75 litres of lemonade. What fraction of the punch is made up by each ingredient?
		b How much fruit crush will you need to mix with 2 litres of lemonade?
		Hints and tips Set up a table.
		c You have half a litre of fruit crush. How much lemonade will you need?
	8	The recipe for a pudding is 125 g of sugar, 150 g of flour, 100 g of margarine and 175 g of fruit. What fraction of the pudding is made up by each ingredient?
MR	9	Andy plays 16 bowls matches. He wins $\frac{3}{4}$ of them. He plays another <i>x</i> matches and wins them all. The ratio of wins to losses is now 4 : 1. Work out the value of <i>x</i> .
MR	10	Three brothers share some money. The ratio of Mark's share to David's share is 1 : 2. The ratio of David's share to Paul's share is 1 : 2. What is the ratio of Mark's share to Paul's share?
PS	11	Three brothers, Jarek, Jerzy and Justyn, share a block of chocolate in the ratio of their ages. Jarek gets half of the bar. Jerzy gets $\frac{3}{5}$ of the rest. a What ratio, Jarek : Jerzy : Justyn, of the bar of chocolate does each brother get? b Justyn is 8 years old. How old is Jarek?
PS	12	Three cows, Gertrude, Gladys and Henrietta, produced milk in the ratio $2:3:4$. Henrietta produced $1\frac{1}{2}$ litres more than Gladys. How much milk did the three cows produce altogether?

Dividing amounts in a given ratio

To divide an amount in a given ratio, you first look at the ratio to see how many parts there are altogether.

For example, 4:3 has 4 parts and 3 parts giving 7 parts altogether.

The whole amount is 7 parts.

You can work out 1 part by dividing the whole amount by 7.

Then you can work out 3 parts and 4 parts from 1 part.

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Divide £28 in the ratio 4 : 3.

Method 1: Using a table

Set up the first column then continue the columns as multiples.

First part	4	8	12	16
Second part	3	6	9	12
Total	7	14	21	28

So £28 divided in the ratio 4:3 is £16 : £12.

Method 2: Division to find one part 4 + 3 = 7 parts altogether. So 7 parts = £28. 1 part = £4 Dividing by 7. 4 parts = $4 \times £4 = £16$ 3 parts = $3 \times £4 = £12$ So £28 divided in the ratio 4 : 3 is £16 : £12.

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You can also use fractions to divide an amount in a given ratio. First, express the whole numbers in the ratio as fractions with the same common denominator. Then multiply the amount by each fraction.

Method 2: Using fractions

Change the ratio to fractions.

So Peter receives $\pounds 40 \times \frac{2}{5} = \pounds 16$ and Hitan receives $\pm 40 \times \frac{3}{5} = \pm 24$.

Peter's share = $\frac{2}{(2+3)} = \frac{2}{5}$

Hitan's share = $\frac{3}{(2+3)} = \frac{3}{5}$

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Example

Divide £40 between Peter and Hitan in the ratio 2 : 3.

Method 1: Using a table

Set up the first column then work out what you need to multiply 5 by, to get 40.

Do the same thing to the other rows.

Peter	2	$2 \times 8 = 16$
Hitan	3	$3 \times 8 = 24$
Total	5	$5 \times 8 = 40$

So Peter receives $2 \times 8 = \pm 16$

and Hitan receives $3 \times 8 = \pounds 24$

Exercise 7B

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	1	Divide eh amount accordir	ng to the given ratio.				
		a 400 g in the ratio $2:3$	b 280 kg in the ratio 2 : 5	c 5 hours in the ratio 7 : 5			
		d £100 in the ratio 2 : 3 : 5	e £240 in the ratio 3 : 5 : 12	2 f 600 g in the ratio 1:5:6			
	2	The ratio of female member number of members of the	ers to males at Lakeside Gard group is 250.	dening Club is 7 : 3. The total			
		a How many members are	e female?				
		b What percentage of mer	mbers are male?				
	3	A supermarket aims to stoe They stock 500 kg of break	ck branded goods and their fast cereal.	own goods in the ratio 2 : 3.			
		a What percentage of the	cereal stock is branded?				
		b How much of the cereal	stock is their own?				
	4	Over the years 1981–1992, the Illinois Department of Health tested a total of 357 horses for rabies. They reported that the ratio of horses with rabies to those without was 1 : 16.					
	•	How many of these horses	had rabies?				
	5	Rewrite each of these scale	es as a ratio in its simplest fo	orm.			
		a 1 cm to 4 km	b 4 cm to 5 km	c 2 cm to 5 km			
		d 4 cm to 1 km	e 5 cm to 1 km	f 2.5 cm to 1 km			
	6	Map A has a scale of 2 cm t 10 km.	to 5 km. Map B, of the same	area, has a scale of 1 cm to			
		a Rewrite these scales as r	atios in their simplest form.				
		b How long is a path that i	measures 0.8 cm on map A?				
		c How long should a 12 kr	n road be on map B?				
MR		d A river is 1.2 cm long on	map B. How long will it be o	on map A?			
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	7 Kati	lo, speed and proportion					

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Calculating with ratios when only part of the information is known



Two business partners, Lubna and Adama, divided their total profit in the ratio 3 : 5. Lubna received £2100. How much did Adama get?

Method 1: Set up a table

Lubna	3	3 ÷ 3 = 1	$1 \times 2100 = 2100$
Adama	5	$5 \div 3 = 1\frac{2}{3}$	$1\frac{2}{3} \times 2100 = 3500$

Method 2: Using fractions

Lubna's £2100 was $\frac{3}{8}$ of the total profit. (Check that you know why.)

 $\frac{1}{8}$ of the total profit = £2100 ÷ 3 = £700

So Adama's share, which was $\frac{5}{8}$, was $\pm 700 \times 5 = \pm 3500$.

7.1 Ratio

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Example 6

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Exercise 7C

	1	Derek, aged 15, and Ricki, aged 10, shared all the conkers they found in the woods in the same ratio as their ages. Derek had 48 conkers.
		a Write down and simplify the ratio of their ages.
		b How many conkers did Ricki have?
		c How many conkers did they find altogether?
	2	A blend of tea is made by mixing Lapsang with Assam in the ratio 3 : 5. I have a lot of Assam tea but only 600 g of Lapsang. How much Assam do I need to make the blend, if I use all the Lapsang?
	3	The ratio of male to female spectators at ice hockey games is 4 : 5. At the Giants' last match, 4500 men watched the match. What was the total attendance at the game?
	4	A teacher always arranged the content of every lesson for Year 10 as 'teaching' and 'practising learnt skills' in the ratio 2 : 3.
		a If a lesson lasted 35 minutes, how much teaching would he do?
	•	b If he decided to teach for 30 minutes, how long would the lesson be?
	5	Three business partners, Kevin, John and Margaret, put money into a business in the ratio 3 : 4 : 5. They shared any profits in the same ratio. Last year, Margaret made £3400 out of the profits. How much did Kevin and John make last year?
	6	a Iqra is making a drink from lemonade, orange and ginger ale in the ratio 40 : 9 : 1. If Iqra has only 4.5 litres of orange, how much of the other two ingredients does she need to make the drink?
		b Another drink made from lemonade, orange and ginger ale uses the ratio 10:2:1. Which drink has a larger proportion of ginger ale, Iqra's or this one? Show how you work out your answer.
MR	7	On a plane the ratio of business to premium to economy class seats is $1:6:30$.
Ŭ		A family of 8 book all of the business seats. How many seats are there on the plane altogether?
PS	8	A group of boys and girls is waiting for school buses. 25 girls get on the first bus. The ratio of boys to girls at the stop is now 3 : 2. 15 boys get on the second bus.
		There are now the same number of boys as girls at the bus stop. How many students altogether were originally at the bus stop?
	9	A jar contains 100 cc of a mixture of oil and water in the ratio 1 : 4. Enough oil is added to make the ratio of oil to water 1 : 2. How much water must be added to make the ratio of oil to water 1 : 3?
EV	10	A teacher asked her class to choose a number in the 10 times table then divide it in the ratio 1 : 3 : 5.
		Zeke chose 10. Yoko chose 50. Will chose 90.
		a Who made the most sensible choice? Why?
		b Zeke correctly worked out the values and wrote them down as $1.1:3.3:5.5$.
		Yoko correctly worked out the values and wrote them down as 5.56 : 16.67 : 27.78.
	•	What mistake have they both made?
10	7 Rati	io, speed and proportion

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7.2 Speed, distance and time

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This section will show you how to:

- recognise the relationship between speed, distance and time
- calculate average speed from distance and time
- calculate distance travelled from the speed and the time taken
- calculate the time taken on a journey from the speed and the distance.

The relationship between **speed**, **time** and **distance** can be expressed in three ways:

speed = $\frac{\text{distance}}{\text{time}}$ distance speed distance = speed \times time time =

In problems relating to speed, you generally mean average speed, as it would be unusual to maintain one exact speed for the whole of a journey.

This diagram will help you remember the relationships between distance (D), time (T) and speed (S).



Paula's average speed = $\frac{\text{distance she drove}}{\text{time she took}} = \frac{270}{5} = 54$ miles per hour (mph)

Sarah drove from Sheffield to Peebles in $3\frac{1}{2}$ hours at an average speed of 60 mph. How far is it from Sheffield to Peebles?

Example Since:

Example

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distance = speed \times time

the distance from Sheffield to Peebles is given by:

 $60 \times 3.5 = 210$ miles

Note: You need to change the time to a decimal number and use 3.5 (not 3.30).

Sean is going to drive from Newcastle upon Tyne to Nottingham, a distance of 190 miles. Example He estimates that he will drive at an average speed of 50 mph. How long will it take him?

Sean's time = $\frac{\text{distance he drove}}{\text{his average speed}} = \frac{190}{50} = 3.8$ hours

Change the 0.8 hours to minutes by multiplying by 60, to give 48 minutes.

So, the time for Sean's journey will be 3 hours 48 minutes.

Remember: When you calculate a time and get a decimal answer, as in Example 9, do not mistake the decimal part for minutes. You must either:

- leave the time as a decimal number and give the unit as hours, or
- change the decimal part to minutes by multiplying it by 60 (1 hour = 60 minutes) and give the answer in hours and minutes.

Key words	
average	distance
speed	time

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Exercise 7D

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A cyclist travels a distance of 90 miles in 5 hours. What was her average speed?

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- How far along a motorway would you travel if you drove at 70 mph for 4 hours?
- 3 I drive to Bude, in Cornwall, from Sheffield in about 6 hours. The distance from Sheffield to Bude is 315 miles. What is my average speed?

The distance from Leeds to London is 210 miles. The train travels at an average speed of 90 mph. If I catch the 9:30 am train in London, at what time should I expect to arrive in Leeds?

Hints and tips Remember to convert time to a decimal. If you are using a calculator, for example, 8 hours 30 minutes is 8.5 hours.

Hints and tips km/h means kilometres per hour. m/s means metres per second.

5 Copy and complete this table.

	Distance travelled	Time taken	Average speed
a	150 miles	2 hours	
b	260 miles		40 mph
С		5 hours	35 mph
d		3 hours	80 km/h

The most common Ordnance Survey maps have a scale of 1:50 000.

- **a** How far is the actual distance represented by a distance of 1 cm on the map?
- **b** Ed plans a cycle ride. He estimates the distance on his Ordnance Survey map to be 78 cm. He plans to leave at 9 am and stop for about 30 minutes for a break. He will cycle at an average speed of 15 km/h. About what time will he be back? Show your working.

Hints and tips 1 km = 1000 m = 100 000 cm

Callum drives home from his son's house in 2 hours 15 minutes. He says that he drives at an average speed of 44 mph.

- **a** Change the 2 hours 15 minutes to a decimal.
- **b** How far is it from Callum's home to his son's house?

The distance between Paris and Le Mans is 200 km. The express train between Paris and Le Mans travels at an average speed of 160 km/h.

- **a** Calculate the time taken for the journey from Paris to Le Mans. Give your answer as a decimal number of hours.
- **b** Change your answer to part **a** to hours and minutes.

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This timetable shows a train journey from Sheffield to London by the Midland mainline. The distance travelled is 150 miles.

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Depart	Arrive	Travel by	Train company	Duration
11:29 Sheffield	13:30 London St	Train	East Midlands	02h 01
	Pancras Intl		Trains	

This timetable shows a train journey from Sheffield to London by the East Coast mainline. The distance from Sheffield to Doncaster is 20 miles and from Doncaster to London is 160 miles.

Depart	Arrive	Travel by	Train company	Duration
11:10 Sheffield	11:35 Doncaster	Train	Transpennine Express	00h 25
11:46 Doncaster	11:35 London Kings Cross	Train	East Coast	01h 42

a Work out the average speed of each journey.

b Work out the average speed of the train journey from Doncaster to London.

A train travels at 50 km/h for 2 hours. Then it slows down to travel the last 30 minutes of its journey at 40 km/h.

- a What is the total distance of this journey?
- **b** What is the average speed of the train over the whole journey?

Hints and tips Remember that there are 3600 seconds in an hour and 1000 metres in a kilometre. So to change from km/h to m/s multiply by 1000 and divide by 3600.

11 Change each speed to metres per second.

a 36 km/h **b** 12 km/h **c** 60 km/h

lints and tips To change from m/s to km/h multiply by 3600 and divide by 1000.

12 Change each speed to kilometres per hour.

1 25 m/s	b 12 m/s	c 0.5 m/s	
Hints and tips	To convert a decimal frac multiply by 60.	ction of an hour to minutes, just	

13 A train travels at an average speed of 18 m/s.

The train sets off at 07:30 on a 40 km journey.

At approximately what time will it reach its destination?

At 9:00 am cyclist A sets off on a trail at an average speed of 16 km/h.

At 10:00 am cyclist B sets off from the same place, in the same direction at an average speed of 24 km/h.

Approximately what time will cyclist B catch up with cyclist A?

Hints and tips Set up a table to show how far the cyclists have gone every 15 minutes after 10:00 am.

5 Rebecca says: 'If I travel for 10 minutes at 50 mph, then 10 minutes at 70 mph, then my average speed must be 60 mph.'

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Nick says: 'If I travel for 10 miles at 40 mph, then 10 miles at 60 mph, then my average speed for the 20 miles will be 50 mph.'

Are they both correct? Show your working.

Josh and Nell need to travel from A to B.Across town the distance is 20 miles.By motorway it is 50 miles.

by motor way it is 50 miles.

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(EV)

The speed limit in town is 30 mph and on the motorway it is 70 mph.



They work out the time it will take them to go via the town and via the motorway if they travel at the speed limit.

Josh decides to go via the town. Nell decides to go via the motorway.

Who is more likely to get there first? Show your working and explain any assumptions that you make.

7.3 Direct proportion problems

This section will show you how to:

 recognise and solve problems that involve direct proportion. direct proportion unit cost

unitary method

Key words

Suppose you buy 12 items that each cost the same. The total amount you spend is 12 times the cost of one item.

The total cost is in **direct proportion** to the number of items bought. The cost of a single item (the **unit cost**) is the constant factor that links the two quantities.

Direct proportion is not only concerned with costs. Any two related quantities can be in direct proportion to each other.

To solve any problem involving direct proportion, start by finding the single unit value. This is called the **unitary method**. You can do this by using a table or just working out the single unit value. These methods are very similar to each other, as you will see from Examples 10 and 11. The table can be useful if you have to do more complicated calculations.

Hints and tips Before solving a direct proportion problem, think about it carefully to make sure that you know how to find the required single unit value.

10	If eight pens cost £2.64, what is the cost of five pens?				
Method 1: Set up a table $\div 8$ $\times 5$		×5	Method 2: Unitary method First, find the cost of <i>one</i> pen.		
	Pens	8	1	5	This is $\pm 2.64 \div 8 = \pm 0.33$.
	Cost	2.64	0.33	1.65	So, the cost of five pens is $\pm 0.33 \times 5 = \pm 1.65$.

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7 Ratio, speed and proportion

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Example 11

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b

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a How many packed lunches can she make from 20 loaves?

b How many loaves will she need to make packed lunches for 60 people?

Emma uses eight loaves of bread to make packed lunches for 18 people.

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Method 1: Set up a table

	÷ 8		20
Loaves	8	1	20
Packed lunches	18	2.25	45

From 20 loaves she can make packed lunches for 45 people.

	÷ 18		50
Loaves	8	$\frac{8}{18} = \frac{4}{9}$	$\frac{4}{9} \times 60 = 26\frac{2}{3}$
Packed lunches	18	1	60

To make packed lunches for 60 people she will require 27 loaves. Remember that she will need to round up, to a whole number of loaves.

	÷ 3		10
Loaves	8	$8 \div 3 = 2\frac{2}{3}$	$26\frac{2}{3}$
Packed lunches	18	$18 \div 3 = 6$	60

Method 2: Unitary method

First, find how many lunches she can make from one loaf.

From one loaf she can make $18 \div 8 = 2.25$ lunches.

- **a** So, with 20 loaves she can make $2.25 \times 20 = 45$ lunches.
- **b** Work out how many loaves she needs for one packed lunch.

 $8 \div 18 = \frac{4}{9}$ or 0.444...

So for 60 packed lunches she will need $\frac{4}{9} \times 60 = 26\frac{2}{3}$ loaves, so she will need 27 loaves.

Exercise 7E

- If 30 matches weigh 45 g, what would 40 matches weigh?
- Five bars of chocolate cost £2.90. Find the cost of nine bars.
- Eight men can chop down 18 trees in a day. How many trees can 20 men chop down in a day?
- Find the cost of 48 eggs when 15 eggs can be bought for £2.10.
- 70 maths textbooks cost £875.
 - a How much will 25 maths textbooks cost?
 - **b** How many maths textbooks can you buy for £100?

nts and tips Remember to work out the value of one unit each time. Always check that answers are sensible.



7.4 Best buys

This section will show you how to:

- find the cost per unit mass
- find the mass per unit cost
- use the above to find which of two products is the cheaper.

When you wander around a supermarket and see all the different prices for the many differentsized packets, it is rarely obvious which are the **best buys**. However, with a calculator you can easily compare **value for money** by finding either the cost per unit mass *or* the mass per unit cost.

To find:

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cost per unit mass, divide cost by mass

• mass per unit cost, divide mass by cost.

The next two examples show you how to do this.

Note that in everyday speech everyone says: 'How much does this weigh?' or 'I weigh 58 kg.' In fact, weight is the force that pulls you towards the Earth's centre. If you were standing on the Moon, your weight would be about one-sixth of what it is on Earth, but your **mass** would stay the same. In this section, you will use mass to represent the amount of a quantity.

7 Ratio, speed and proportion

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Key words	5
best buy	better value
mass	value for money

Example 12 A 300 g tin of cocoa costs £1.20. Find the cost per unit mass and the mass per unit cost.

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First change £1.20 to 120p. Then divide, using a calculator.

Cost per unit mass $120 \div 300 = 0.4p$ per gram

Mass per unit cost $300 \div 120 = 2.5$ g per penny

A supermarket sells Whito soap powder in two different-sized packets. The medium size contains 800 g and costs £1.60 and the large size contains 2.5 kg and costs £4.75. Which is the better buy?

Example 13 Find the mass per unit cost for both packets.

Medium: $800 \div 160 = 5$ g per penny

 $2500 \div 475 = 5.26$ g per penny Large:

From these it is clear that the large size gives more 'mass per penny', which means that the large size is the better buy.

Sometimes it is easier to use a scaling method to compare prices and find better value.

Which of these boxes of fish fingers is better value? Example 14 12 is a common factor of 24 and 36 so work out the cost of 12 fish fingers. For the small box, 12 fish fingers cost $\pm 3.40 \div 2 = \pm 1.70$. For the large box, 12 fish fingers cost $\pm 4.95 \div 3 = \pm 1.65$. So the large box is better value.

Sometimes it is easier to use a table to compare the cost of one item in each case.



7.4 Best buys

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Exercise 7F



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Worked exemplars



Jonathan is comparing two ways to travel from his flat in London to his parents' house.

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Tube, train and taxi

It takes 35 minutes to get to the railway station by tube in London. The train journey from London to Doncaster takes 1 hour 40 minutes. From Doncaster it is 15 miles by taxi at an average speed of 20 mph.

Car

The car journey is 160 miles at an average speed of 50 mph. Which journey takes longer: tube, train and taxi or car?

This question assesses 'communicating mathematically' so it is important that you show your methods clearly.		
For the taxi: time = distance \div speed = $\frac{15}{20}$ = 0.75 hour (or 45 minutes) D S T	First, work out the time taken by the taxi. It is not essential to show the formula 'time = distance ÷ speed' but it is useful to draw the triangle that shows the relationship. Be careful with time as a decimal. 0.75 hours = 45 minutes	
Total time for the journey by tube, rail and taxi = 35 minutes + 1 hour 40 minutes + 45 minutes = 3 hours	Second, work out the total time for tube, train and taxi.	
For the car: time = distance ÷ speed = 160 ÷ 50 = 3.2 hours (or 3 hours 12 minutes)	Third, work out the time taken by the car. 0.1 hours = 6 minutes so 0.2 hours = 12 minutes	
Travelling by car takes 12 minutes longer.	Finally compare the times taken. It is essential to write a final conclusion. Do not assume 'it is obvious'. Communicating mathematically requires more than just a list of calculations. Some words to explain what you are working out are needed. Imagine that you will pass your answer to a friend or relative and ask yourself: Would they be able to make sense of it?'	

7 Ratio, speed and proportion

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2 A, B, C and D are four points on a number line.



AB:BC=7:3

BC: CD = 2:5

Work out the ratio AC : CD.

Give your answer in its simplest form.

This questions requires you to 'interpret and communicate information', so you will need to extend the information beyond what is stated explicitly.

This table shows the given ratios.	Set up a table and write in the
AB 7	information you are given.
BC 3 2	
CD 5	
BC has the same value in the first and second columns.	Now complete the third column. Multiply AB and BC in the first column by
AB 7 14	2 to give $AB = 14$ and $BC = 6$.
BC 3 2 6	Multiply BC and CD in the second column by 3 to give $BC = 6$ and $CD = 15$
CD 5 15	$\frac{1}{10000000000000000000000000000000000$
Add AB and BC to find AC.	Combine AB and BC to get AC = 20
AB 7 14 20 4	(column 4) and cancel AC and CD by a factor of 5 (column 5).
BC 3 2 6	
CD 5 15 15 3	
Hence the ratio AC : CD is 4 : 3.	

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Ready to progress?

I can simplify a ratio.

I can calculate average speeds from data. I can calculate distance from speed and time. I can calculate time from speed and distance. I can compare prices of products to find the 'best buy'.

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I can solve direct proportion problems. I can use ratio to solve problems.

Review questions

- The total cost of these three pens is £1.20. Work out the total cost of eight of these pens. Give your answer in pounds.
- 2 These are the ingredients for making apple pie for eight people.

Bill makes an apple pie for five people.

a Work out how much flour he needs.

Jenny makes an apple pie for 18 people. **b** Work out how much milk she needs.

- Maura travelled 80 miles in 1 hour 40 minutes. Work out Maura's average speed. Give your answer in miles per hour.
- A car travels for 5 hours. Its average speed is 45 km/h. Work out the total distance the car travels.
- Ron drives 270 km in 3 hours 45 minutes. Work out Ron's average speed.
- The ratio of the totals of the numbers in Box A and Box B is 2:3.

Swap a number from each box so that the ratio of the totals of the numbers is now 9:11.

Show your working.

Apple pie for 8 people 240 g flour 5 eggs 320 g apples 210 ml milk 105 g butter

Box A		Box B	
4	5	6	7
9	,		3
10	12	15	19

7 Ratio, speed and proportion

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